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The influence of adiabatic heating/cooling on magnetohydrodynamic systems

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Abstract

We have studied the influence of the dissipation number on the behaviour of a magnetohydrodynamic system in a rotating liquid Cartesian box in the presence of a finitely conducting solid box beneath the liquid. The dissipation number appears in dimensionless description of the three physical mechanisms in the heat equation: the adiabatic heating/cooling, the dissipative heating and the Joule heating. We have demonstrated that the adiabatic heating/cooling can strongly suppress the horizontal gradient of temperature, if the surface temperature is of the same order as the temperature drop over the liquid layer. This effect stabilizes the convection pattern. We hypothesize that the adiabatic heating/cooling could be the substantial stabilizing mechanism in systems with a high Rayleigh number, which seem to be suitable for the description of the magnetohydrodynamics of the Earth's core.

Keywords: Dynamo theory; Magnetoconvection; Dissipation number

1. Introduction

The magnetohydrodynamic dynamo problem represents a complicated non-linear dynamical system described by a set of partial differential equations (Roberts, 1992; Braginsky and Roberts, 1995), which expresses fundamental physical laws of conservation (conservation of mass, momentum and energy) together with Maxwell's equations for the magnetic induction. The advent of massively parallel supercomputers has enabled the modelling of the magnetohydrodynamic system's evolution under the conditions, which are in several aspects close to those in the Earth, and to obtain the Earth-like behaviour of the magnetic field (Glatzmaier and Roberts, 1995, 1997; Kuang and Bloxham, 1997). One of the problems is that due to our lack of knowledge of the core material properties, the considered span of parameters controlling the system is very wide, which requires to perform complex investigations throughout the parameter space. However, the extreme computational cost of the modelling of the full system in spherical geometry as well as problems with reaching sufficiently high resolution in turbulent regimes result in monitoring just one or few parameter configurations and thus much remains to be done to understand the geodynamo.

This is the reason why parameter studies of simplified systems can help in understanding the basic physics of nonlinear magnetohydrodynamics. For ex-

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ample, Busse et al. (1998) studied the influence of the Prandtl number, Brestenský et al. (1998) dealt with the magnetoconvection instabilities in dependence on the Roberts number, Walker and Barenghi (1997) or Cupal (1998) concentrated on the situation created by a low Ekman number and Matthews (1997) concerned with the magnetoconvection for the Rayleigh number close to its critical value, which can be appropriate for astrophysical applications. In these studies, the heat equation was considered in the Boussinesq approximation, which means that only heat conduction and advection were taken into account. However, in the extended Boussinesq approximation the adiabatic as well as Joule heating and dissipation terms occur. In the dimensionless form, they are expressed by means of the dissipation number (Christensen, 1989). The influence of the Joule heating and dissipation is probably not very important but the adiabatic heating/cooling can influence the convection substantially as it acts against the buoyancy force and represents thus a strong stabilizing factor in convection models with a relatively high dimensionless surface temperature (Mistr, 1996).

The problem of convection stability may be of a great importance in planetary dynamo studies because it can be demonstrated that geomagnetic field structures similar to those observed in geomagnetism can be reached in dynamo models with zero dissipation number and with the Rayleigh numbers close to the critical value (Christensen et al., 1998). Estimates of the outer core viscosity span almost 15 orders of magnitude with the bounds 10^{-3} Pa s and 10^{12} Pa s (Secco, 1995) but the other physical parameters, which are necessary for the evaluation of the Rayleigh and Ekman numbers, are much better constrained (Cardin and Olson, 1994; Braginsky and Roberts, 1995). If we take into account that the critical Rayleigh number for thermal convection in a rotating spherical shell with zero dissipation number and the Earth-like Prandtl number is about $8E^{-17/15}$, where E is the Ekman number (Cardin and Olson, 1994), we get that the Rayleigh number of the outer core is at least seven orders of the magnitude higher than its critical Rayleigh number. The question then arises why the magnetohydrodynamics of the Earth generates the similar field to that obtained from simulations with very low Rayleigh numbers. Therefore, the natural conjecture is that a stabilizing physical mechanism is present in the real planet.

The aim of this study is to demonstrate the stabilizing influence of the dissipation number on the vigour of a magnetohydrodynamic system. To simplify the computations and to reach a higher resolution, we modelled thermal convection heated from below in a rotating liquid Cartesian box. The convection is coupled with the magnetic field. To simulate the effect of the Earth's inner core on the evolution of the magnetic field, the computations were performed in the presence of a finitely conducting solid box just beneath the liquid box, whereas an insulator was assumed to fill the space above the liquid, and reflecting boundary conditions were used at the sidewalls. Although the employed geometry is unrealistic, it was shown in many studies that the basic physics can be demonstrated in Cartesian geometries (e.g., Soward, 1980; Skinner and Soward, 1988; Jones and Wallace, 1992). We believe that the studied system is capable of answering the question as to the potential importance of adiabatic heating/cooling in planetary dynamos.

2. Description of the model

We have studied numerically the time-evolution of self-consistent magnetohydrodynamic system in two rotating three-dimensional rectangular boxes, where a homogeneous gravitational field is acting downward parallel to the z-axis (see Fig. 1). The

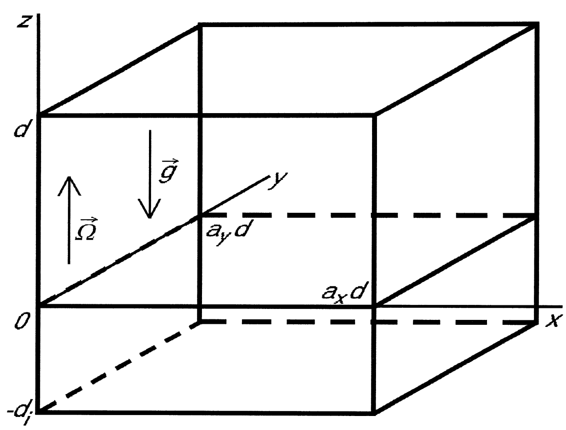


Fig. 1. Computational domain.

upper box of sizes $(a_x d \times a_y d \times d)$ is filled by a fluid, which is considered to be homogeneous, Newtonian and incompressible. The convection is driven by the temperature drop between the bottom and the top boundary of the upper box. The model includes the Coriolis force with rotation axis parallel to the z-axis, the inertial force and the Lorentz force. The laws of conservation of mass, momentum and energy and the magnetic induction equation:

$$\operatorname{div} \vec{v} = 0, \tag{1}$$

$$\rho_0 \frac{\partial \vec{v}}{\partial t} = \rho_0 \nu \nabla^2 \vec{v} - \rho_0 \vec{v} \cdot \nabla \vec{v} - \nabla \Pi + \rho_0 \alpha (T - T_0) g \vec{e}_z - 2 \rho_0 \Omega \vec{e}_z \times \vec{v} + \frac{1}{\mu_0} (\operatorname{curl} \vec{B}) \times \vec{B}, \tag{2}$$

$$\rho_0 c_p \frac{\partial T}{\partial t} = k \nabla^2 T - \rho_0 c_p \vec{v} \cdot \nabla T + \rho_0 \nu \left(\nabla \vec{v} + (\nabla \vec{v})^T \right) : \nabla \vec{v} + \frac{\lambda}{\mu_0} (\operatorname{curl} \vec{B})^2 - \rho_0 \alpha g T \vec{v} \cdot \vec{e}_z, \tag{3}$$

$$\frac{\partial \vec{B}}{\partial t} = \lambda \nabla^2 \vec{B} + \operatorname{curl}(\vec{v} \times \vec{B}), \tag{4}$$

determine the evolution of the flow velocity $\vec{v}$, the pressure deviation from the hydrostatic state Π, the temperature T, and the magnetic induction $\vec{B}$. Maxwell's equation:

$$\operatorname{div} \vec{B} = 0 \tag{5}$$

is treated as an initial condition of the simulations and, owing to Eq. (4), it is then satisfied during the time-evolution. The meaning of the symbols is summarized in Table 1.

Table 1
Review of used symbols, physical parameters appearing in the magnetohydrodynamic equations and the definition of dimensionless numbers

Symbol	Meaning	SI Unit
d	vertical size of the upper box	m
d_i	vertical size of the lower box	m
a_x, a_y	horizontal sizes of the box relative to d	1
$\vec{e}_x, \vec{e}_y, \vec{e}_z$	unit base vectors (orthogonal)	1
T_0	temperature at the top boundary	K
ΔT	vertical temperature drop	K
ρ_0	reference density at T_0	kg m^{-3}
$\vec{\Omega}$	Earth's rotation	rad s^{-1}
ν	kinematic viscosity	m^2 s^{-1}
$\vec{g}$	gravitational acceleration	m s^{-2}
κ	thermal diffusivity	m^2 s^{-1}
α	thermal expansivity	K^{-1}
β	isothermal compressibility	kg^{-1} m s^2
c_p	specific heat at a constant pressure	m^2 s^{-2} K^{-1}
λ	magnetic diffusivity	m^2 s^{-1}
E	Ekman number, $E = \nu / \Omega d^2$	1
Pr	Prandtl number, $Pr = \nu / \kappa$	1
Pm	Prandtl magnetic number, $Pm = \nu / \lambda$	1
Ra	Rayleigh number, $Ra = \alpha g d^3 \Delta T / \kappa \nu$	1
Dn	dissipation number, $Dn = \alpha g d / c_p$	1

The extended Boussinesq approximation is employed, i.e., the specific entropy of the fluid depends not only on the temperature but also on the hydrostatic pressure, and the Grüneisen's ratio $G = \alpha / (\rho_0 \beta c_p)$ is set to infinity. Since the isothermal compressibility β does not influence the form of the heat equation (Eq. (3)), and since β and the dissipation number Dn (see Table 1) are thermodynamically independent parameters (Tritton, 1988), the most important consequence of the infinite Grüneisen's ratio is the incompressible form (Eq. (1)) of the continuity equation. While not as complicated as fully compressible models the extended Boussinesq approximation implies that the adiabatic heating term $(-\rho_0 \alpha g T \vec{v} \cdot \vec{e}_z)$ arises in the energy equation (Christensen, 1989). Moreover, the dissipative heating $\rho_0 \nu (\nabla \vec{v} + (\nabla \vec{v})^T) : \nabla \vec{v}$ and the Joule heating $(\lambda / \mu_0)(\operatorname{curl} \vec{B})^2$ terms are also included. Scaling the lengths, time, velocity, temperature deviation $T - T_0$, pressure and magnetic induction by the factors d, d^2/λ, λ/d, ΔT, $\rho_0 \nu \lambda / d^2$ and $\sqrt{2 \rho_0 \Omega \mu_0 \lambda}$, respectively, yields the dimensionless form of Eqs. (2)–(4):

$$\frac{\partial \vec{v}}{\partial t} = Pm \nabla^2 \vec{v} - \vec{v} \cdot \nabla \vec{v} - Pm \nabla \Pi + \frac{(Pm)^2 Ra}{Pr} T \vec{e}_z - \frac{2 Pm}{E} \vec{e}_z \times \vec{v} + \frac{2 Pm}{E} (\operatorname{curl} \vec{B}) \times \vec{B}, \tag{6}$$

$$\frac{\partial T}{\partial t} = \frac{Pm}{Pr}\nabla^2 T - \vec{v}\cdot\nabla T + \frac{DnPr}{RaPm}\left(\nabla\vec{v} + (\nabla\vec{v})^T\right):\nabla\vec{v} + \frac{2\,DnPr}{ERaPm}(\mathrm{curl}\,\vec{B})^2 - Dn\left(T + \frac{T_0}{\Delta T}\right)\vec{v}\cdot\vec{e}_z, \tag{7}$$

$$\frac{\partial\vec{B}}{\partial t} = \nabla^2\vec{B} + \mathrm{curl}(\vec{v}\times\vec{B}). \tag{8}$$

The lower box is considered to be made of the same, but motionless, material. In other words, we will only deal with the (dimensionless) equation:

$$\frac{\partial\vec{B}}{\partial t} = \nabla^2\vec{B} \tag{9}$$

in this box.

The behaviour of the system is controlled by the six dimensionless parameters: Ekman, Prandtl, Prandtl magnetic, Rayleigh and dissipation numbers and by the ratio of the top surface temperature to the vertical temperature drop $T_0/\Delta T$ (see Table 1). The upper (lower) box is the Cartesian representation of the outer (inner) core in our model. We also employ boundary conditions which are simpler than those in the Earth. We take into account impermeable, free-slip boundary conditions at all boundaries of the upper box, i.e.:

$$\left(v_n = \frac{\partial v_t}{\partial n} = \frac{\partial v_s}{\partial n}\right)_{\mathrm{sidewalls,\,top,\,bottom}} = 0. \tag{10}$$

As it was already discussed by Kuang and Bloxham (1997), who have been using the same boundary conditions, ''this may be a better approximation to the dynamical regime of the Earth's core as it effectively reduces the boundary layer thickness to zero because it is unlikely that the thin viscous boundary layers in the Earth's core are important in generating a planetary scale magnetic field''. The subscript n corresponds to the normal component of a vector, the subscripts t and s denote the tangential components of a vector and $\partial/\partial n$ is the derivative along the outer normal to the boundary.

We prescribe the temperature values at the top and at the bottom of the upper box, while no heat leaves the box through the sidewalls:

$$(T)_{\mathrm{top}} = 0, \tag{11}$$

$$(T)_{\mathrm{bottom}} = 1, \tag{12}$$

$$\left(\frac{\partial T}{\partial n}\right)_{\mathrm{sidewalls}} = 0. \tag{13}$$

The diffusion equation is solved in the lower layer, allowing only the z-component to be non-zero at its bottom plane:

$$\left(B_t = B_s = \frac{\partial B_n}{\partial n}\right)_C = 0. \tag{14}$$

We use the reflecting conditions at both the upper and lower box sidewalls, i.e.:

$$\left(B_n = \frac{\partial B_t}{\partial n} = \frac{\partial B_s}{\partial n}\right)_{\mathrm{sidewalls}} = 0. \tag{15}$$

Although the electrical conductivity of the lower mantle of the Earth can be responsible for some observable effects (Busse, 1992), we considered the space above the upper layer as an insulator to simplify the model. This implies that the magnetic field in the mantle is governed by the equation:

$$\nabla^2\vec{B} = 0. \tag{16}$$

If we expand $\vec{B}$ into the Fourier series:

$$\vec{B}(x,y,z) = \sum_{p,q=0}^{\infty}\hat{\vec{B}}_{pq}(z)\,\mathrm{e}^{[2\pi i((px/a_x d)+(qy/a_y d))]}, \tag{17}$$

Eq. (16) yields:

$$\frac{d^2\hat{\vec{B}}_{pq}(z)}{d^2 z} - 4\pi^2\left[\left(\frac{p}{a_x d}\right)^2 + \left(\frac{q}{a_y d}\right)^2\right]\hat{\vec{B}}_{pq}(z) = 0, \quad \forall p,q. \tag{18}$$

The solution of Eq. (18), that does not diverge for $z \to \infty$, has the following form of decaying exponential function:

$$\hat{\vec{B}}_{pq}(z) = \hat{\vec{B}}_{pq}(d) \times \exp\left[-2\pi\sqrt{\left(\frac{p}{a_x d}\right)^2 + \left(\frac{q}{a_y d}\right)^2}\,(z-d)\right]. \tag{19}$$

All magnetic boundary conditions are thus homogeneous in our model, which implies that the magnetic field can be generated only by motions of the liquid in the upper layer.

3. Numerical methods

In order to satisfy the momentum equation (Eq. (6)) together with the auxiliary divergence-free condition (Eq. (1)), we have employed the vorticity:

$$\vec{\omega} = \mathrm{curl}\,\vec{v}. \tag{20}$$

Applying the curl operation to Eq. (6) we get:

$$\frac{\partial \vec{\omega}}{\partial t} = Pm\nabla^2\vec{\omega} + \vec{\omega}\cdot\nabla\vec{v} - \vec{v}\cdot\nabla\vec{\omega} + \frac{(Pm)^2 Ra}{Pr}\mathrm{curl}\left(T\vec{e}_z\right) + \frac{2\,Pm}{E}\vec{e}_z\cdot\nabla\vec{v} + \frac{2\,Pm}{E}\left[\vec{B}\cdot\nabla\left(\mathrm{curl}\,\vec{B}\right) - \left(\mathrm{curl}\,\vec{B}\right)\cdot\nabla\vec{B}\right]. \tag{21}$$

Eq. (21) replaces Eq. (6) during time integration. In each time step, the velocity vector is computed from the vorticity by applying the Fourier transform to Eqs. (20) and (1), solving the set of three linear algebraic equations in the spectral domain and returning back by the inverse FT.

The systems of Eqs. (21), (7) and (8) is solved on a $(64\times64\times64)$ regular grid, i.e., the vertical resolution is twice the horizontal one and is the same throughout the entire box. The inner core box, where only the magnetic diffusion equation (Eq. (9)) is solved, has the same horizontal size and the same resolution as the outer core box, its vertical size is 42 layers. The spatial derivatives were performed by the eighth order finite difference scheme (Fornberg, 1996). The boundary conditions are satisfied by symmetric or antisymmetric expansion of the quantities at four grid points outside the computational domain. The magnetic induction at the top of the upper box is expanded in the insulator by 2D Fourier transform according to Eq. (19).

The time integration is performed by the second order Runge–Kutta method. The time step is variable and is computed in order to satisfy the Courant–Friedrichs–Levi criterion. The FFT algorithm (Press et al., 1992) is used to compute 2D and 3D Fourier transforms.

4. Results

We have studied the behaviour of the magnetohydrodynamic system described in the previous sections for $a_x = a_y = 2$ and the three different sets of parameters which are summarized in Table 2.

In Case I, the law of energy conservation (Eq. (7)) was simplified by neglecting the adiabatic, the viscous and the Joule heating. The system evolved close to a steady state due to the low Rayleigh number. The typical flow and temperature patterns are in Figs. 2 and 3. They are highly symmetric with one large upwelling along the diagonal of the box. This large upwelling creates lateral temperature gradients in the middle depth comparable to the vertical temperature gradient in the lower and upper boundary layers. The magnetic induction was initially set to be weak (about four orders lower than the dimensionless velocity) homogeneous vertical field. Although the intensity of the magnetic field increased during the simulation, the influence of the Lorentz force on the convection pattern was not significant. The orientation of the magnetic field lines remained dominantly vertical, see a typical snapshot in Fig. 4, because the magnetic field in the lower solid box was kept close to a homogeneous state. However, the distortion of the field lines by horizontal motions can be clearly seen.

Table 2
The choice of dimensionless parameters controlling the studied system

Parameter	Case I	Case II	Case III
E	2×10^{-2}	2×10^{-2}	2×10^{-2}
Pr	1	1	1
Pm	1	1	1
Ra	10^4	10^4	10^5
Dn	0	0.2	0.2
$T_0/\Delta T$	–	4	4

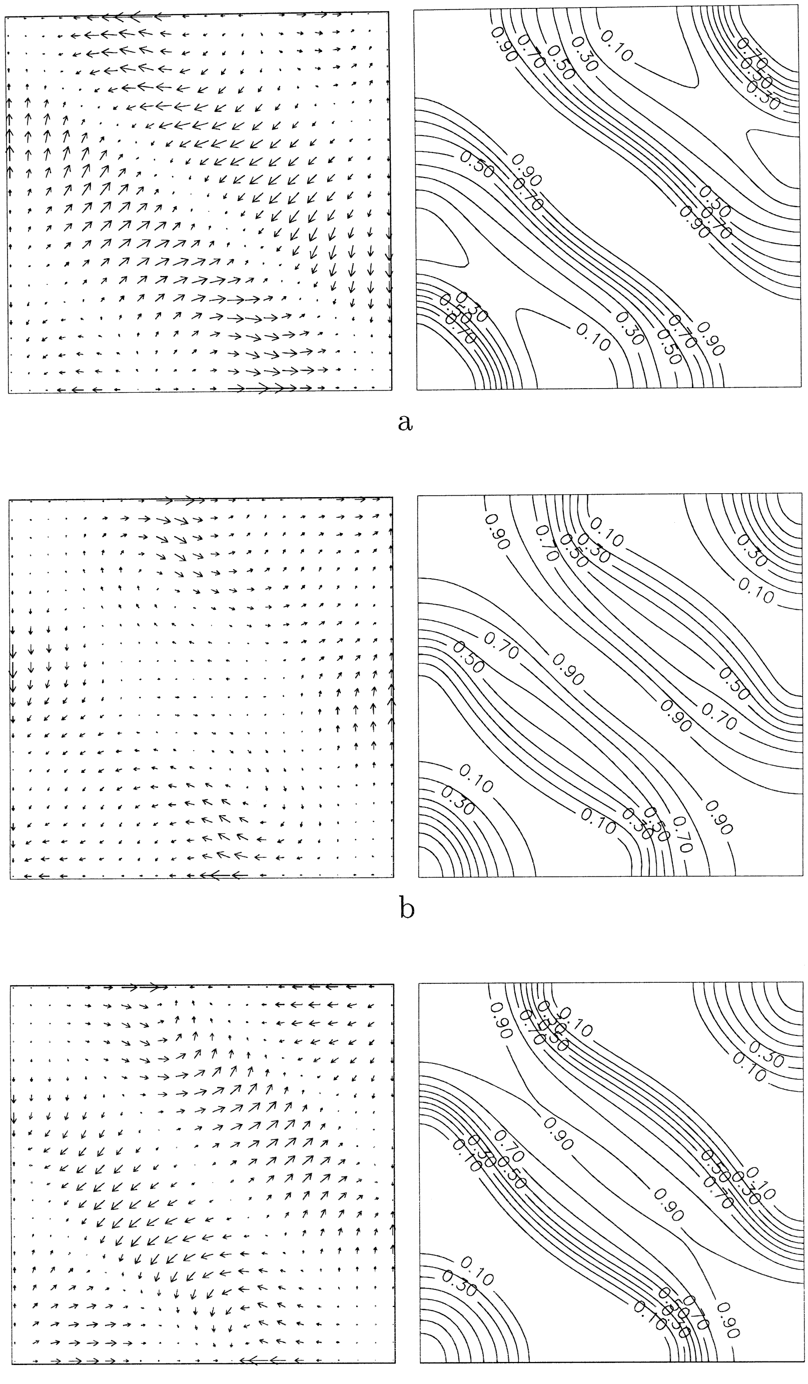


Fig. 2. Case I: Horizontal velocity components and temperature in the three horizontal cross-sections: (a) $z = 0.15$, (b) $z = 0.5$, (c) $z = 0.85$.

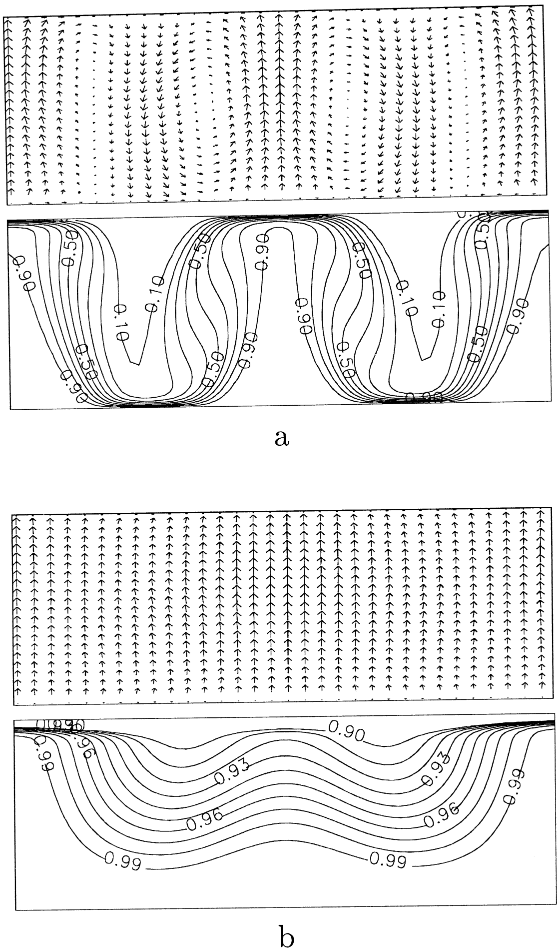


Fig. 3. Case I: Vertical velocity components and temperature in the two vertical diagonal cross-sections: (a) perpendicular to the major upwelling (from $x=0$, $y=0$ to $x=2$, $y=2$), (b) along the major upwelling (from $x=0$, $y=2$ to $x=2$, $y=0$).

In the Case II the effect of the adiabatic heating/cooling together with the viscous dissipation and the Joule heating was included by putting $Dn=0.2$. This corresponds to the estimated values of α and c_p, presented by Braginsky and Roberts (1995), which yield $Dn=0.12$ near the inner core boundary and $Dn=0.5$ near the core–mantle boundary. The convection pattern dramatically changed, which is visible in Figs. 5 and 6. It is again symmetric but the two major upwellings are present at the two opposite sides of the box and they are followed by the two downwellings at the remaining sides. Moreover, the magnitude of velocity was about five times lower than in the Case I. The most striking is the change of the temperature field. It became vertically stratified and close to the conductive solution with only small horizontal gradients. The upwellings are made of only slightly warmer material than their surroundings. The dominantly vertical dependence of temperature is caused by the balance between the adiabatic heating/cooling term $Dn(T+T_0/\Delta T)\vec{v}\cdot\vec{e}_z$ and the heat advection term $\vec{v}\cdot\nabla T$ in the energy equation (Eq. (7)). The ratio of the surface temperature T_0 to the temperature drop over the convecting layer ΔT plays the substantial role in the adiabatic heating/cooling in our simulations. We have chosen this ratio to be equal to 4, which corresponds to the core–mantle boundary temperature 4000 K and the inner core boundary temperature 5000 K. This choice is consistent with the iron melting experiments (Boehler, 1996). This relatively high value of the ratio is the reason why the role of adiabatic heating/cooling is much more important in the core than in the mantle convection. The viscous dissipation contributed to decreasing the vigour of the flow. The run was started from the state described in Case I; the magnitude of the dissipation term in the heat equation was almost comparable to the magnitude of the adiabatic heating term but it became less important during the evolution as demonstrated in Fig. 7. The magnetic field evolved similarly as in the previous case with very weak feedback to the convection through the Lorentz force and the Joule heating. The field lines

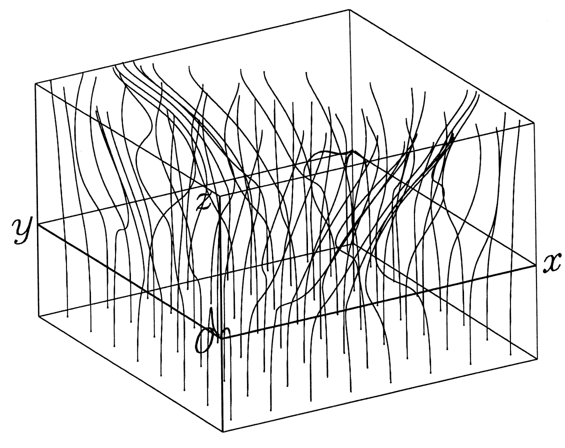


Fig. 4. Case I: The magnetic field lines in the lower and upper boxes.

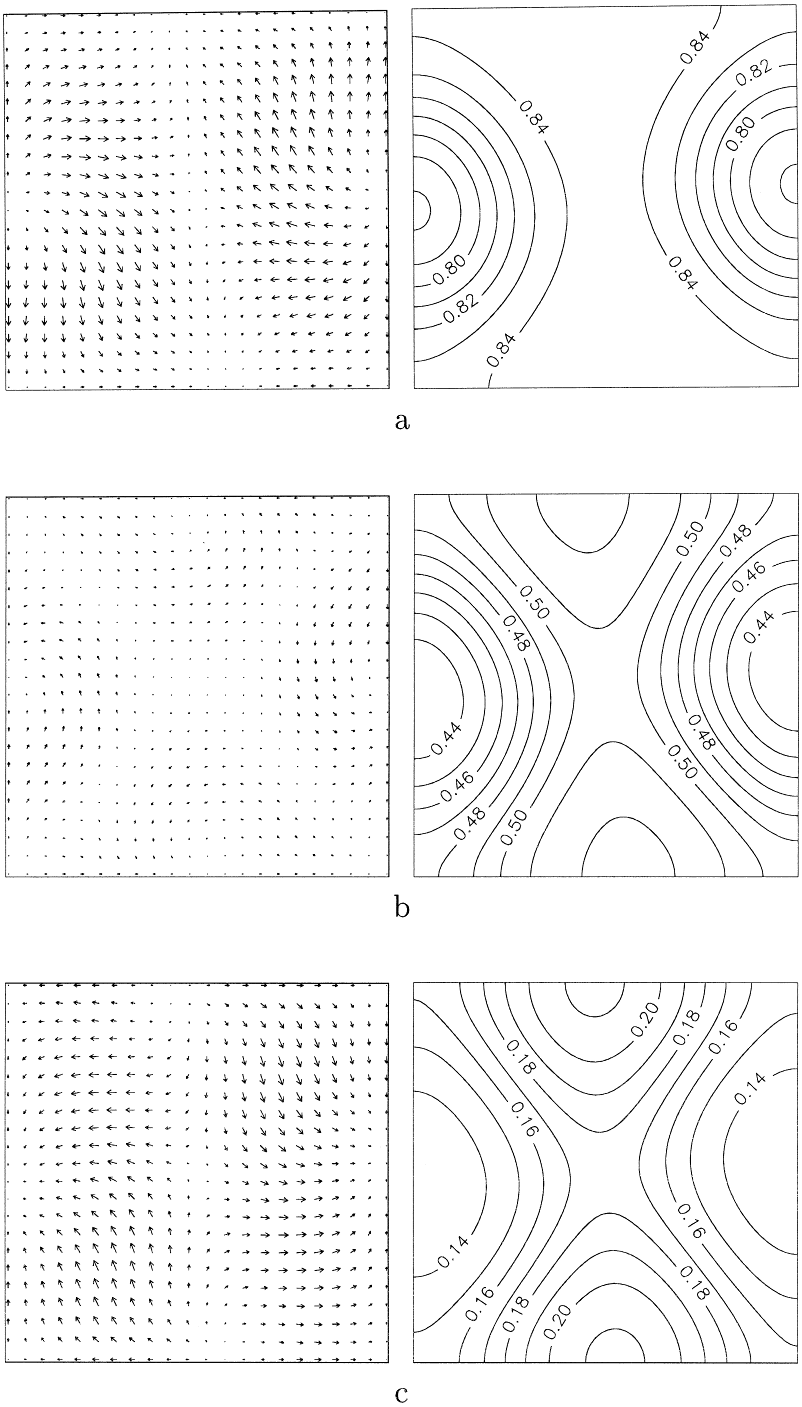
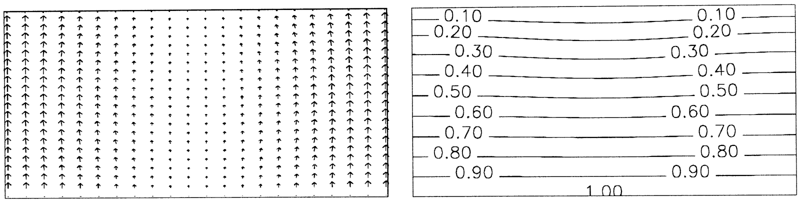


Fig. 5. Case II: Horizontal velocity components and temperature in the three horizontal cross-sections: (a) $z = 0.15$, (b) $z = 0.5$, (c) $z = 0.85$.



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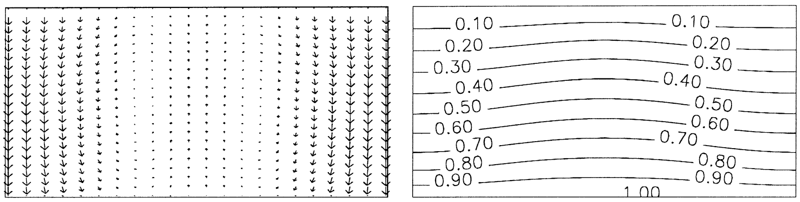

b

Fig. 6. Case II: Vertical velocity components and temperature in the two vertical cross-sections: (a) perpendicular to the x-axis at $x = 1.0$, (b) perpendicular to the y-axis at $y = 1.0$.

are in Fig. 8. Hollerbach and Jones (1995) emphasized the stabilizing role of the finitely conducting solid inner core in the evolution of the geomagnetic field. In our case it is clearly visible, that a completely different convection pattern generates the magnetic field, which is similar to that in the previous case. This suggests that the role of the conducting solid layer may be substantial not only for the stability of the field but also for its morphological characteristics.

To study the influence of the Rayleigh number, we increased this number by one order of magnitude in the Case III. The temperature remained vertically stratified and the magnitude of its horizontal variations increased only slightly. The consequence was that the buoyancy force just increased by one order and the kinetic energy increased by two orders of magnitude. This demonstrates the key stabilization effect of adiabatic heating/cooling on the temperature field, which remains close to the conductive solution for various Rayleigh numbers.

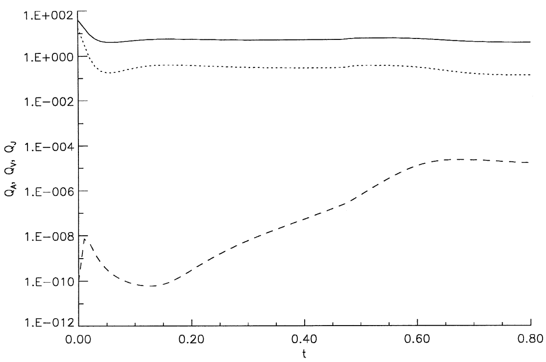

Fig. 7. Case II: Time evolution of the absolute value of the adiabatic (solid line), viscous (dotted line) and Joule (dashed line) heating averaged over the volume of the upper box.

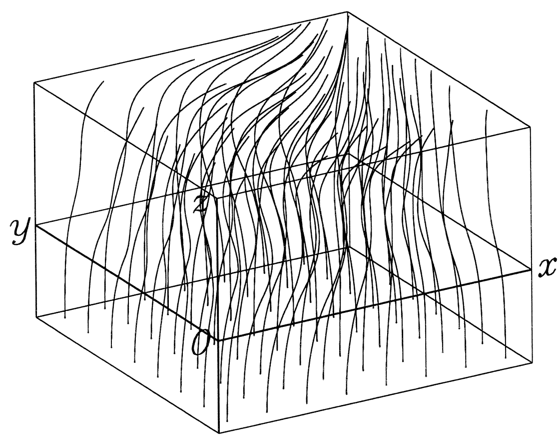

Fig. 8. Case II: The magnetic field lines in the lower and upper boxes.

5. Conclusions

The adiabatic heating/cooling can play the substantial role in the thermal convection under the core conditions. The problem is that the planetary dynamos are probably driven partly by thermal buoyancy and partly by compositional buoyancy, and thus the applications of our results to planetary models may be questionable. For example, Lister and Buffet (1995) conjectured that the thermal convection was dominant in the early Earth but now contributes only about 20% of the dynamo power. Nevertheless, the potential vertical stratification of temperature close to the conductive profile could be one of the reasons, why the seismic tomography is not able to detect any lateral variations of P wave velocities in the Earth's core (Morelli and Dziewonski, 1987). We have shown that the suppression of lateral temperature variations can be very efficient, which may lead to enhancement of the stability of temperature pattern. We hypothesize that this could be the explanation of the fact, demonstrated by Christensen et al. (1998), that the Earth-like features of the magnetic field can be reached by models with low Rayleigh numbers.

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