Bullen’s parameter $\eta$: 
a link between seismology and geodynamical modelling

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Abstract

Valuable information concerning the style of mantle density stratification can be gleaned from the spatial distribution of Bullen’s parameter. By means of numerical modelling of 2-D cartesian convection and by monitoring this with the 2-D field of the local Bullen’s parameter values, which were obtained by post-processing the convection results, we show that the local adiabaticity is seriously influenced by the presence of an endothermic phase transition at 670 km depth. In this situation the upper mantle suffers much more from non-adiabatic effects than the lower mantle. We have also employed the 3-D distributions of density and seismic velocities from the model of Ishii and Tromp based on free oscillation observations and constructed 3-D local Bullen’s parameter for a spherical Earth. In the lower mantle we find that there is a striking similarity in the range of magnitudes of the local Bullen’s parameter calculated from convection and those inferred from the splitting of seismic free oscillations. The morphologies of subadiabatic regions in the deep mantle under Africa and the central Pacific would suggest a thermal–chemical nature of the superplumes. Their vertical extent is limited to around 400 km above the core–mantle boundary (CMB). Underneath the Icelandic area in the North Atlantic such an subadiabatic region in the deep mantle above the CMB cannot be discerned. We conclude that the mantle is not as adiabatic as has commonly been held. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The concept of adiabaticity in mantle convection is well-established and entrenched in the geosciences and is important for several reasons:

(1) an adiabatic interior is often associated with a well-stirred vigorously convecting state [1],
(2) petrological problems involving the assumed thermodynamic paths of rising diapirs [2,3],
(3) the concept of potential temperature in estimating mantle temperatures of komatitites [4,5].

On the basis of arguments drawn from simple convective configuration [1], most geochemists and geophysicists have taken this adiabatic concept dogmatically. However, previous investiga-
tions [6,7] have shown that with increasing complexity in mantle convection, such as viscosity stratification in the lower mantle and phase transition, the mantle is not locally adiabatic everywhere. Lagrangian tracers were used in [7] for delineating the state of adiabaticity of the tracers inside the descending flow during flushing events.

Bullen’s parameter $\eta$ is a dimensionless Eulerian quantity introduced many years ago [8] for quantifying the departure of a globally averaged one-dimensional profile based on local seismic and density information from an adiabatic profile. Matyska and Yuen [9,10] and Bunge et al. [11] have initiated work on the non-adiabatic nature of mantle convection by calculating Bullen’s parameter from numerical simulations. The models considered up to now have been relatively simple and do not have all of the realistic physics, which can cause partial layering in mantle convection. In this work we will go one step further and will include the effect of phase transition. We will also link the convective modelling efforts on determining Bullen’s parameter $\eta$ with recent work yielding the 3-D distribution of density and seismic velocities in the mantle [12]. We plan to bring to the fore three main points in this paper:

1. construction of one-dimensional profiles of Bullen’s parameter from 2-D numerical simulations of mantle convection,
2. construction of 2-D maps of the local Bullen’s parameter from 2-D mantle convection models and compare situations for with and without the presence of an endothermic phase change at 670 km depth,
3. construction of 3-D maps of the local Bullen’s parameter from density and seismic velocity data [12] inverted from the splitting of free oscillations due to lateral anomalies of these fields.

We first give a general discussion of Bullen’s parameter cast in multidimensions. This is followed by a description of the numerical model of thermal convection. The next section is concerned with the fields delineating Bullen’s parameter from convection modelling, which is followed by the 3-D construction of Bullen’s parameter in the mantle from normal-mode results. Finally we will finish with a discussion of the implications of this work for making prognosis of features in time-dependent mantle convection and on the nature of deep mantle plumes.

2. Bullen’s parameter

We begin with a discussion of Bullen’s parameter in terms of the well-known Adams–Williamson [13] equation cast in one dimension:

$$\frac{d\rho}{dr} = -\frac{\rho g}{\Phi}$$  \hspace{1cm} (1)

where $\rho$ is the density, $r$ is the radial distance, $\Phi = \frac{v_P^2 - 4/3v_S^2}{\rho} = K/\rho$ is the seismic parameter ($v_P$ is the velocity of P-waves, $v_S$ is the velocity of S-waves and $K$ is the bulk modulus) and $g$ is the gravity acceleration. This equation yields the density gradient in a homogeneous adiabatic self-compressed region (e.g. [14]). The dimensionless Bullen’s parameter $\eta$ was introduced as a useful test of the chemical homogeneity and adiabaticity in the mantle by comparing a real density gradient with the adiabatic gradient [8]:

$$\eta = -\frac{\Phi}{\rho g} \frac{d\rho}{dr}$$  \hspace{1cm} (2)

It is a common practice in geophysics to apply this definition to 1-D depth-dependent density profiles for deriving information concerning the adiabaticity of the mantle. For example, the PREM model [15] is nearly adiabatic in the lower mantle ($\eta$ varies between 0.97 and 1.01) but exhibits substantial deviations from adiabaticity in the upper mantle. However, if we have a seismic model in a form $\rho = \rho_0(r) + \delta\rho(r, \psi, \phi)$, $\Phi = \Phi_0(r) + \delta\Phi(r, \psi, \phi)$, where $\rho_0(r)$ and $\Phi_0(r)$ are reference spherically symmetric models, $\delta\rho$ denotes 3-D density anomalies and $\delta\Phi$ represents 3-D anomalies of the seismic parameter depending not only on the radial variable $r$, but also on the colatitude $\psi$ and the longitude $\phi$, we can employ this model to obtain the 3-D local Bullen’s parameter with the equation:
Here \( \eta (r, \vartheta, \varphi) = -\frac{\Phi_0 (r) + \delta \Phi (r, \vartheta, \varphi)}{[\rho_0 (r) + \delta \rho (r, \vartheta, \varphi)]g(r) \partial} \partial r \)

\[ \eta (r, \vartheta, \varphi) = \eta_0 (r) - \frac{\Phi_0 (r)}{\rho_0 (r)g(r)} \left[ \frac{\delta \rho (r, \vartheta, \varphi)}{\rho_0 (r)g(r)} \partial \right] \]

We will consider only the depth dependence of \( g \) in the computations of local Bullen’s parameter. In usual situations, \( |\delta \rho| \ll \rho_0 \) and \( |\delta \Phi| \ll \Phi_0 \), therefore the linearized expression of Bullen’s parameter is:

\[ \eta (r, \vartheta, \varphi) = \left[ \frac{\Phi_0 (r)}{\rho_0 (r)} \right] \left[ \eta_0 (r) + \delta \Phi (r, \vartheta, \varphi) \right] \]

where \( \eta_0 (r) \) is the profile of the Bullen’s parameter associated with the reference model.

Let us assume now that the reference models \( \rho_0 (r) \) and \( \Phi_0 (r) \) correspond to an adiabatic state. If we consider only the non-adiabatic changes of density due to the thermal expansion and also take into account that adiabatic temperature increase with depth is \( \alpha \delta T g / c_p \), where \( \alpha \) is the thermal expansivity, \( T \) is the absolute temperature and \( c_p \) is the isobaric specific heat (e.g. [14]), we can arrive first at Birch’s modification of the 1-D Williamson–Adams equation [16]:

\[ \frac{d \rho}{dr} = -\rho \frac{\Phi_0}{\rho_0} - \rho (\frac{dT}{dr} + \frac{\alpha T g}{c_p}) \]

Here \( \rho \) and \( T \) denote the 1-D horizontally averaged density and geotherm, respectively. In this case the 1-D Bullen’s parameter can easily be obtained from the temperature profile by substituting Eq. 5 into Eq. 3:

\[ \eta (r) = 1 + \frac{\alpha \Phi_0}{\rho_0} \left( \frac{dT}{dr} + \frac{\alpha T g}{c_p} \right) \]

where we neglected \( \delta \rho = \rho - \rho_0 \) and \( \delta \Phi = \Phi - \Phi_0 \) in the substitution into the fraction on the right-hand side of Eq. 3.

However, the temperature distribution in the Earth’s mantle is a 3-D function, which strongly varies laterally, but the corresponding seismic parameter and density variations are small. This is the reason why it is reasonable to deal with the 3-D local Bullen’s parameter expression which is obtained from Eq. 4:

\[ \eta (r, \vartheta, \varphi) = \left[ 1 + \frac{\delta \Phi (r, \vartheta, \varphi)}{\Phi_0 (r)} + \alpha (r) (T (r, \vartheta, \varphi) - T_0 (r)) \right] \times \left[ 1 + \frac{\alpha (r) \Phi_0 (r)}{g(r)} \left( \frac{\partial T (r, \vartheta, \varphi)}{\partial r} + \frac{\alpha (r) T (r, \vartheta, \varphi) g(r)}{c_p} \right) \right] \]

where \( T_0 \) is an adiabatic reference geotherm. As will be demonstrated numerically in the next sections, the deviations of Bullen’s parameter from the adiabatic value of unity can reach tens of percents. However, magnitudes of \( \delta \rho \rho_0 \) and \( \delta \Phi \Phi_0 \) can reach only several percent. These terms may thus be neglected in thermal convection models, which are employed for evaluating the approximate order of possible deviations from adiabaticity of the thermal fields in mantle convection. Moreover, by neglecting these two terms, one also avoids the necessity for constructing a reference adiabatic geotherm \( T_0 (r) \) and the subsequent handling of the thermal dependence of the seismic parameter \( \Phi \).

In numerical simulations of mantle convection, dimensionless quantities are usually employed. We will here rewrite the last two equations for the case of 2-D cartesian geometry, where dimensionless depth \( z \) and horizontal coordinate \( x \) are scaled by the thickness of the mantle \( d = 2890 \) km. The dimensionless temperature \( T^* \) is introduced simply by the relation \( T^* = (T^* + T^*) \Delta T \), where \( \Delta T \) is the temperature drop over the mantle and \( T^* \) is the dimensionless surface temperature given by the ratio of the actual surface temperature to \( \Delta T \). We also scale the depth-dependent thermal expansivity, \( \alpha^* = \alpha \alpha_0 \), where \( \alpha_0 \) is the reference surface value. The 1-D and 2-D Bullen’s parameters can then be expressed by means of these dimensionless quantities in the form:
and after neglecting the second and the third term on the right-hand side of Eq. 7:

\[
\eta(x, z) = 1 - \frac{\alpha(z) \Phi_0(z) \Delta T}{gd} \\
\left(\frac{dT^*(z)}{dz} - D \alpha^*(z) (T^*_s + T^*(x, z))\right)
\]

and, after neglecting the second and the third term on the right-hand side of Eq. 7:

\[
\eta^*(z, T^*) = \exp(4z) \times \min \left\{ 10.0, \max \left\{ 0.1, \exp\left(10.0 \left(\frac{0.6}{0.2 + T^*} - 1\right)\right) \right\} \right\}
\]

where \( \eta^* \) is the dimensionless viscosity and \( T^* \) is the dimensionless temperature. This formula yields respectively an increase of a factor of 55 due to the depth dependence and a factor of 100 due to the temperature dependence. One single endothermic change depicting the spinel to perovskite transition, with the buoyancy parameter in the momentum equation \( P = (\Delta \rho / \alpha_s \rho^2 g d) (d \rho / d T) \) (see also [21]) equal to \( -0.1 \) was included at the depth \( z = 0.23 \) corresponding to the 670 km depth in the mantle (\( \Delta \rho \) is the density jump due to the phase change under consideration and \( d \rho / d T \) is its Clapeyron slope). The surface dissipation number was chosen to be \( D = 0.5 \), the dimensionless internal heating was equal to 12, which corresponds to heating with chondritic abundance of radioactive elements (around \( 6 \times 10^{-12} \) W/kg) [22], and the surface temperature \( T^*_s \) was 0.08. A surface Rayleigh number of \( 10^7 \) has been used. For a complete description of the governing equations, the reader is referred to [10] with the modification of the stream function equation according to [19,23] and the employment of the effective thermal expansivity according to [21].

For an aspect-ratio six box, we have used 769 \times 129 points for the temperature equation. A second-order Runge–Kutta time-stepping scheme was used to advance the temperature equation, which has both adiabatic and viscous heating. We used four times more dense points along each direction for the temperature equation than for the fourth-order partial differential equation for the stream function. Equations for the stream function were solved by the conjugate gradient method so that the iteration errors were substantially lower than the changes of the stream function during time-stepping. At each time step some iterations are required to satisfy the convergence criterion. Two models were considered – one with the endothermic phase change and one without the phase transition. We are interested to show

\[
\alpha^*(z) = \frac{8}{(2 + z)^3}
\]
the difference in the state of adiabaticity between these two models.

4. Results from convection modelling

Fig. 1 shows the 2-D temperature fields for the model with the endothermic phase transition at five time steps $t = 0.0343, 0.0360, 0.0370, 0.0380$ and $0.0390$, spanning a period of around 1 Ga. We can see that during this time there is a richness and variety in the dynamics of time-dependent mantle convection with a phase transition, which is a well-studied phenomenon [24]. In Fig. 2 we show the corresponding 2-D fields portraying the Bullen's parameter for the same instants.

![Temperature Fields](image)

**Fig. 1.** Five snapshots of temperature in five subsequent dimensionless times $t = 0.0343, 0.0360, 0.0370, 0.0380$ and $0.0390$, under the presence of the phase change boundary.
of time and the associated one-dimensional $\eta(z)$, obtained from horizontally averaged temperature in the first and the fourth panels of Fig. 1, is displayed in Fig. 3. We have chosen these particular time instants in order to demonstrate the situation before, during and after penetration of the phase boundary by a flushing event. As expected, there are two superadiabatic boundary layers at the top and the bottom and one superadiabatic boundary layer associated with the partial layer-

Fig. 2. The snapshots of the local Bullen parameter $\eta$ corresponding to the snapshots of the temperature field in Fig. 1. The contour interval is 0.1. The yellow color portrays the ‘adiabatic’ regions, where the local Bullen’s parameter $\eta$ is inside the interval $1 \pm 0.05$. The regions with superadiabatic local vertical gradient of temperature are blue and those with subadiabatic local vertical gradient of temperature are orange-red.
ing of convection due to the phase change. However, the most remarkable features are the distinct subadiabatic layers above both the phase change and at the thermal boundary layer near the core-mantle boundary (CMB). They are caused by the downwellings of cold material sinking into the surrounding mantle, which is relatively hot due to the internal heating. The bottom subadiabatic layer is fed by flushing events and thus it weakens during the time intervals of layered convection. On the other hand, the subadiabatic layer above the phase change is strongest just before flushing events, when much cold material is amassed there. The temperature dependence of viscosity strengthens the cold downwellings, which supports the accumulation of cold material, thus resulting in the creation of subadiabatic regions.

We have demonstrated here that the presence of an endothermic phase transition at 670 km depth does make a great impact on the local adiabaticity of the upper mantle, much more so than in the lower mantle. As one can clearly recognize, the upper layer is nearly non-adiabatic everywhere just prior to the onset of the flushing events. Cold downwellings in the upper layer remain superadiabatic and, therefore, the subadiabaticity is partly suppressed in the 1-D depth-dependent geotherms. The geotherms also clearly show how strongly the upper mantle can be heated up and the lowermost part of the lower mantle can be cooled down after the penetration of the phase change boundary. Both situations are clearly very non-adiabatic by all accounts.

In order to demonstrate quantitatively the influence of phase change boundary, we show in Figs. 4 and 5 convection without any phase change and with the same depth and temperature dependence of viscosity and depth dependence of thermal expansivity. The surface Rayleigh number was decreased to $2 \times 10^6$ to obtain approximately the same characteristic dimensions of plumes and downwellings. We doubled the resolution of the stream function, which was required because of the thinning of well-developed downwellings at their upper part. Let us emphasize here that these two snapshots are the two different situations: (i) the rapid descent of a cold blob into the hot adjacent material and (ii) a cold downwelling falling to a previously cooled part above the CMB. The highest temperature gradients inside the convective layer are again associated with the cold downwellings. In case (ii) they can again create a typical subadiabatic boundary layer above the CMB. As expected, the rest of the mantle remains nearly adiabatic. An interesting effect is caused by the roots of the plumes. The roots of the strong plumes are adiabatic (see also [10]), those of the weak plumes are only slightly superadiabatic and thus the bottom boundary layer (CMB) is laterally heterogeneous in the sense of adiabaticity. These adiabatic or slightly superadiabatic regions divide up the surrounding strongly superadiabatic boundary layer into several sections.
5. 3-D distribution of Bullen’s parameter obtained from the density and seismic parameter fields derived from free oscillations

We now employ the 3-D distribution of density and seismic parameter extracted from the splitting of normal-mode spectra [12] for deriving the distribution of the corresponding 3-D Bullen’s parameter in the lower mantle. As the PREM model [15] has been used as the reference model, we can directly use Eq. 3 to derive the distribution of Bullen’s parameter corresponding to their model, i.e. we take into account not only the 3-D density distribution $\rho$ but we also construct the 3-D field
describing the seismic parameter $\Phi$. However, we have found from numerical tests that the influence of lateral variations of $\Phi$ is negligible in comparison to the influence of vertical gradient of $\delta \rho$. In the lower mantle the PREM is almost adiabatic, i.e. $\eta_0 \approx 1$, and thus one can easily see from Eq. 4 that deviations from adiabaticity are caused mainly by the vertical gradients of density anomalies, since the magnitudes of $\delta \rho/\rho_0$ and $\delta \vartheta/\vartheta_0$ are only several percent. On the other hand, the PREM is not adiabatic in the upper mantle, i.e. $\eta_0 \neq 1$, but lateral variations of $\eta$ are again generated mainly by the vertical gradient of $\delta \rho$. The distribution of Bullen’s parameter can therefore help to forecast the places with a possibility for a potential gravitational instability, as the contrast between heads of cold downwellings (or hot plumes) and the surrounding material below (above) is marked by a superadiabatic temperature gradient.

The distribution of Bullen’s parameter for the deep lower mantle is shown in Fig. 6. We emphasize that the range of the scales associated with Bullen’s parameter is the same as in Figs. 2 and 4, where the results for the convection models were presented. This comparison points to a close physical link between results from global seismology and geodynamical modelling in that the same range of Bullen’s parameter is obtained in both circumstances. This is the first time such a comparison has been displayed. This figure truly shows that the superplumes under Africa and the Pacific Ocean are subadiabatic in character because of the density increases with depth. This result is consistent with the fact that plumes have intrinsically a chemical nature. Kennett et al. [25] have inverted simultaneously for both the bulk sound velocity and shear wave velocity and have also inferred from their changes with depth in the lower mantle that chemical anomalies are likely in the deep portion close to the CMB. However, Romanowicz [26] claimed that density models based on free oscillation splitting are not well resolved in the deepest mantle and thus high density anomalies corresponding to low seismic velocities, under Africa and the Pacific Ocean, may not be a robust feature. On the other hand, it is interesting that the African superplume seems to have the two cylindrical cores, reaching up to a depth of 2400 km, which is consistent with the older S-wave tomographic models by Su and Dziewon- ski [27]. We refer the reader also to the 3-D images of this model depicted in the spherical geometry in [28].

The PREM model is not adiabatic at all in the upper mantle and thus the average value of Bullen’s parameter need not be close to zero as demonstrated in Fig. 7 for the depths 300 km and 500 km. Let us point out here that the scale for the depth 500 km is shifted by a factor of 0.5 to subadiabatic magnitudes. The increase of the PREM density with depth is extremely high in the depth range.
interval 400–600 km, where the dependence of density on depth was modelled by means of a continuous linear function. The main physical reason for such a strong subadiabatic density increase is probably a series of minor density ‘jumps’ due to phase transitions present in this depth interval (e.g. [29,30]). Nevertheless, it could also be partly caused by convection layering, which results in accumulation of cold heavy masses above the 670 km boundary as demonstrated in Figs. 1–3.

The interesting feature is that underneath Iceland Bullen’s parameter is above its mean value in the shallower depths but it is relatively low in the transition zone and below the 670 km interface. According to our recent study [10], such a change of Bullen’s parameter is consistent with the transition from a plume head to its limb. In the deep mantle underneath the Icelandic region a continuation of the anomaly indicating a plume root cannot be discerned. It may be caused by the resolution of the density model employed or the mantle upwelling beneath Iceland does not extend down to the lower mantle [31].

6. Discussion and concluding remarks

In the last several years there has been a resurgence of interest in thermal-chemical convection in the deep lower mantle and in the idea of a chemically stratified portion in the bottom 1000 km of the lower mantle [32,33]. This idea of thermal chemical plumes in the deep mantle is not new but has already been invoked by many people due to the presence of the heterogeneous D’ layer [12,34–37]. Recently Karato and Karki [38] have argued on the basis of mineral physics that chemical heterogeneities are required in the lower mantle deeper than around 2000–2300 km depth. This is consistent with the earlier inference based on seismic tomography [25,35,39] and the recent inference of the distribution of density and seismic velocities [12].
The 3-D spatial distribution of Bullen’s parameter, which involves the radial derivative of the density, can yield valuable information about the style and nature of 3-D mantle convection, whether it is strictly thermal or thermal-chemical at certain depth. Our intention in this paper is to show that we can glean some special information concerning deep-mantle geodynamics from investigating maps of Bullen’s parameter and correlate them with the numerical results from mantle convection modelling.

The 3-D distribution of Bullen’s parameter inferred from normal-mode seismology is consistent with a thermal-chemical plume in the bottom 400–600 km in the lower mantle where subadiabaticity prevails. Subadiabaticity ($\eta$ greater than 1) is consistent with heavy but hot material being pulled up partially, as first inferred by Yuen et al. [35] from the seismic tomographic model of Su and Dziewonski [27]. A knowledge of the density distribution under particular tectonic provinces such as Africa is important, as it also allows one to calculate the regional dynamical topography from the inferred global viscosity profiles [40].

The impact of an upper-mantle phase change on the state of adiabaticity in the mantle is quite dramatic and has also been found in 3-D spherical-shell convection with an endothermic phase change at 670 km [41]. It is indeed very difficult to find large areas in the upper mantle, which are adiabatic, even for partially layered mantle convection. In the lower mantle there is a greater tendency for adiabaticity to prevail for thermal convection. But if there is some amount of thermal-chemical anomalies in the deep mantle [35,38] or a high viscosity hill at a depth of around 1800 [42,43], which would impede both the vertical up- and downwellings going through a very viscous zone [44,45] then we may expect that the lower mantle may be contaminated by a large non-adiabatic regions, which may not be detectable from the present resolution of free oscillation data.

We are thus led to the conclusion that perhaps...
the whole mantle is not as adiabatic as has commonly been held by many geophysicists. Such a change in view would seriously impact many fields in the geosciences, such as petrology, geochemistry and mineral physics.

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