Supporting Information for “Complex earthquake sequences on simple faults”

Camilla Cattania

1Department of Geophysics, Stanford University, Stanford, CA

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Introduction

The Supplementary Material consists of the following parts:

• Supplementary text S1 describes the numerical methods and the analytical derivations behind the expressions presented in the main text.

• Figure S1 shows the number of ruptures per cycle as a function of $W/L_c$, where $L_c$ is one of various characteristic lengths associated with rate-state friction and commonly used in the literature.
• Figure S2 establishes a linear relation between the nucleation length \( L_\infty \) and the distance for creep penetration within the velocity weakening region when instability occurs.

• Figure S3 shows the stress intensity factor for a crack driven by end point displacement at depth and propagating updip, with and without a free surface.

• Figure S4 shows the timing of ruptures within cycles as a function of \( W/L_\infty \) for simulations with \( a/b = 0.75 \). The decrease of the time to the first nucleation is evident, and well explained by the theoretical expression derived in the main text (eq.9).

Text S1 - Methods

1. Numerical simulations

In order to test a wide range of \( W/L_\infty \), I run rate-state simulation on 2-D vertical antiplane faults. I use the pseudo-dynamic rupture code Fdra (Fault Dynamics Radiation-damping Approximation; Segall and Bradley (2012a)), an efficient boundary element code to model slip on planar faults with rate-state friction. Fault slip is controlled by the equation of motion:

\[
\tau_{el}(x) - \tau_f(x) = \frac{\mu}{2c_s}v(x),
\]

with \( \mu \) the shear modulus and \( \tau_{el} \) is the elastostatic shear stress due to loading from the boundary and quasi-static elastic interactions between fault elements. Sliding resistance \( \tau_f \) is expressed by the rate-state equations (Dieterich, 1979; Marone, 1998) and the “ageing” law:

\[
\tau_f = \sigma \left[ f_0 + a \ln \left( \frac{V}{V_s} \right) + b \ln \left( \frac{V_0\theta}{d_c} \right) \right]
\]
\[ \frac{d\theta}{dt} = 1 - \left( \frac{V\theta}{d_c} \right) \]  

(3)

where \( \bar{\sigma} = \sigma - p \) is the effective normal stress, \( p \) the pore pressure, \( V \) the sliding velocity, \( \theta \) a state variable and \( d_c \) a characteristic slip distance; \( f_0 \) is the friction coefficient at a reference slip velocity \( V^* \), and \( a, b \) are material dependent rate-and-state parameters. The steady-state strength at a given slip velocity \( V \) is

\[ \tau_{ss}(V) = \sigma \left[ f_0 + (a - b) \log \frac{V}{V^*} \right] \]  

(4)

I intentionally used a uniform distribution of frictional properties to test for the role of \( W/L_\infty \) alone. An antiplane fault of vertical extent \( W \) with velocity weakening frictional parameters overlays a velocity-strengthening fault of extent \( W_{vs} = W \). The system is loaded by slip at \( V_{pl} = 10^{-9} \text{ m/s} \) at the bottom of the VS region. For all simulations I used \( \sigma = 50 \text{ MPa}, d_c = 0.1\text{mm} \). I tested two values of \( b \) (0.01 or 0.02), and varied \( a \) to cover a wide range of \( a/b \).

Earthquakes are defined as the slip occurring above a threshold slip velocity of 0.1 m/s. Cell size needs to be sufficiently small, in particular to avoid slip complexity arising as an artifact for under resolved models (Lapusta et al., 2000): it is important to resolve the nucleation length and the cohesive zone accurately. The latter is given by \( \Lambda = C L_b \), with \( L_b = \mu'd_c/\sigma b \) and \( C \) a factor of order one (Perfettini & Ampuero, 2008). I chose cell size \( \Delta x \) such that \( \min(L_\infty, L_b)/\Delta x \geq 7.7 \). This criterion was further tested by running a subset of the simulations with double resolution; note that the constraint on \( L_b/\Delta x \) alone was not sufficient for low \( a/b \). To reduce computational requirements, I halved the
resolution in the VS region, and verified that this discretization yields the same results as setting the same cell size everywhere.

1.1. Limit cycles

I exclude the initial part of each simulation (spin up); it is important to do this carefully since the first few cycles often present high variability in rupture dimensions and interevent time, even for small $W/L_\infty$. For cycles with zero or few partial ruptures, a limit cycle is easy to identify by eye and is reached after few (2-3) full ruptures. On the other hand, simulations with large $W/L_\infty$ did not reach a limit cycle at all (see Fig. 1 in the main text). For the simulation with $a/b = 0.75$, $W/L_\infty = 415$ I simulated 14 full ruptures (after removing an early phase which exhibited a different statistical distribution, which I considered the spin-up phase), and I verified that the statistical properties across a cycle are stationary: using the first 7 or the last 7 cycles produces only minor differences in the frequency-size distributions. It is possible that, waiting longer, a limit cycle would eventually have been reached; alternatively, the differences in stress field due to the large number of partial ruptures could make cycles intrinsically less repeatable.

2. $L_{\text{crit}}$ for rate-state faults (ageing law)

In the main text I showed that the number of earthquakes per cycle is determined by the ratio $W/L_{\text{crit}}$, where $L_{\text{crit}} \sim (L_n \Delta \tau / K_c)^2$ with $L_n$ proportional to the nucleation dimension. For rate-state ageing law, this expression can be simplified due to the form of $L_n$. Rubin and Ampuero (2005) derived the nucleation length $L_\infty$ by considering the stability of a constant stress drop crack overcoming a toughness $K_c$, analogous to eq. 1

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in the main text, with $K_i = 0$. Taking $K \Delta \tau \sim \Delta \tau \sqrt{L} = K_c$, gives

$$L_\infty \sim \left( \frac{K_c}{\Delta \tau} \right)^2,$$

so that $L_{\text{crit}} \sim L_\infty$.

### 3. Estimating $\alpha$ for vertical antiplane faults

In the main text I presented a simple scaling argument for the number of events per cycles (eq. 6). Here I derive various constants of proportionality for the case of vertical antiplane faults with ageing rate-state friction.

A full treatment of this geometry would be as follows: the displacement boundary condition is applied at the bottom of the VS region. The latter slips at $V \sim V_p$ during most of the seismic cycle, and has a displacement profile leading to a constant stress $\tau = \tau_{ss}(V_p)$ (steady state strength at plate rate). Stress in the VW region responds to changes in the slip profile in the VS region. However, I find that approximating the effect of the VS region by assuming uniform slip and considering a dislocation at the VW-VS transition seems to be sufficient. I tested variable sizes for the VS region: $W_{VS} = nW$ with $0, 1, 2, 5, 8, 12$, and verified that the average number of ruptures per cycle is not affected by this choice. As expected, setting $n = 0$ radically changes the time-dependent behavior: there is no afterslip, so the temporal clustering disappears and it always takes the same amount of time to nucleate each event.

The number of ruptures per cycle is given by

$$\alpha(W/L_\infty) = \frac{S_{\text{full,fs}}}{S_n} = \frac{K_c \sqrt{2W/\pi \phi}}{\Delta \tau L_n},$$

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where $S_{full,fs} = \phi S_{full}$ is the slip required for a full rupture including the effect of the free surface, which is quantified by the factor $\phi$ as described below. Previously I assumed that the creep penetration length required for instability is proportional to the minimum dimension for instability for a constant stress drop crack in a uniform stress field ($L_n \sim L_\infty$). This intuitive result is confirmed by Supplementary Figure S2: $L_n = (2.9 \pm 0.6)L_\infty$. This is about 50% larger than the value Rubin and Ampuero (2005) found for simulations with a uniform loading rate. The discrepancy, also found by Rubin (2008) for inplane dipping faults, is presumably caused by the stabilizing effect of loading from the edge, since a nucleating crack grows away from the load point.

The stress increase behind the creep front $\Delta \tau$ is the difference between the steady-state stress at seismic velocity $V_{co}$ ahead of the creep front, and steady-state stress at creep rate $V_{pl}$ behind the creep front: $\Delta \tau = (b - a)\sigma \log(V_{co}/V_{pl})$. The fracture toughness is related to fracture energy $G_c$ by

$$K_c = \sqrt{2\mu' G_c},$$

(7)

And the fracture energy for the aging law, in the no-healing approximation and constant coseismic slip velocity, is given by (Rubin & Ampuero, 2005)

$$G_c = \frac{d_c b \sigma}{2} \left[ \log \left( \frac{V_{co} \theta_i}{d_c} \right) \right]^2,$$

(8)

so that

$$\frac{K_c}{\Delta \tau} = \frac{\sqrt{d_c b \sigma \mu' \log(V_{co} \theta_i/d_c)}}{(b - a)\sigma \log(V_{co}/V_{pl})}$$

(9)

where $\theta_i$ is the state variable just outside the crack tip. Due to healing, strength and fracture energy grow with time. Interseismically, I take $\dot{\theta} = 1$ in eq. 3 and hence $\theta_i = t$,
where \( t \) is the time since the last full rupture. Since the dependence on time is weak, I set a constant value of \( \theta_i = 1 \) year (see also Cattania and Segall (2019)). Setting \( V_{co} = 1 \) m/s, \( V_{pl} = 10^{-9} \) m/s and \( d_c = 10^{-4} \) m results in

\[
\frac{K_c}{\Delta \tau} \approx 1.3 \frac{\sqrt{d_c b \sigma \mu'}}{(b-a) \sigma} = 2.3 \sqrt{L_\infty}.
\]  

with \( L_\infty \) the nucleation half-length derived by Rubin and Ampuero (2005):

\[
L_\infty = \frac{b}{\pi (b-a)^2} \frac{\mu' d_c}{\sigma}.
\]  

Finally, I estimate the effect of the free surface on the stress intensity factor from loading, expressed by the factor \( \phi \) in eq. 6. For an antiplane vertical fault loaded at depth \( z = W \), the free surface can be included by adding a mirror dislocation at \( z = -W \) (Segall (2010); Fig. S3). As before, I consider a crack extending from \( z = W \) (load point) to \( z = d \) (crack tip). The stress intensity factor for this configuration is given by Tada, Paris, and Irwin (2000):

\[
K_{t,fs} = \frac{\mu' S}{2 \kappa F(\kappa) \sqrt{\pi d}}
\]  

where \( \kappa = \sqrt{1-(d/W)^2} \) and \( F(\kappa) \) the complete elliptic integral of the first kind. For convenience, I normalize this expression by the SIF without free surface at \( d = 0 \), that is \( K_t(W) = \mu' S/\sqrt{2\pi W} \), which I used to derive eq. 3 in the main text. Fig. S3 shows the normalized SIFs with and without free surface as a function of crack length \( x = W - d \). Without free surface, the SIF is a monotonically decreasing function of \( x \), minimum at \( x = W \). With free surface, the SIF increases as the crack approaches the top of the fault, with a minimum value at \( x = 0.65W \) where \( K_{t,fs} = 1.4K_t(W) \). One would therefore expect no ruptures with a length between 0.65\( W \) and \( W \), and Fig. S3 confirms that these
are in fact very rare. Since the free surface increases the SIF by a factor of 1.4, the load point displacement required to satisfy the condition $K_{l,fs} = K_c$ is $S_{full,fs} = S_{full}/1.4$, or $\phi = 0.71$ in eq. 6.

Finally, setting $L_n = (2.9 \pm 0.6)L_\infty$, $\phi = 0.71$ and eq. 10 in eq. 6 yields

$$\alpha(W/L_\infty) \approx \frac{1.3}{2.9 \pm 0.6} \sqrt{W/L_\infty} = 0.45^{+0.12}_{-0.08} \sqrt{W/L_\infty}. \quad (13)$$

The transition from single ruptures to multiple ruptures per cycle is obtained by solving $\alpha = 1$, which gives $W = (5 \pm 2)L_\infty$.

4. Frequency-magnitude distribution

In the main text, section 3 I showed that to ensure consistency with eq. 6, a truncated power law distribution for the survival function of rupture lengths must be of the form $N(l) \sim l^{-1/2}$. Here I start from this result and derive a frequency-magnitude distribution for ruptures on 2-D faults in a 3-D medium. In order to infer the distributions of seismic moments from simulations of 1-D faults in a 2-D medium, one must make further assumptions on the rupture geometry. Each simulated event is replaced with multiple 3-D events with an along strike rupture length proportional to along dip length. A simulated rupture of length $l$ (along dip) can contain $\sim L/l$ such events, which modifies the survival function by a factor $L/l$: $N_{3D}(l) \sim l^{-3/2}$. Taking $M_0 \sim \Delta \tau l^3$ (constant aspect ratio and constant stress drop $\Delta \tau$) gives $N_{3D} \sim M_0^{-1/2}$, equivalent to a $b$ value of 3/4. In fig. 3 (main text) I compare this with magnitude distributions estimated from the simulations.

Here moments are calculated by

$$M_0 = \mu l^2 \bar{s} \quad (14)$$
where $\bar{s}$ is the average slip. I calculate the 3-D survival function from 2-D simulations assuming that $L/l$ ruptures occur in place of a rupture of length $L$; and since the value of $L$ doesn’t affect the probability distribution function after normalization, for simplicity I take $L = 1$. Note that here the rupture width of each event was assumed to be the same as the rupture length; assuming a different aspect ratio would modify the moments and the number of events per unit length by a constant factor, without affecting the normalized probability distributions.

5. Interevent time distributions

5.1. Functional form for observed seismicity

Corral (2004) showed that inter-event times $\Delta t$ for different regions and magnitude ranges follow the same distribution, after rescaling them by the local seismicity rate $\lambda_i$. This implies that the probability density function for a region can be written as

$$P_i(\Delta t) = \lambda_i f(\lambda_i \Delta t) \tag{15}$$

where the suffix indicates a particular catalog, and the universal function $f$ was found to be close to a generalized gamma distribution. A distribution similar to the one observed by Corral (2004) can be obtained by the superposition of Poissonian background events and aftershock sequences (Hainzl et al., 2006). Similarly, Saichev and Sornette (2007) obtained the distribution of inter-event times assuming Omori-Utsu decay of seismicity rates: $(t + c)^{1+\theta}$, where $\theta \gtrsim 0$ and $c$ is constant. The normalized inter-event times $\lambda \Delta t$ have the following probability density function:

$$f(x) = \left( n\epsilon^\theta x^{-1-\theta} + [1 - n + n\epsilon^\theta x^{-\theta}]^2 \right) \varphi(x, \epsilon) \tag{16}$$
with

\[ \varphi(x, \epsilon) = \exp \left( -(1 - n)x - \frac{ne^\theta}{1 - \theta} x^{1 - \theta} \right) \]  

(17)

where \( n \) is the fraction of triggered events and \( \epsilon = c\lambda \). At short recurrence intervals \( \lambda \Delta t \ll 1 \) this function is a power-law decay inherited from the Omori-Utsu law (main text, Fig. 4). Independent events are Poissonian, and hence have an exponential inter-event time distribution, as can be seen setting \( n = 0 \); this defines the shape of the function for \( x \gtrsim 1 \). I set \( n = 0.9, \theta = 0.03 \) and \( \epsilon = 0.76 \), which Saichev and Sornette (2007) found to be a good fit to observed regional catalogs. Note that the seismicity decay derived in section 6 implies \( \theta = 0 \), but positive values are required for mathematical tractability.

Given the wide range spanned by inter-event times, probability density functions need to be estimated using a variable bin width. This is achieved by binning the sorted inter-event times in bins containing \( N = 20 \) events each, and diving \( N \) by each bin width to estimate the PDF. I verified that alternative methods, such as exponentially growing bin width used by Corral (2004), yields similar results.

6. Afterslip driving nucleation rates

The first nucleation after a full rupture occurs when the creep front reaches a distance proportional to \( L_\infty \) (Supplementary Figure S2), corresponding to a critical amount of slip at the VS-VW transition (eq. 2 in the main text). The creep front propagates at different speeds depending on \( W \), due to the different afterslip evolution in the velocity strengthening region. A full rupture penetrates a certain distance \( (L_p) \) into the VS region. This length determines the stiffness of the area initially experiencing afterslip, which in
turn controls its temporal evolution: a larger $L_p$ produces faster afterslip, as shown below using a simple spring-slider model. The penetration length is proportional to $W$, as one might expect. A simple fracture energy argument balancing the stress intensity factor of a constant stress drop crack in the VW region with a negative stress drop crack in the VS region yields the estimate $L_p = W/3$. In practice, $L_p$ is smaller (between $0.05W - 0.1W$), possibly due to neglecting the contribution of fracture energy to rupture arrest.

A spring-slider model developed by Perfettini (2004) and Perfettini and Ampuero (2008) for a velocity strengthening region with constant stiffness, reaching coseismic speed $V_{co}$ during the earthquake, predicts that afterslip evolves according to

$$S(t) = t_0 V_{pl} \log \left[ \frac{V_{co}}{V_{pl}} (e^{t/t_0} - 1) + 1 \right]$$

(18)

with $t_0 = \sigma(a - b)_{vs}/kV_{pl}$, and $(a - b)_{vs}$ is the value in the VS region; $k(L_p)$ is the spring stiffness, given by $k = \mu'/L_p$ (Ampuero & Rubin, 2008). Note that the “background” creep velocity $V_{pl}$ in this expression is not exactly the loading velocity, but it must be lower in order for the total slip across a cycle to equal the imposed displacement. However, since this term appears in a logarithm, this correction is negligible. The time to the first earthquake is obtained by solving $S(t) = S_n$, which yields

$$T_n = t_0 \log \left[ \frac{V_{pl}}{V_{co}} (e^{S_n/t_0 V_{pl}} - 1) + 1 \right].$$

(19)

All simulations were run with $(a - b)_{vs} = (b - a)_{vw}$. Therefore, for simplicity, I derive the nucleation time for this particular case; a more general treatment would result in similar expressions, with factors of $(a - b)_{vs}/(b - a)_{vw}$, and use the notation $a - b$ to refer to the velocity strengthening region in what follows. Combining eq. 19 with eq. 2 (main text)
and $\Delta \tau = (b - a) \sigma \log(V_{co}/V_{pl})$ gives, in the limit of $(V_{co}/V_{pl})^{\pi L_n/L_p} \gg 1$,

$$T_n = \frac{\sigma (a - b) L_p}{\mu V_{pl}} \log \left( \left( \frac{V_{co}}{V_{pl}} \right)^{\frac{L_n}{L_p}} + 1 \right). \quad (20)$$

I test this against simulations with $b = 0.02$, $a = b \pm 0.005$: Supplementary Figure S4 shows that this expression agrees reasonably well with the simulations, reproducing the sharp decrease in the time of the first interseismic earthquake with increasing $W$. It can be easily verified that as $L_p \to 0$ (no afterslip), eq. 19 becomes equal to $S_n/V_{pl}$. In this case there would be no temporal clustering, because nucleation rates are constant; since $L_p$ is proportional to $W$, this explains the periodicity observed at small $W$. A more quantitative argument consists of comparing the amount of slip accumulated during afterslip to the critical slip for nucleation $S_n$ (eq. 2 in the main text). Taking the limit of eq. 18 at $t \gg t_0$ shows that the total amount of slip due to afterslip is $S_{after} = t_0 V_{pl} \log(V_{co}/V_{pl})$, so that

$$\frac{S_{after}}{S_n} = \frac{t_0 V_{pl} \log(V_{co}/V_{pl}) \mu'}{\pi \Delta \tau L_n} = \frac{L_p}{\pi L_n}. \quad (21)$$

The scaling of the rupture penetration length with $W$ ($L_p \sim (0.075 \pm 0.025)W$) and the creep penetration length with $L_\infty$ ($L_n \sim (2.9 \pm 0.6)L_\infty$) imply that afterslip can lead to temporal clustering for $W/L_\infty$ larger than $\sim 10^2$.

Finally, I derive the temporal evolution of seismicity rates from the evolution of afterslip. Instead of calculating the time to each successive nucleation, seismicity rates are estimated as $R = \dot{S}/S_n$ (since each slip increment $S_n$ gives one event), and by differentiating eq. 18 to find

$$R(t) = \frac{\dot{S}}{S_n} = \frac{V_{pl}}{S_n} \frac{e^{t/t_0}}{V_{pl}/V_{co} + e^{t/t_0} - 1}. \quad (22)$$
This expression, analogous to one derived by Perfettini (2004), is similar to the Omori-Utsu evolution of seismicity rates, eventually going back to the constant background rate \( R_0 = V_{pl}/S_n \). The seismicity rate at \( t = 0 \) is \( R = (V_{co}/V_{pl})R_0 \), and remains constant at short times \( (t/t_0 < V_{co}/V_{pl}) \); at later times, seismicity rate has a power-law decay: \( R = (t_0/t)R_0 \), until returning to the background value at \( t/t_0 \sim 1 \). This is consistent with the observed inter-event time distributions for large \( W \) (Fig. 4 in the main text), characterized by a \( 1/t \) decay.

References


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Figure S1. Number of ruptures per cycle as a function of $W/L_{\text{crit}}$, with $L_{\text{crit}}$ corresponding to various characteristic lengths. $W/L_{\infty}$ appears to be the most effective at predicting the number of events per cycle for $a/b \gtrsim 0.5$, when nucleation occurs as crack-like expansion with a critical half-length $L_{\infty}$ (Rubin & Ampuero, 2005).
Figure S2. Position of the creep front updip of the VW-VS transition for all simulations (median value for all ruptures in each simulation) vs. $L_\infty$. The light grey band represents one standard deviation.
Figure S3. (a) Stress intensity factor as a function of normalized crack length with a without free surface (solid and dotted line respectively), normalized by stress intensity factor with a crack of length $W$ calculated without free surface. (b) Cartoon showing the configuration for each SIF. (c) number of ruptures vs. normalized rupture length for all simulations: partial ruptures exceeding $0.65W$ are forbidden by energy balance, since the SIF increases with crack length above this value; they are in fact rather rare.
Figure S4. Timing of earthquakes since the previous full rupture, as a function of $W/L_\infty$, for simulations with $a/b = 0.75$. The dotted line shows the theoretical minimum duration between full ruptures (eq. 9 in the main text) and the solid line the time to the first nucleation (eq. 8). The range in the latter (grey area) is obtained from $L_n = (2.9 \pm 0.6)L_\infty$ and $L_p = (0.075 \pm 0.025)W$, approximately the range of values observed in the simulations.