Spectral models for ground motion prediction in the L’Aquila region (central Italy): evidence for stress-drop dependence on magnitude and depth

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SUMMARY
In this study we derive a spectral model describing the source, propagation and site characteristics of $S$ waves recorded in central Italy. To this end, we compile and analyse a high-quality data set composed of more than 9000 acceleration and velocity waveforms in the local magnitude ($M_l$) range 3.0–5.8 recorded at epicentral distances smaller than 120 km. The data set spans the time period from 2008 January 1 to 2013 May 31, and includes also the 2009 L’Aquila (moment magnitude $M_w$ 6.1, $M_l$ = 5.8) sequence. This data set is suitable for the application of data-driven approaches to derive the empirical functions for source, attenuation and site terms. Therefore, we apply a non-parametric inversion scheme to the acceleration Fourier spectra of the $S$ waves of 261 earthquakes recorded at 129 stations. In a second step, with the aim of defining spectral models suitable for the implementation in numerical simulation codes, we represent the obtained non-parametric source and propagation terms by fitting standard parametric models. The frequency-dependent attenuation with distance $r$ shows a complex trend that we parametrize in terms of geometrical spreading, anelastic attenuation and high-frequency decay parameter $k$. The geometrical spreading term is described by a piecewise linear model with crossover distances at 10 and 70 km: in the first segment, the spectral ordinates decay as $r^{-1.01}$ while in the second as $r^{-1.68}$. Beyond 70 km, the attenuation decreases and the spectral amplitude attenuate as $r^{-0.64}$. The quality factor $Q(f)$ and the high-frequency attenuation parameter $k$, are $Q(f) = 290f^{0.16}$ and $k = 0.012 \text{s}$, respectively, the latter being applied only for frequencies higher than 10 Hz. The source spectra are well described by $\omega^2$ models, from which seismic moment and stress drops of 231 earthquakes are estimated. We calibrate a new regional relationship between seismic moment and local magnitude that improves the existing ones and extends the validity range to 3.0–5.8. We find a significant stress drop increase with seismic moment for events with $M_w$ larger than 3.75, with so-called scaling parameter $\varepsilon$ close to 1.5. We also observe that the overall offset of the stress-drop scaling is controlled by earthquake depth. We evaluate the performance of the proposed parametric models through the residual analysis of the Fourier spectra in the frequency range 0.5–25 Hz. The results show that the considered stress-drop scaling with magnitude and depth reduces, on average, the standard deviation by 18 per cent with respect to a constant stress-drop model. The overall quality of fit (standard deviation between 0.20 and 0.27, in the frequency range 1–20 Hz) indicates that the spectral model calibrated in this study can be used to predict ground motion in the L’Aquila region.

Key words: Earthquake ground motions; Earthquake source observations; Seismic attenuation; Site effects.
INTRODUCTION

The Abruzzo region (central Italy) is an area characterized by high seismic hazard, where low to moderate magnitude earthquakes frequently occur (Rovida et al. 2011). The most recent seismic sequence started on 2009 April 6 with the extensively studied L’Aquila earthquake ($M_w 6.1$, $M_s 5.8$) and lasted for about one year (Chiaraluce et al. 2011; Valeroso et al. 2013).

In an effort to improve ground motion prediction capabilities in the area, several studies were conducted with the aim of evaluating the different components of the spectral model (i.e. attenuation, source parameters, source scaling and site effects). Different techniques, data sets and/or spatial/temporal windows were adopted in these studies, and while some of them provided a comprehensive set of parameters required for the definition of ground motion, others investigated only specific components. A generalized inversion technique (GIT) was applied by Bindi et al. (2009), hereinafter BIN09, to the records of 13 events of the 2009 L’Aquila seismic sequence in order to isolate the source, propagation and site contributions of ground motion. While BIN09 provided rapid insights into the spectral composition of ground motions in the aftermath of the main shock, the data set was however very limited as it was only composed of the waveforms of the main shock and 12 aftershocks with $M_w \geq 4$ at epicentral distances smaller than 200 km. Ameri et al. (2011), hereinafter AME11, provided an entire set of spectral terms to predict ground motions after applying the GIT technique to a data set significantly larger than BIN09, containing the records of 100 aftershocks in the local magnitude range $3.1 < M_l < 5.3$ and hypocentral distance range $8-50$ km. Malagnini et al. (2011), hereinafter MAL11, studied the attenuation of a wide area of the central Apennines using a data set of 170 foreshocks and aftershocks of the L’Aquila sequence ($M_w \geq 2.8$) at distances up to 200 km. Different from the previously mentioned studies, they used a spectral ratio technique to investigate the regional source scaling and employed their attenuation and source scaling results to predict ground motions.

Source scaling was also investigated by Calderoni et al. (2013), hereinafter CAL13, through the empirical Green’s functions (EGF) approach applied to broad-band signals in the distance range 100–250 km. Both MAL11 and CAL13 found evidence for a stress drop increase for large size earthquakes, contrasting the results of BIN09 and AME11. In particular, CAL13 argued that the contradicting results between their work and BIN09 and AME11 could be partially related to the different methods used to estimate the stress drops and, in particular, to the EGF efficiency in separating source and propagation compared to the GIT.

In order to shed light on these issues and provide the most comprehensive set of spectral parameters for ground motion prediction in the region to-date, we apply the GIT approach to the largest available data set for the area composed of more than 15,000 acceleration and velocity waveforms (about 450 events and 400 stations), recorded in the time span 2008 January 1–2013 May 31 and including the 2009 L’Aquila seismic sequence. This data set provides optimal spatial coverage up to 100 km and reliably spans the magnitude range 3–6.

Following a non-parametric GIT scheme as BIN09 and AME11, we evaluate the complete suite of spectral parameters required for ground motion prediction in a wide region of central Italy. We use parametric models to interpret the non-parametric spectral attenuation curves (in terms of geometrical spreading and anelastic attenuation) and source spectra (in terms of an omega-square model; Brune 1970). The obtained quality factor, stress drops and seismic moments are compared and discussed to the estimates by different authors for this area. Finally, we reproduce the observed spectral ordinates by performing a forward computation using the parametric models derived in this study and we evaluate their performance through the residual analysis.

DATA SET

The Abruzzo data set is constructed by selecting earthquakes in a geographical area in central Italy constrained by latitude [41 N; 43 N] and longitude [12 E; 15 E]. The bulk of the data originates from the 2009 L’Aquila seismic sequence (Fig. 1). Differently from previous studies (i.e. BIN09, AME11 and MAL11) that analysed subsets of acceleration or velocity data, we exploit all records available from broad-band, short period and accelerometric sensors, belonging to different networks, installed in the area in the time span 2008–2013. The considered networks are: the National Seismic Network, RSN, operated by INGV (Istituto Nazionale di Geofisica e Vulcanologia); the Mediterranean Network, Mednet, operated by INGV; the Rapid Response Networks operated by INGV, RESIF (Reseau Sismologique et Geodesique Francais) and the University of Genoa; the National Accelerometric Network, RAN, operated by the Department of Civil Protection, DPC. We complete the data set by including data from temporary networks deployed for seismic microzonation (Working Group SM-AQ 2010), site effect studies (Bergamaschi et al. 2011) and aftershock monitoring (Margheriti et al. 2011). Details on instruments and stations of each network are listed in Table 1.

The final data set consists of more than 15,000 records from about 450 events recorded at about 400 stations, in the time period 2008 January 1–2013 May 31 (Fig. 1). The local magnitude range spans from 2.5 to 5.8 and the epicentral distances range up to a maximum of 150 km. The majority of the events were generated by normal faults in the upper crust (i.e. hypocentral depth <15 km).

We carry out an accurate data selection and processing through the following steps:

(i) Visual inspection of waveforms, instrumental correction and manual picks of $P$- and $S$-wave onset; this procedure allows us to identify clipped signals and other specific problems such as signal distortion, spikes, lack of data on the coda, etc.

(ii) Earthquake location using Hypoellipse (Lahr 1989).

(iii) $M_l$ estimation (Wood–Anderson equivalent).

(iv) Bandpass filtering: for velocimetric data, we use a fixed frequency band (0.1–40 Hz); for accelerometric data, we adopt the processing procedure of the Italian strong-motion database ITACA (Pacor et al. 2011; Paolucci et al. 2011).

(v) Identification of unreliable recordings by comparing the geometric mean of peak ground acceleration (PGA) and velocity (PGV) with the predictions from a ground motion attenuation relation valid for the Italian territory (Bindi et al. 2011b): if the difference between predicted and observed strong-motion parameters exceeds 1.5 log units, the waveforms are not further considered.

The local magnitudes are re-computed from synthetic Wood–Anderson seismograms using a local magnitude scale calibrated for the central Italy (M. Cattaneo, personal communication, 2012), in order to have a consistent estimate of the magnitude size for all events. Since the earthquake focal depth is a key parameter for the analysis and interpretation of the inversion results, we put a great deal of effort toward obtaining reliable estimates of this parameter through manual interpretation of the seismograms recorded by temporary networks deployed in the epicentral area. Earthquakes are
Selection of the analysed subset

From the whole compiled data set, we extract a data subset applying the following criteria:

(i) Hypocentral depths shallower than 20 km and local magnitudes in the range 3–5.8 (the latter refers to the magnitude of the main shock of the L’Aquila sequence).

(ii) Hypocentral distances in the range 1–120 km, since most of the records are related to small-magnitude events and the data coverage for distances larger than 120 km is not complete.

(iii) Removing stations closer than 1 km, to avoid unbalanced distributions of regional paths.

(iv) Fourier spectra having at least 50 per cent of points that satisfy the SNR criteria defined above.

(v) Events recorded by a minimum of 5 stations and stations having at least 10 records.

The selected data set is composed of 261 events and 129 stations (Fig. 3a). Due to the frequency-dependent SNR threshold used to extract the spectral ordinates and the variable window lengths, the number of available spectral amplitudes varies with frequency,
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*Indicates that the soil category is assigned on the base of geological information.
Selection of the analysed subset

Figure 2. Left: Time histories converted in acceleration from a short period—EH (top), strong motion—HN (middle) and broad-band—HH (bottom) sensors; the selected S-wave window is indicated in grey and for each record; the sensor code, the station code, the hypocentral distance and magnitude are reported. Right: Fourier amplitude spectra computed for the S-wave window (continuous line) and the pre-event noise window (dashed line).

decreasing from about 9250 in the frequency range 1–10 Hz to about one-third for lower and higher frequencies. To preserve a good data redundancy for the least-squares inversion and considering the limit on the spectral resolution imposed by the window lengths, we select the frequency range 0.4–25 Hz for the following analysis. The number of available spectral amplitudes is 7160 and 5200 at 0.4 and 25 Hz, respectively.

According to geological information, most of the recording stations can be classified as rock (EC8-A class), stiff soil (EC8-B class) or soft soil (EC8-C) sites (Table 1). Site amplification effects are relevant for the L’Aquila data set (e.g. Bergamaschi et al. 2011; Puglia et al. 2011; Di Giulio et al. 2014), since the epicentral area is located inside a river valley filled with quaternary deposits of various depths. Geological conditions are so variable that over a distance of few tens of metres the shear wave velocities of the shallower soil layers may differ substantially, as shown by the soil profiles available for some strong-motion stations from the Italian accelerometric archive (ITACA, http://itaca.mi.ingv.it; Luzi et al. 2008; Pacor et al. 2011).

The selected data set provides an optimal spatial coverage up to 100 km, in the magnitude range 3–5, as shown by the map of the path rays (Fig. 3b) and magnitude–distance distribution (Fig. 4a). Fig. 4(b) shows the distribution of the records as a function of magnitude, hypocentral distance, depth and FAS window duration. The bulk of the data are from small-magnitude events ($M_l < 4$) and hypocentral distances in the range 5–80 km. Magnitudes larger
than 5 relate to the three largest events of the L’Aquila sequence. Most recordings originate from events located at depths larger than 6 km. The S-wave windows are generally short (<10 s) since the waveforms were mostly recorded close to the epicentres on rock and stiff soil sites. The selected window lengths range between 4 s and 19.5 s, with median of 6 s (Fig. 4b).

In Fig. 5, the acceleration spectral amplitudes at 0.4 and 5.9 Hz are plotted versus magnitude and hypocentral distance. Besides the expected attenuation with distance and amplitude increase with magnitude, Fig. 5 shows clear frequency-dependent attenuation rates and dispersion of the spectral values, both increasing with frequency. To isolate in these trends the source, propagation and site
Selection of the analysed subset

METHOD

The GIT is a well-known method to isolate frequency-dependent attenuation characteristics, source spectra and site response functions (Andrews 1986; Castro et al. 1990).

The acceleration spectral amplitude of ground motion can be written as:

\[
\log_{10} U_{ij} (f, M_i, r_{ij}) = \log_{10} S_i (f, M_i) + \log_{10} A (f, r_{ij}) + \log_{10} G_j (f)
\]

where \( U_{ij} (f, M_i, r_{ij}) \) represents the amplitude of the FAS at the \( j \)th station resulting from the \( i \)th earthquake with magnitude \( M_i \); \( r_{ij} \) is the hypocentral distance; \( S_i (f, M_i) \) is the source spectrum of the \( i \)th earthquake, \( A (f, r_{ij}) \) is the attenuation operator and \( G_j (f) \) is the site response function of the \( j \)th station.

The solution of the system described by eq. (1) can be performed either following a parametric approach, where the unknown functions related to source and propagation are expressed in terms of standard models (e.g. Kawase 2006; Tsuda et al. 2006, 2010; Salazar et al. 2007; Drouet et al. 2008, 2011), or following a non-parametric inversion scheme (e.g. Castro et al. 1990; Parolai et al. 2000, 2004; Oth et al. 2008, 2009). In this study, given the good data coverage available, we follow a non-parametric inversion scheme that allows us to capture possible modulations of the attenuation with distance (e.g. effects of later arrivals) and to highlight deviations.
from standard source models. The inversion is performed in a single step (Oth et al. 2011), for the vector composition of the horizontal components, $H$, considering 73 frequencies from 0.4 to 25 Hz, equally spaced in logarithmic scale. The hypocentral distance range from 1 to 120 km is discretized into 60 bins, each 2 km wide. The system is solved in a least-squares sense (Paige & Saunders 1982) and the vertical amplification function obtained by GIT (red curve) for the three selected reference stations (MNS, RM06 and RM08); the number rec of $S$-phases available for each station is also provided.

In order to remove two unresolved degree of freedom affecting eq. (1), two constraints have to be added. The attenuation is applied from a reference distance of 5 km (i.e. $A(f, 5) = 1$ for any frequency) and, consequently, the source spectra are shifted to the same reference distance. Moreover, to eliminate the linear dependence between the source and site terms, the site amplification function for one or more reference stations is constrained to one. For the selection of the reference stations, we perform a preliminary inversion for the vertical component, setting the average response of all sites to 1, and we also compute the horizontal to vertical spectral ratio ($H/V$) for all stations. The reference stations were then selected with the following criteria: (i) amplifications smaller than 2 on vertical component of the site response function resulting from GIT; (ii) flat horizontal-to vertical spectral ratio $H/V$, or characterized by small-amplitude peaks ($<2.5$). Three stations were finally selected as reference sites (RM06, RM08 and MNS) and Fig. 6 shows the $H/V$ and vertical component amplification results for these stations.

Before discussing the results of the non-parametric inversion, we analyse the goodness of fit of the non-parametric models by computing the residual distributions, shown in Fig. 7. The residuals, computed as the observed minus predicted spectral values, do not show any trend with magnitude and distance, and they have almost zero mean with standard deviations of about 0.2 for all considered distance and magnitude bins. We conclude that the performed non-parametric inversion provides empirical functions that well describe the overall trends present in the data.

**RESULTS**

**Attenuation functions**

Fig. 8(a) shows the non-parametric attenuation functions at different frequencies as a function of the hypocentral distance. The attenuation functions show a clear dependence on frequency and, generally, they decrease monotonically with distance up to 100 km, with a rate increasing with frequency. A rapid decay is observed, for all frequencies up to about 70 km.

In order to obtain those parameters useful for ground motion predictions and simulation (Boore 1983), we fit the non-parametric attenuation functions with a model including the geometrical spreading and the frequency-dependent attenuation term in the form:

$$A(f, r_i) = G(r) \exp \left[\frac{-\pi f (r - r_0)}{Q(f) v_s} \right] \exp [-\pi k f]$$

where $G(r)$ is the geometrical spreading attenuation, $v_s$ is the shear wave velocity, $r_0$ is the reference distance, $Q$ is the apparent quality factor, assumed frequency-dependent and $k$ is parameter describing the high-frequency attenuation, as proposed by Anderson & Hough (1984).

It is known that the estimates of the geometrical spreading and the quality factor are inherently difficult to obtain and are affected by errors that cannot be solved without a clear understanding on the attenuation mechanisms in the propagation medium (Mitchell 2010). These uncertainties arise because, while the empirical curves are the results of complex phenomena that occur in the earth crust, the models used to fit them are generally very simple. The main problem in reproducing the attenuation curves by eq. (2) is the existing trade-off between geometrical spreading and frequency-dependent quality factor $Q$ (Xie 2010; Atkinson 2012; Babaie Mahani & Atkinson 2012; Castro et al. 2013; McNamara et al. 2014).

To face this problem, different inversion strategies can be implemented: (i) to fix the geometrical spreading exponent $n$ for specific distance range and evaluate the frequency dependence of $Q$ (Castro et al. 2013; McNamara et al. 2014); (ii) to assume geometrical spreading variable with distance and frequency while $Q$ is assumed constant and independent on frequency (Morozov 2010); (iii) to consider both $n$ and $Q$ depending on frequencies (Bindi et al. 2004). Independently on the adopted inversion scheme, without other external constrains, the trade-off between the quality factor and geometrical spreading remains in any case unresolved.

In this study, we follow a two-step approach: in the first step, we determine the frequency-independent geometrical spreading model $G(r)$ while, in the second, we consider the data corrected for $G(r)$ to derive the frequency-dependent attenuation model that includes the quality factor $Q$ and the high-frequency decay parameter $k$.  

**Figure 6.** Comparison between the horizontal to vertical ($H/V$) spectral ratio (black and grey area indicate mean and the ± one standard deviation, respectively) and the vertical amplification function obtained by GIT (red curve) for the three selected reference stations (MNS, RM06 and RM08); the number rec of $S$-phases available for each station is also provided.
Selection of the analysed subset

Figure 7. Residuals (grey symbols), computed as log10 (observation/prediction), at 1 Hz (a) and 10 Hz (b) as a function of distance (top) and magnitude (bottom); the mean residuals ± one standard deviation computed for different distance and magnitude bins are shown as black dots with error bars.

To minimize the effect of the anelastic attenuation, we perform the first regression by selecting the non-parametric curves around 1 Hz (i.e. from 0.8 to 1.2 Hz) and considering a tri-linear hinged model with crossover distances at 10 and 70 km, in the following form:

\[ G(r) = \begin{cases} 
\left( \frac{r}{r_0} \right)^{n_1}, & \text{for } r \leq 10 \text{ km} \\
\left( \frac{r_0}{10} \right)^{n_1} \left( \frac{10}{r} \right)^{n_2}, & \text{for } r > 10 \text{ km and } r \leq 70 \text{ km} \\
\left( \frac{r_0}{10} \right)^{n_1} \left( \frac{10}{r_0} \right)^{n_2} \left( \frac{70}{r} \right)^{n_3}, & \text{for } r > 70 \text{ km}
\end{cases} \]

(3)

The hinge distances are selected on visual inspection of the empirical curves in Figs 8(a) and (b) and performing preliminary trial-and-error regressions to minimize the residuals. The best least-squares solution corresponds to \([n_1; n_2; n_3] = [1.08; 1.64; 0.64]\), as shown in Fig. 8(b) together with the empirical data. This model predicts strong attenuation up to 70 km; beyond this distance the decay rate decreases, since the signals are dominated by reflected and refracted waves that attenuate slower. AME11 already suggested a distance decay in epicentral area described as \(r^{-3}\) and interpreted either as an effect of \(S\) waves travelling through the fault zone of the L’Aquila main shock, or to the geometrical spreading of the near-field terms.

As discussed in Yenier & Atkinson (2015), a geometrical spreading faster than \(r^{-1}\) at short and intermediate distances is not an uncommon feature, considering that the real propagation medium is far from to be homogenous. A strong geometrical spreading at short and intermediate distances is observed in other worldwide regions, such as Western North America (Atkinson 2004; Babaie Mahani & Atkinson 2012) and Central and Eastern America (Yenier & Atkinson 2015). Numerical simulations in a typical layered medium also indicate that the distance decay in the epicentral area can be described by geometrical spreading with exponent greater than 1 (Frankel 1991; Chapman & Godbee 2012).

The entire set of non-parametric curves corrected for the geometrical spreading given in eq. (3) are used to find the attenuation parameters \(Q(f) = Q_o f^\alpha\) and \(k\), fitting the following model:

\[ AK(f) = \begin{cases} 
\exp \left( \frac{-\pi f (r - r_0)}{\rho U_0 k r} \right) \exp (-\pi k (f - 10)), & \text{for } f \geq 10 \text{ Hz} \\
\exp \left( \frac{-\pi f (r - r_0)}{\rho U_0 k r} \right), & \text{otherwise}
\end{cases} \]

(4)

Taking the logarithm of both sides of eq. (4), we evaluate the \(Q_o\), \(\alpha\) and \(k\) values through a least-squares regression in the frequency range 0.4–25 Hz. The obtained best-fit model is given by:

\[ Q(f) = (290 \pm 3) f^{0.16 \pm 0.01} \text{ and } k = (0.012 \pm 0.001) \]

(5)
Due to the trade-off between geometrical spreading and anelastic attenuation, a meaningful comparison of our $Q$-frequency relations with other studies is only possible when the estimates of $Q$ are made using the same distance attenuation function. For this reason, in Fig. 8(c) and Table 2 we compare the whole attenuation model, that is, composed by geometrical spreading, quality factor and high-frequency terms as inferred in this work, with those from previous studies for L’Aquila region.

Within 50 km from the hypocentre, our model predicts similar attenuation rate to BIN09, although the quality factor and the distance decay are parametrized in different way. In comparison to MAL11, the main differences are observed for low frequencies and at distances larger than 20 km, where our model attenuates faster.

Source spectra

In order to derive the source parameters (i.e. seismic moments and corner frequencies) of the earthquakes in our data set, we fit the non-parametric acceleration source spectra to the omega-square model (Brune 1970):

$$ S(f) = \frac{\bar{R}_{\theta\phi} F V}{4\pi \rho \beta r_0} M_0 (2\pi f)^2 \frac{1}{1 + \left( \frac{f}{f_c} \right)^2} $$

(6)

where $\bar{R}_{\theta\phi}$ is the average radiation pattern for $S$ waves, in this study set to 0.55; $V = \sqrt{2}$ is the partition of $S$-wave energy into the two horizontal components; $F = 2$ is the free-surface amplification and $\rho = 2.8$ g cm$^{-3}$ is the density; $\beta$ is the shear wave velocity; $r_0$ is the reference distance in kilometres; $M_0$ denotes the seismic moment and $f_c$ is the corner frequency. For each event, we derive $M_0$ and $f_c$ through a non-linear regression by applying an iterative least-squares algorithm. $M_0$ is then computed from seismic moment using the relation of Hanks & Kanamori (1979). Finally, the seismic
moment and the corner frequency are used to determine the stress drop $\Delta \sigma$ using standard relationships (Brune 1970):

$$\Delta \sigma = \frac{7M_0}{16R^7},$$

where

$$R = \frac{2.34 \cdot \beta}{\pi f_c},$$

is the source radius.

### Seismic moment

The non-parametric source functions obtained from the inversion are shown in Fig. 9 for six events approximately spanning the considered magnitude range. Each spectrum is compared with the corresponding omega-square model showing that the non-parametric spectra can be satisfactorily explained using Brune’s source model, with very few exceptions (e.g. event 090409131936 in Fig. 9a). Concerning the main shock of the L’Aquila seismic sequence (event 090406013242), the empirical source function shows a bump in the frequency band 0.5–5 Hz. This shape could be explained by the presence of forward directivity effects at stations located to the southeast of the epicentre (Ameri et al. 2012). The observed spectra, corrected for attenuation and site functions inferred from the inversion (Fig. 9b), confirm the asymmetric distributions of spectral amplitudes with the largest values in the southeastern region with respect to the epicentre. CAL 13 and Calderoni et al. (2015) identified directivity effects in other events of L’Aquila sequence by applying the spectral ratio technique. However, these directivity effects are not observed in the source spectra as inferred by GIT since the non-parametric curves represent a sort of average source spectra as seen by the network. Since we selected only well-sampled events (i.e. earthquakes recorded by at least 10 stations) and with a good azimuthal coverage, differences in the spectral amplitudes at each single station due to directivity effects are expected to be averaged out, allowing for a robust estimation of the corner frequencies and seismic moment.
Following Oth et al. (2011), we verify if the empirical source spectra are affected by the effect of the average high-frequency decay $k$ through the fit to the Anderson & Hough (1984) model, for frequencies higher than 10 Hz. The obtained values are normally distributed with an average $k = -0.003 \pm 0.01$ s. This means that the high-frequency decay does not affect the source spectra and is included in the attenuation and the site terms, an observation that has also been made in a range of other studies using this methodology (e.g. Oth & Kaiser 2014).

The computed moment and local magnitudes are used to investigate the $M_w-M_l$ scaling (Fig. 10). A continuous positive curvature is expected when the scaling between $M_w$ and $M_l$ is analysed over a broad magnitude range (Bakun 1984; Hanks & Boore 1984). Therefore, a quadratic term is generally considered to fit the scaling over a broad magnitude range with a single relation (e.g. Ben-Zion & Gomberg 2014). For common events, the $M_w$ estimates of this study are compared to the moment magnitudes $M_w^{\text{GIT}}$ computed by Herrmann et al. (2011) using a moment tensor analysis (blue crosses in Fig. 10a). For magnitudes smaller than 4.8, the $M_w^{\text{GIT}}$ is, on average, 0.30 units smaller than the $M_w$ from GIT, while the two moment magnitude estimates agree for larger earthquakes. Differences of the same magnitude order (about 0.22) were also discussed by Scognamiglio et al. (2010) and Herrmann et al. (2011) comparing their results with the moment magnitude estimates by Pondrelli et al. (2010) using a regional moment tensor analysis.

Fig. 10 reports the difference between $M_w$ and $M_l$ versus $M_l$ for this study, compared to the differences obtained from empirical relationships derived for other regions. To this aim, we consider the models derived in a wide $M_l$ range (lower than 1 and up to 5.9) for Switzerland (Goertz-Allmann et al. 2011), European continental regions (Grüenthal et al. 2009) and northwestern Turkey (Parolai et al. 2007). Since a systematic vertical offset can appear when different models are compared, even for the same region, due to differences in some model parameters (e.g. velocity model or earthquake locations), in Fig. 10(b) the models from other regions are arbitrarily shifted to make the comparison among the magnitude scaling of the different relations easier.

For $M_l > 4.5$, the $M_w-M_l$ scaling in the L’Aquila region is almost flat (in the $M_l$ range 4.5–6, $M_w$ varies from 4.59 to 6.06), in agreement with the scaling observed for the other regions shown in Fig. 10. For smaller magnitudes, the curvature of the $M_w-M_l$ scaling obtained in this study is significantly stronger than the one observed for Switzerland and central Europe. On the other hand, when compared to the scaling obtained for another seismic sequence occurred in a shallow crustal active region (i.e. the 1999 Izmit earthquake in northwestern Turkey), the two scaling behaviours reasonably agree. Although a more representative statistical sample is needed to draw any definitive conclusions, Fig. 10(b) suggests that regional dependences could affect the $M_w-M_l$ scaling.

### Source scaling of the L’Aquila sequence

To investigate the source properties in terms of stress drop, we restrict the analysis to the epicentral area of the sequence and select 231 events in a spatial window with latitude range 42.0–42.8 and longitude range 12.6–13.6.

Stress drops were calculated through eqs (7) and (8), assuming a variation of $S$-wave velocity with depth. The adopted velocity–depth function is the 1-D model by Bianchi et al. (2010), obtained from receiver function analyses and used by Ameri et al. (2012) to simulate the recorded ground motions of L’Aquila main shock. The estimated stress drops have a mean equal to 2.6 MPa, with the bulk of the observations ranging between 1 and 5 MPa (Fig. 11a), and can be described by a lognormal distribution with mean 0.25 and standard deviation 0.32 in log units. The stress-drop range is comparable to what was observed for single crustal seismic sequences...
in other regions of the world, for instance in Japan (Oth 2013) and Canterbury, New Zealand (Oth & Keiser 2014), in contrast with the large scatter observed for global data (Allmann & Shearer 2007), where the stress drops span up to 4 order of magnitude.

The range of our estimates is also in agreement with BIN09, MAL11 and CAL13, while the range of variability of AME11 is the largest (Table 3). We compare the stress drops obtained with the EGF approach (CAL13) with the ones obtained in this study (Fig. 11b) and find a persistent bias, roughly independent on magnitude. The EGF approach is based on single station spectral ratios, and therefore stress drops of events are relative one to the other, as absolute values are obtained fixing the stress drop of a reference event. CAL13 selected the event of 2009 April 7 (12:29, $M_w$ 3.59) as reference. The stress drop of this event is 1.3 MPa (see their Table 2), approximately half of the value obtained in this study (2.16 MPa). The mean bias (0.26) equals the log of the ratio between the two estimates for the reference event.

Fig. 12 shows the stress-drops plotted versus hypocentral depth and moment magnitude: despite the large scatter, a depth-dependence of the average stress drop can be observed, with increasing average values for depths shallower than 6 km, between 6 and 10 km and deeper than 10 km. A significant increase of stress drop with magnitude is also observed with an average value of 1 MPa at $M_w$ 3 and 10 MPa at $M_w$ 5.8.

Earthquake source scaling has been a persistently controversial issue over the past decades. Aki (1967) was the first to hypothesize the self-similarity of earthquakes, implying that the stress drop is constant independent of earthquake size. Large data sets from different seismotectonic regions usually do not show a breakdown in self-similarity (e.g. Shearer et al. 2006; Allmann & Shearer 2007; Oth et al. 2010), yet studies on single seismic sequences often provide indications that smaller events have lower stress drops than larger ones (e.g. Mayeda & Malagnini 2010; Mereu et al. 2013; Malagnini et al. 2014). For the Colfiorito (1997–1998) and L’Aquila (2009) seismic sequences, Malagnini et al. (2008, 2011) found a clear departure from self-similarity, given by stress drops around 22 MPa for the largest events ($M_w$ 6.0) and around 5 MP for smaller magnitudes (e.g. $M_w$ 4.0). Similarly, CAL13 recognized an increasing

<table>
<thead>
<tr>
<th>$M_w$</th>
<th>Number of event</th>
<th>$\Delta\sigma$ (MPa)</th>
<th>Method</th>
</tr>
</thead>
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<td>AME11</td>
<td>3.0–5.3</td>
<td>112</td>
<td>0.3–60.0</td>
</tr>
<tr>
<td>BIN09</td>
<td>4.1–6.3</td>
<td>13</td>
<td>2.4–16.8</td>
</tr>
<tr>
<td>MAL11</td>
<td>2.8–6.1</td>
<td>170</td>
<td>1.0–15.0</td>
</tr>
<tr>
<td>CAL13</td>
<td>3.3–6.1</td>
<td>64</td>
<td>0.5–10.0</td>
</tr>
<tr>
<td>This study</td>
<td>3.3–5.9</td>
<td>231</td>
<td>0.2–25.0</td>
</tr>
</tbody>
</table>

**Table 3.** Moment magnitude and stress-drop variability for L’Aquila region (GIT: generalized inversion technique; EGF: empirical Green’s function technique).
Stress-drop models

We investigate the source scaling using two models for the stress-drop dependency on seismic moment. In the first model (Model A), the stress drop is assumed to be linearly dependent on the seismic moment, that is:

$$\log_{10}\Delta\sigma = a + b \log_{10} M_0,$$

where $a$ and $b$ are the regression coefficients.

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where $a$ and $b$ are the regression coefficients.

In order to account for the features observed in Fig. 12, we test also a model including both the stress-drop dependence on depth and its dependence on seismic moment through a piecewise linear function on logarithmic scales. We follow a mixed effect approach (e.g. Pinheiro & Bates 2000), where the depth is categorized into three intervals, indicated with $k = D1, D2, D3$ and introduced in the model as random effect over the offset parameter in the form

$$\log_{10}\Delta\sigma = f (M) + \delta_k + \varepsilon,$$

where

$$f (M) = \begin{cases} 
  a & \text{for } M_0 < M_0^{ref} \\
  a + b \log_{10}(M_0/M_0^{ref}) & \text{otherwise}
\end{cases}$$

In eq. (12), $a$ (offset) and $b$ (slope) are the fixed-parameter of the model, $M_0^{ref}$ is the hinge seismic moment used to define the piecewise linear function (i.e. a constant stress-drop model below the hinge magnitude and a linear increasing model above the hinge magnitude). In eq. (11), $\delta_k$ are the random effects on the offset parameter $a$ and $\varepsilon$ is the residual distribution. The random effects depend on three a-priori selected depth categories $D1, D2, D3$. After testing several combinations of depth categories and threshold magnitude, we adopt a regression model (Model B) defined by a hinge seismic moment corresponding to $M_0 = 3.75$ and by the
Figure 15. Site amplification functions for 12 selected sites. Empirical site amplifications obtained from the GIT for horizontal (black thick line) and vertical (black dotted lines) components and from $H/V$ ratio (white lines) are plotted. The grey shaded areas indicate the mean and the standard deviations of the $H/V$ from earthquakes recordings. The number of records for each station is also reported.
Figure 16. Comparison of observed and simulated Fourier spectra (FAS) for 5 events (see Table 5) and 15 sites. Red curve: observed spectra (geometrical mean of horizontal components). Thin grey lines: theoretical FAS computed with eq. (11) and the Monte Carlo method. Thick black and dashed lines: mean ± 1 standard deviation of theoretical spectra. Thick grey line indicates theoretical FAS computed assuming constant stress drop (2.6 MPa).
following three depth categories: $D_1 = [0–6]$, $D_2 = [6–10]$, $D_3 \geq 10$ km. Inversion results for Model B reported in Fig. 14 shows that the depth grouping has a clear effect on the offset $\alpha$. The regression results and the standard deviation of Models A and B are listed in Table 4. The introduction of the random effect depending on the hypocentral depth improves the fit of the regression, reducing the standard deviation of the residuals by about 10 per cent. The results indicate that the source scaling for the L’Aquila region can be described by a model which includes a stress-drop dependence on seismic moment for magnitude larger than 3.75, with an overall offset controlled by the hypocentral depth.

Site functions

Examples of site response functions that result from the non-parametric inversion are displayed in Fig. 15, whereas the entire set of amplification functions is reported in the Electronic supplement. When independent estimates were available from other studies, we also verified the consistency of the results. This is the case of stations AQV and AQK (Puglia et al. 2011) and NOR (Bindi et al. 2011a), for which the amplification levels and the fundamental frequencies are in agreement with the results from standard spectral ratios.

The site functions are also compared with $H/V$ ratios computed directly from the FAS used in the non-parametric inversion. On average, when no amplification on the vertical component occurs, the two estimates agree in term of shapes (e.g. FIAM, AQV, CAFR, TERO). On the other hand, when the vertical ground motion is amplified, the two curves can significantly differ and the $H/V$ curves cannot be used as estimators of the site response.

Examples are stations AVZ, NOR, AQK and TRTR for amplification of the vertical at low-frequency, GUMA and OFFI for amplification at intermediate frequency, RM16 and CING for vertical amplification at high frequency. As already observed by other authors (Ameri et al. 2011; Bergamaschi et al. 2011), the empirical functions found in this study confirm the large variability of the site response in the Abruzzo region that cannot be taken into account using simple site-classification based on geological information (Table 1). Furthermore, this study also confirms the relevance of the amplification on the vertical motions, indicating that in the L'Aquila region, the $H/V$ ratio should be used with caution for evaluating the local site response.

The site functions are also compared with $H/V$ ratios computed directly from the FAS used in the non-parametric inversion. On average, when no amplification on the vertical component occurs, the two estimates agree in term of shapes (e.g. FIAM, AQV, CAFR, TERO). On the other hand, when the vertical ground motion is amplified, the two curves can significantly differ and the $H/V$ curves cannot be used as estimators of the site response.

SPECTRAL PARAMETERS FOR GROUND MOTION PREDICTION

The reliability of the spectral models used to parametrize attenuation $A(f, r)$ and the source spectra $S(f)$, including the stress-drop models, are tested by simulating the observed acceleration spectra for L'Aquila data set in the distance range 5–120 km. The empirical amplification functions obtained by the inversion are assumed as the response of soil sites.

The source spectra are modeled by an omega-square model with seismic moments estimated from the inversion and stress drop given by the empirical relationship in eq. (11). The frequency-dependent attenuation with distance is described by the global attenuation model of eq. (2), where the geometrical spreading is as in eq. (4). The quality factor $Q(f)$ and the high-frequency decay parameter $k$ are given in eq. (5).

In order to take into account the uncertainties associated with the model parameters, we assume quality factor and stress drop as drawn from respective lognormal distributions. The empirical amplification functions are also represented as lognormal distribution, with mean and standard deviation obtained from the bootstrap analysis applied in the non-parametric inversion. Based on these distributions, the theoretical FAS of each site–source couple is evaluated through 500 Monte Carlo simulations.

Fig. 16 shows the comparison between the geometric mean of theoretical and observed spectra for the horizontal components at several stations for the events listed in Table 5 (in the moment magnitude range 3.5–5.2 and hypocentral depths 4–12 km). We consider stations characterized by different site responses, such as stations located on rock (FIAM, LNSS, TERO and the reference site RM08), stations showing amplification over a broad frequency band (ORC, INTR), or affected by strong site effects at high (NCRA, BRS), intermediate (GUMA, AQV, FAGN) or low (NOR, AVZ, OFFI) frequencies. In Fig. 16, the theoretical FAS computed using a fixed stress drop (2.6 MPa) are also reported. In general, the simulated spectra (black curves) using an increasing stress drop with magnitude and depth fit the observations reasonably well, especially in case of the strongest events, where the improvement of using variable stress drop is more evident (e.g. earthquakes 73 and 43 in Table 5).

To evaluate the reliability of the proposed spectral models, we compute the residuals between the observed data and the simulated spectral amplitudes. The residuals distributions at 1 and 10 Hz are shown in Figs 17 and 18 for variable and constant stress-drop models, respectively. The fitted Gaussian distributions show a mean distribution around 0 with standard deviations slightly increasing with frequency, varying from 0.2 and 0.27 (Table 6). The residuals computed using a constant stress drop show a clear dependence on magnitude, confirming that a stress-drop magnitude scaling is necessary to better fit the observed data. These results indicate that the proposed source, site and attenuation models are suitable to describe the seismic motion recorded in the L’Aquila region.

CONCLUSIONS

We analysed a data set composed of more than 9000 acceleration or velocity seismic records selected in the Abruzzo region in the time period 2008 January 1–2013 May 31, covering the local magnitude range 3.0–5.8 and epicentral distances shorter than 120 km.
This data set was selected after a validation procedure implying the removal of unreliable records and the evaluation of the SNR. All events were relocated and the local magnitude estimated adopting a specific velocity model for the region. We applied a non-parametric generalized inversion to provide the complete set of spectral terms (e.g. attenuation, source parameters, source scaling and site effects) to predict ground motions in the area. Attenuation was parametrized by geometrical spreading as...
well as anelastic and scattering attenuation, while the source was interpreted using an omega-square model (Brune 1970) to estimate seismic moment and stress drop. The results were finally validated through a comparison with previous studies based on generalized inversion or other independent techniques (e.g. EGF).

The main results can be summarized as follows:

(i) Strong attenuation of $S$ waves occurs at short and intermediate distances at all frequencies. We parametrize the empirical attenuations curves in terms of geometrical spreading, anelastic attenuation and high-frequency decay parameter $k$. The geometrical spreading

Figure 18. As Fig. 17, but considering a constant stress-drop value (2.6 MPa).
term is described by a piecewise linear model with crossover distances at 10 and 70 km (eq. 3 and Table 2). In the distance range 5–70 km the exponents of the geometrical spreading are greater than 1. Similar findings are also found in other worldwide regions and interpreted as a consequence of complex propagation effects in layered propagation medium. The associated quality factor $Q(f)$ and the high-frequency attenuation parameter $k$, are $Q(f) = 290f^{0.10}$ and $k = 0.012$ s, respectively, the latter being applied only for frequencies higher than 10 Hz. The proposed model is valid for a wide magnitude and distance range ($M_0 = 3–5.8$ and $R = 5–120$ km).

(ii) The omega-square model adequately represents the source spectra and a quadratic scaling between $M$ and $f$ is proposed (eq. 9). When compared to the scaling obtained in previous studies for different regions, a regional dependence of the scaling is observed, being the curvature below magnitude 4.5 more pronounced for the two considered shallow crustal active regions (i.e. northwestern Turkey and central Italy) than the curvature observed for stable and continental regions (central and northern Europe).

(iii) Stress drop variability within the 2009 L’Aquila sequence spans two orders of magnitude ($0.1 < \Delta\sigma < 25$ MPa) and is comparable with the range observed for seismic sequences in other Italian and worldwide regions. The average value is 2.6 MPa.

(iv) The stress drop can be described by a model that includes a dependence on seismic moment for magnitude larger than 3.75, and with an offset controlled by earthquake depth (eq. 11). The coefficients of the model are reported in Table 4.

(v) The site functions obtained by GIT inversion confirm the large variability of the site response in central Italy, with significant amplification over the entire frequency band. The vertical ground motions can be strongly amplified at low or high frequencies. The amplification of vertical ground motions explains the main discrepancies between the GIT site functions and $H/V$ ratios.

The reliability of the parametric spectral models for attenuation $A(f, r)$ and source spectra $S(f)$ were tested simulating the observed acceleration spectra. To take into account the site effects, the corresponding empirical transfer functions were considered. The overall fit, evaluated through the residuals, indicates that the proposed source, site and attenuation models are suitable to describe the seismic motion recorded in the L’Aquila region.

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**REFERENCES**


**Table 6.** Standard deviations at selected frequency of residual distributions.

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<th>Model</th>
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<th>10 Hz</th>
<th>15 Hz</th>
<th>20 Hz</th>
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<tr>
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<td>0.22</td>
<td>0.23</td>
<td>0.25</td>
<td>0.27</td>
</tr>
<tr>
<td>Constant stress drop</td>
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<td>0.27</td>
<td>0.28</td>
<td>0.30</td>
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</tr>
</tbody>
</table>


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