Dynamics of Metal–Silicate Equilibration in a Terrestrial Magma Ocean

Martina Ulvrová

Nicolas Coltice , Fabien Dubuffet
Jakub Velímský , Ondřej Šrámek
Yanick Ricard , Ondřej Souček

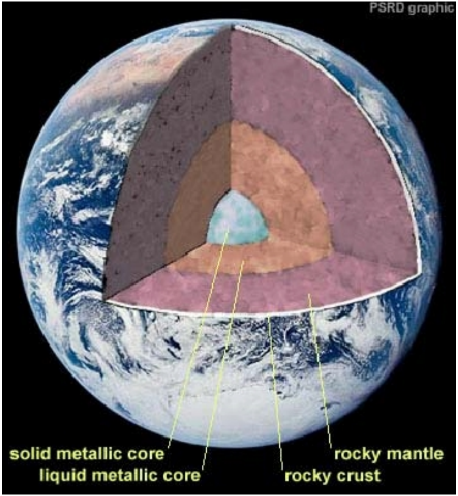
Department of Earth Sciences, University Lyon 1
Department of Geophysics, Faculty of Mathematics and Physics, Charles University in Prague

March 5, 2008

Outline

- Motivation
- Introduction - core formation and the early Earth
- Problem formulation
- Numerical realization
- Experiments and key parameters
- Prospects

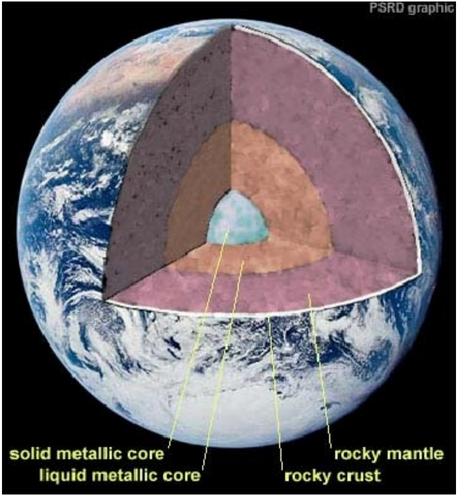
Motivation



- Understanding of the process of core formation
- The metal droplet scenario

(Stevenson, Origin of the Earth, 1990)

Motivation



- Understanding of the process of core formation
- The metal droplet scenario

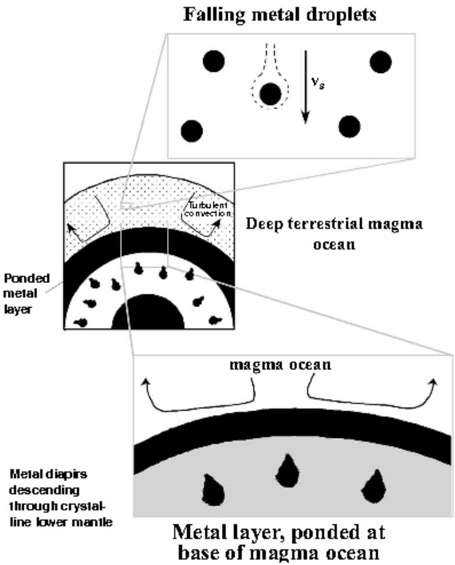
(Stevenson, Origin of the Earth, 1990)

Primitive Earth

- Accreted from chondritic material
- Magma ocens were episodicaly formed
- Early differentiation involved the separation of silicate an Fe-rich metal
- Magma ocean condition
  - pressure $30 - 60\ GPa$
  - temperature $> 2200\ K$

  (Chabot, Geochimica et Cosmoch., 2005)

Core formation scenario



Chemical constraints

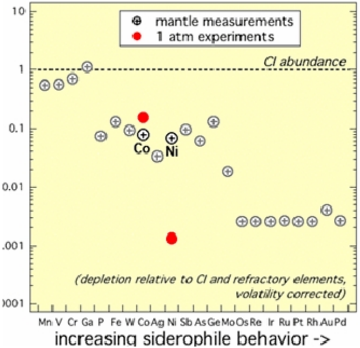


Figure: Concentration of siderothetale elements in Earth's mantle.

(Taylor, Planet. Sc. Research, 2005)

Formulation - The model

Basic assumptions

- Coexistance of two immiscible liquids of Fe–alloy and silicate
- Stationary situation $\Rightarrow$ no acceleration
- Axisymmetrical problem $\Rightarrow$ two independent variables (θ, r)

Formulation - The model

Basic assumptions

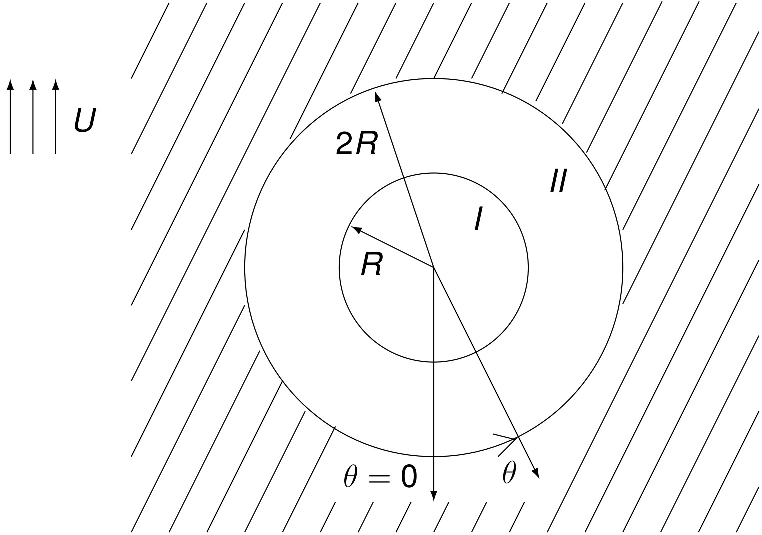
- Coexistance of two immiscible liquids of Fe–alloy and silicate
- Stationary situation $\Rightarrow$ no acceleration
- Axisymmetrical problem $\Rightarrow$ two independent variables (θ, r)

Formulation - The model

Basic assumptions

- Coexistance of two immiscible liquids of Fe–alloy and silicate
- Stationary situation $\Rightarrow$ no acceleration
- Axisymmetrical problem $\Rightarrow$ two independent variables (θ, r)

Geometry of a falling drop



Basic equation

Conservation equations for chemical exchange

- in metal

$$\frac{\partial C^I}{\partial t} = -\nabla \cdot \boldsymbol{F}^I = \nabla \cdot (D_{Fe} \nabla C^I - \boldsymbol{v}^I C^I)$$

- in silicates

$$\frac{\partial C^{II}}{\partial t} = -\nabla \cdot \boldsymbol{F}^{II} = \nabla \cdot (D_{Si} \nabla C^{II} - \boldsymbol{v}^{II} C^{II})$$

Velocity field

Conservation equations

- in metal

$$\nabla \cdot \boldsymbol{v}^{I} = 0$$

$$-\nabla P^{I} + \mu_{Fe} \nabla^2 \boldsymbol{v}^{I} = \boldsymbol{0}$$

- in silicates

$$\nabla \cdot \boldsymbol{v}^{II} = 0$$

$$-\nabla P^{II} + \mu_{Si} \nabla^2 \boldsymbol{v}^{II} = \boldsymbol{0}$$

Boundary condition

$$v_r^{II} = v_r^{I} = 0$$

$$v_\theta^{II} = v_\theta^{I}$$

$$\sigma_{r\theta}^{II} = \sigma_{r\theta}^{I}$$

Velocity field

In the blob

$$v_r^I = \left(1 - \frac{r^2}{R^2}\right) \frac{U\mu^E \cos\theta}{2(\mu^I + \mu^E)}$$

$$v_\theta^I = \left(\frac{2r^2}{R^2} - 1\right) \frac{U\mu^E \sin\theta}{2(\mu^I + \mu^E)}$$

In the magma

$$v_r^{II} = \left(-1 - \frac{\mu^I}{2(\mu^I + \mu^{II})}\frac{R^3}{r^3} + \frac{2\mu^I + 3\mu^{II}}{2(\mu^{II} + \mu^I)}\frac{R}{r}\right) U\cos\theta$$

$$v_\theta^{II} = \left(1 - \frac{\mu^I}{4(\mu^I + \mu^{II})}\frac{R^3}{r^3} - \frac{2\mu^I + 3\mu^{II}}{4(\mu^{II} + \mu^I)}\frac{R}{r}\right) U\sin\theta$$

Falling velocity

$$U = \frac{2(\rho^E - \rho^I)gR^2}{3\mu^E}\frac{\mu^E + \mu^I}{2\mu^E + 3\mu^I}$$

Streamlines

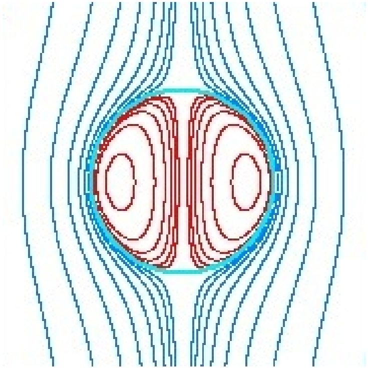


Figure: Streamlines for steady state.

Boundary Condition

- Interface

$$[\boldsymbol{F} \cdot \boldsymbol{n}]_{-}^{+} = 0 \rightarrow \left[D\frac{\partial C}{\partial r}\right]_{-}^{+} = 0$$

$$K = \frac{C^{I}}{C^{II}}$$

- External boundary

$$\theta \in (0, \frac{\pi}{2}) \rightarrow C(t) = const$$

$$\theta \in (\frac{\pi}{2}, \pi) \rightarrow (\boldsymbol{v} \cdot \nabla C) = 0$$

Time approach - ADI

- Based on Crank–Nicolson scheme with RHS

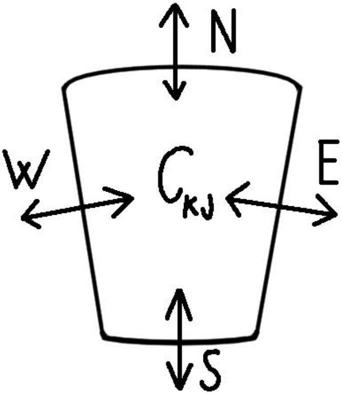
$$\partial_t C = \Lambda C = (\Lambda_r + \Lambda_\theta) C$$

- Solved by Peaceman–Rachford scheme

$$(1 - \frac{\Delta t}{2}\Lambda_r) C^{n+\frac{1}{2}} = (1 + \frac{\Delta t}{2}\Lambda_\theta) C^n$$

$$(1 - \frac{\Delta t}{2}\Lambda_\theta) C^{n+1} = (1 + \frac{\Delta t}{2}\Lambda_r) C^{n+\frac{1}{2}}$$

Spatial approach – finite volume formulation



Time shots

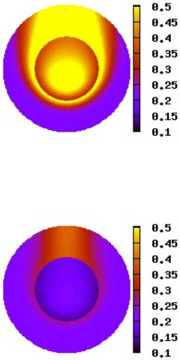


Figure: $u_{int} = 1., u_{ext} = 0.4, u_{out} = 0.2, K = 0.5, H_C = 0.$

Experiments

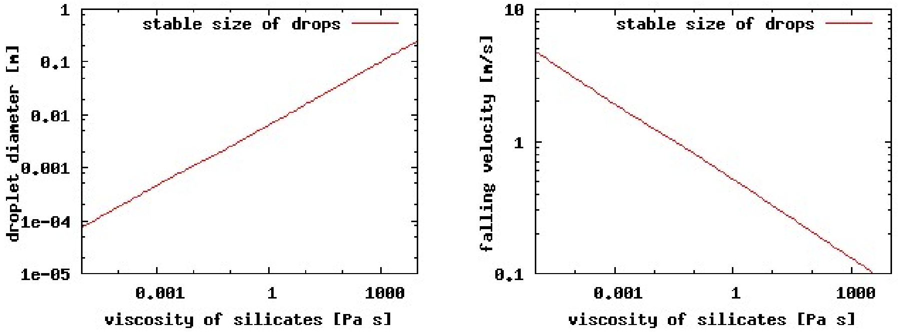
1. $K = const$
   (in the beginning the drop is not in equilibrium)
2. $K = K(depth)$

Experiments

1. $K = const$
   (in the beginning the drop is not in equilibrium)
2. $K = K(depth)$

Stable size of droplets

- $We = \frac{\rho^E R U^2}{2\sigma}$
- $We_c = 4\pi(1 + \frac{1}{S}) \quad , S = \frac{\rho^l}{\rho^E}$



Experiment nb° 1 – Dimensionless form

- $r = Rr^*$
- $\boldsymbol{v} = U\boldsymbol{v}^*$
- $C = C_0 C^*$
- $t = t_0 t^* = \frac{R}{U} t^*$

$$\frac{\partial C^*(r^*, \theta^*, t^*)}{\partial t^*} = \frac{1}{Pe} \nabla^2 C^*(r^*, \theta^*, t^*) - \boldsymbol{v}^{II\,*} \cdot \nabla C^*(r^*, \theta^*, t^*)$$

$$\frac{\partial C^*(r^*, \theta^*, t^*)}{\partial t^*} = \frac{D_{Si}}{D_{Fe}} \frac{1}{Pe} \nabla^2 C^*(r^*, \theta^*, t^*) - \boldsymbol{v}^{I\,*} \cdot \nabla C^*(r^*, \theta^*, t^*)$$

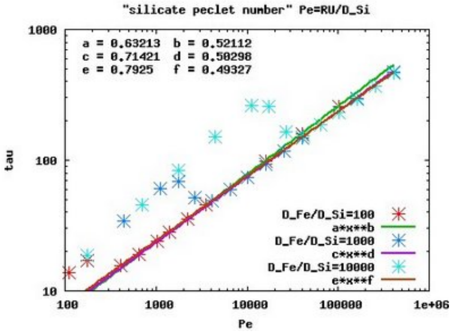
- $Pe = \frac{UR}{D_{Si}}$

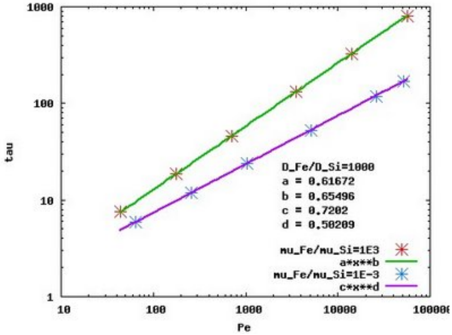
Experiment nb° 1

$$C(t) \sim e^{-\frac{t}{\tau}}$$

two regimes

- $\mu_{Si} > \mu_{Fe} \rightarrow \tau \sim \sqrt{Pe}$
- $\mu_{Si} < \mu_{Fe} \rightarrow \tau \sim Pe^{2/3}$





Future work

- Investigating of experiment with constant K.
  - Turn more runs.
  - Find an analytical solution for evolution of concentration.
  - Describe an influence of crutial parameters.
- Experiment with $K = K(depth)$
- Investigation of the shape of droplets

Future work

- Investigating of experiment with constant K.
  - Turn more runs.
  - Find an analytical solution for evolution of concentration.
  - Describe an influence of crutial parameters.
- Experiment with $K = K(depth)$
- Investigation of the shape of droplets

Future work

- Investigating of experiment with constant K.
    - Turn more runs.
    - Find an analytical solution for evolution of concentration.
    - Describe an influence of crutial parameters.
- Experiment with $K = K(depth)$
- Investigation of the shape of droplets

Thank you.