COMPRESSIBLE CONVECTION IN MODELS WITH COMPLEX MINERALOGY

Hana Čížková
Arie van den Berg
PROJECT EVOLUTION

- 2013: subduction model with complex mineralogy, iron spin transition effect
  
  EBA results suggest strong effects of $\alpha(p,T)$
  but: compressible model needed

- 2014 implementation of TALA + benchmarks

- 2015 evaluation of compressible effects
  - application to large exoplanets

- 2016 back to subduction model?
BASIC EQUATIONS (basal heating)

Extended Boussinesq approximation (EBA)
\[ \nabla \cdot \vec{v} = 0 \]
\[ \nabla \cdot \vec{\sigma} - \nabla p + \rho \ddot{g} = 0 \]
\[ \sigma_{ij} = \eta \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \]
\[ \rho_0 c_p \frac{DT}{Dt} = \nabla \cdot (k \nabla T) - \alpha_0 T g \rho_0 \nu_r + \vec{\sigma} : \nabla \dot{v} \]
\[ \rho = \rho_0 (1 - \alpha(T - T_0)) \]

Truncated anelastic liquid approximation (TALA)
\[ \nabla \cdot (\bar{\rho} \dot{v}) = 0 \]
\[ \nabla \cdot \vec{\sigma} - \nabla p + \rho \ddot{g} = 0 \]
\[ \sigma_{ij} = \eta \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \nabla \cdot \dot{v} \delta_{ij} \right) \]
\[ \rho c_p \frac{DT}{Dt} = \nabla \cdot (k \nabla T) - \alpha \rho g T \nu_r + \vec{\sigma} : \nabla \dot{v} \]

Equation of state:
tabulated \( \rho(p_h,T) \) computed by multi-einstein lattice dynamics method (Jacobs et al., 2013)
CODE BENCHMARKS

1. Analytical solution – 1D channel flow

Heat balance mismatch ~ 1-5%

Reported in TALA models (Jarvis & McKenzie, 1980; Leng & Zhong, 2008; Nakagawa & Tackley, 2010)

Leng & Zhong (2008): consequence of truncated approximation
   - missing dynamic pressure effect in momentum equation
MATERIAL PARAMETERS

Simplified mantle mineralogy – magnesium endmember olivine chemical composition \((\text{Mg}_2\text{SiO}_4)\)
Low-pressure polymorphes \((\alpha,\beta,\gamma \text{ olivine})\) not included
→ Mixture MgO-pv(ppv)

ρ \((\text{kgm}^{-3})\) \hspace{1cm} α \((\text{K}^{-1})\) \hspace{1cm} c_p \((\text{JK}^{-1}\text{kg}^{-1})\)

Thermodynamically consistent
MODEL TERRESTRIAL PLANETS

- rocky planets, mass $M$
- $M/M_E = 1, 2, 4, 8$
- core mass fraction 0.315 (Earth-like)
- purely iron core, equation of state Tachinamy (2011)
- mantle equation of state Jacobs et al. (2013)

<table>
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<th>$M/M_E$</th>
<th>$d$ (km)</th>
<th>$g_{surf}$ (m/s$^2$)</th>
<th>$g_{cmb}$ (m/s$^2$)</th>
<th>$\Delta T$ (K)</th>
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MODEL TERRESTRIAL PLANETS
MODEL SETUP

- Blankenbach-type model
- Aspect ratio 1, free-slip boundaries
- Constant viscosity (+ one case with $\eta(p,T)$)
- Bottom heating
- $Ra = 10^4, 10^5, 10^6, 10^7$

- TALA – $\rho(p,T), \alpha(p,T), c_p(p,T)$

- EBA – a) constant $\alpha$
  b) depth dependent $\alpha$ (parameterised, e.g. Katsura, 2009)
  c) horizontally averaged $\alpha$ from TALA models
EXPANSIVITY MODELS
RESULTS

Comparison of heat flow efficiency (Nu) in TALA and EBA models

Could we get with EBA results close to TALA?

Could we use incompressible models as a reasonable approximation of exoplanetary mantle evolution?
EARTH-LIKE: $M_E$

**EBA $\alpha_{\text{depth}2} 10^6$**

**EBA $\alpha_{\text{aver}} 10^6$**

**TALA $10^6$**
LARGE: $M = 8 \, M_E$

EBA $\alpha_{\text{depth}} \, 10^6$

EBA $\alpha_{\text{aver}} \, 10^6$

TALA $10^6$
AVERAGE GEOTHERMS: $M = M_E$
AVERAGE GEOTHERMS: $M = M_E$
AVERAGE GEOTHERMS: $M = 8M_E$
HEAT FLUX (Nusselt number)

\[ M = M_E \]

\[ M = 8M_E \]
HEAT FLUX (Nusselt number)

TALA - solid line
EBAdepth2 - dashed line
EBAaverage2 - dotted line

Nu vs Ra for M1E, M2E, M4E, M8E
MODEL WITH VARIABLE VISCOSITY $\eta(p,T)$

EBA $\alpha_{\text{aver}} \eta(p,T) \ M8E \ 10^4$

TALA $\eta(p,T) \ M8E \ 10^4$
AVERAGE GEOTHERMS: $M = 8M_E$, $\eta(p,T)$
CONCLUSIONS

• Compressible convection results in systematically warmer geotherm
• Heat transport is more efficient in TALA models than in EBA models
• EBA with depth-dependent expansivity is/is not a reasonable approximation of compressible effects in large exoplanetary mantles