Baroclinic ocean model LSOMG

Libor Šachl, Zdeněk Martinec

1.3. 2017
Seminar topic?

- benchmarks of tidal BT models (+2 time-stepping schemes, +1 computational grid, +2 benchmarks)
- modelling of tides (BT and BC)
- computations of wind-driven ocean-induced magnetic fields
- BC ocean model LSOMG, wind- and buoyancy-driven circulation
Outline

1. Model description
2. Input data
3. Benchmarks and realistic runs
Current section

1. Model description

2. Input data

3. Benchmarks and realistic runs
Motivations for the development

- **GIA**
  - improvement of sea-level equation ⇒ paleocean modelling (bathymetry which is evolving in time)
  - long runs (≈ 20,000 years) ⇒ LSG ocean model (semi-implicit)
- **ESA project “Swarm + Oceans”**
  - present-day circulation
  - a few years of Swarm data
  - tides ⇒ precludes long time steps
Persistent model requirements

- global primitive-equation ocean model
- fully 3D (not only barotropic) ⇒ 3D velocities
- understood on a code level and open for modifications
- usable for geophysical purposes (simplified?)
General characteristics

- primitive-equation 3D ocean model
- global
- z-coordinate
- hydrostatic approximation  
  ⇒ vertical momentum equation reduced to the pressure equation  
  ⇒ vertical velocity computed diagnostically from the continuity equation
- Boussinesq approximation  
  ⇒ reference density is used except for the gravitational force
- shallow ocean (not shallow water!) approximation
<table>
<thead>
<tr>
<th></th>
<th>LSG</th>
<th>LSOMG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal grid</td>
<td>Arakawa E grid</td>
<td>Arakawa C grid</td>
</tr>
<tr>
<td>Horizontal coord.</td>
<td>Spherical</td>
<td>Generalized</td>
</tr>
<tr>
<td>Vertical grid</td>
<td>Manually generated</td>
<td>Semi-auto generated</td>
</tr>
<tr>
<td>Time stepping</td>
<td>no splitting, unstaggered</td>
<td>splitting, staggered</td>
</tr>
<tr>
<td>Barotropic part</td>
<td>implicit</td>
<td>explicit (P-C, gen. FB)</td>
</tr>
<tr>
<td>Tidal forcing</td>
<td>no</td>
<td>yes (from DEBOT)</td>
</tr>
<tr>
<td>Horiz. friction</td>
<td>“rotated” Laplacian</td>
<td>Laplacian, “full” form</td>
</tr>
<tr>
<td>Vert. friction</td>
<td>no</td>
<td>yes (explicit, implicit)</td>
</tr>
<tr>
<td>Nonlinear terms</td>
<td>no</td>
<td>yes (2 formulations)</td>
</tr>
<tr>
<td>Coriolis term</td>
<td>implicit</td>
<td>Adams-Bashforth</td>
</tr>
<tr>
<td>Advection scheme</td>
<td>QUICK</td>
<td>QUICK, LW, DST3</td>
</tr>
<tr>
<td>Programming</td>
<td>F77 fixed format</td>
<td>F90 free format</td>
</tr>
<tr>
<td>Parallelization</td>
<td>no</td>
<td>MPI, (OpenMP)</td>
</tr>
</tbody>
</table>
Acronyms

**LSG** = Large Scale Geostrophic ocean model

**LSOMG** = Large Scale Ocean Model for Geophysics

= Libor Šachl Ocean Model for Geophysics

= Libor Šachl? Oh My God!
Baroclinic ocean model LSOMG

Model description

Primitive equations

\[ u_{,t} + \nabla \cdot (v \otimes v) = -f z \times v - \frac{\nabla_h p}{\rho_0} + F_H + F_V + F_{tid}, \]

\[ p_{,z} = -\rho g, \]

\[ \nabla \cdot v = 0, \]

\[ \eta_{,t} = -\nabla_h \cdot U, \]

\[ C_{,t} + \nabla \cdot (v C) = \nabla \cdot F^C, \]

\[ \rho = \rho(\theta, S, p) \]
Figure: Original Arakawa E-grid (left) and present C-grid (right).
### Perfect E grid?

<table>
<thead>
<tr>
<th>Grid</th>
<th>Coriolis</th>
<th>Pressure gradient</th>
<th>Divergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>B</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>C</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>D</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>E</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

**Table:** Presence of averaging in the discretization of Coriolis, pressure gradient and divergence terms on Arakawa grids A-E.
Figure: Horizontal velocity after 3 days of run. Real bathymetry, nonzero horizontal viscosity.
Figure: Two available distributions of grid points in the vertical.
Figure: Depths (left) and thicknesses (right), 11 layers, analytic form of Madec (2012).
Figure: Depths (left) and thicknesses (right), 22 layers, analytic form of Madec (2012).
Original LSG time stepping

\[ u', \bar{u}, \eta, C \]

Figure: Unstaggered two level time stepping.
**Time stepping in LSOMG**

**Figure:** Staggered time stepping method of Griffies (2004).

**Order:** $\eta, C, \bar{u}, u'$
Friction in ocean models

Griffies (2004):

- Dissipation of $E_k$ is small in the real ocean but huge in models.
- 2 central roles of friction:
  - Parametrization of the effect unresolved scales on the resolved scales.
  - Ad-hoc numerical closure:
    - As low as possible keeping the model stable.
    - Tuning.
Stress-strain rate relation

- generalized Hooke’s law + symmetries
  \[ \frac{\sigma}{\rho} = A : e \]
- transverse isotropy \( \Rightarrow 5 \) coefficients
- \( \nabla \cdot \mathbf{v} = 0 \)
- \( Tr(\sigma) = 0 \)
- quasi-hydrostatic approximation (stable stratification or close to it)
Horizontal friction

\[ F_{H1} = \nabla_h \cdot (A_H \nabla_h u) + \left( M_1 \frac{\partial v}{\partial \xi_1} - M_2 \frac{\partial v}{\partial \xi_2} \right) + (N_1 u + N_2 v) \]

\[ F_{H2} = \nabla_h \cdot (A_H \nabla_h v) - \left( M_1 \frac{\partial u}{\partial \xi_1} - M_2 \frac{\partial u}{\partial \xi_2} \right) + (N_1 v - N_2 u) \]

(Murray and Reason, 2001)
Horizontal friction

\[ F_{H1} = \frac{1}{h_1 h_2 h_3} \left[ \frac{1}{h_2} \frac{\partial}{\partial \xi_1} (h_2^2 h_3 A_H D_T) + \frac{1}{h_1} \frac{\partial}{\partial \xi_2} (h_1^2 h_3 A_H D_S) \right] \]

\[ F_{H2} = \frac{1}{h_1 h_2 h_3} \left[ \frac{1}{h_2} \frac{\partial}{\partial \xi_1} (h_2^2 h_3 A_H D_S) - \frac{1}{h_1} \frac{\partial}{\partial \xi_2} (h_1^2 h_3 A_H D_T) \right] \]

\[ D_T = \frac{h_2}{h_1} \frac{\partial}{\partial \xi_1} \left( \frac{u}{h_2} \right) - \frac{h_1}{h_2} \frac{\partial}{\partial \xi_2} \left( \frac{v}{h_1} \right) \]

\[ D_S = \frac{h_2}{h_1} \frac{\partial}{\partial \xi_1} \left( \frac{v}{h_2} \right) + \frac{h_1}{h_2} \frac{\partial}{\partial \xi_2} \left( \frac{u}{h_1} \right) \]

(Murray and Reason, 2001; Einspigel and Martinec, 2015)
Smagorinsky viscosity

\[ A_H = \left( k_H \frac{d}{\pi} \right)^2 \sqrt{D_T^2 + D_S^2} \]

where

\[ d = \min(\Delta x, \Delta y) \]

Griffies and Hallberg (2000):

\[ k_H \in \langle 3, 4 \rangle \]

Our settings:

\[ k_H = 4 \]
Smagorinsky viscosity

CFL: \[ A_H < \frac{d^2}{4\Delta t} \]

grid Reynolds number: \[ A_H > \frac{Ud}{2} \]  (Bryan et al., 1975)

Munk layer: \[ A_H > \beta \left( \frac{\sqrt{3}}{\pi} d \right)^3 \]  (Munk, 1950)

where \[ \beta = \frac{1}{R} \frac{\partial f}{\partial \phi} = \frac{2\Omega \cos \phi}{R} \]

Our settings:
\[ U = 0.4 \text{ m/s} \]
Vertical friction

\[ F_v = \frac{\partial}{\partial z} \left( A_v \frac{\partial u}{\partial z} \right) \]

Boundary conditions:

\[ A_v \frac{\partial u}{\partial z} = \frac{\tau_{\text{wind}}}{\rho_0} \] at the surface

\[ A_v \frac{\partial u}{\partial z} = \tau_{\text{bottom}} \] at the bottom

where

\[ \tau_{\text{wind}} = \rho_{\text{air}} C_D |u_{10}| u_{10} \]

\[ \tau_{\text{bottom}} = C_g |u| u \]
Baroclinic ocean model LSOMG

Model description

Chosen values

\[ \rho_{air} = 1.3 \, \text{kg/m}^3, \]
\[ C_g = 2.5 \times 10^{-3} \]

Drag coefficient \( C_D \):  
- constant, e.g.: \( 1 \times 10^{-3} \) (Timmermann et al., 2009)  
- dependent on the wind speed, e.g.:  
  \[ 6 \leq |u_{10}| \leq 26 \, \text{m/s} \quad \Rightarrow \quad C_D = \left(0.60 + 0.071 |u_{10}| \right) / 1000 \]
  \[ 3 \leq |u_{10}| \leq 6 \, \text{m/s} \quad \Rightarrow \quad C_D = \left(0.29 + \frac{3.1}{|u_{10}|} + \frac{7.7}{|u_{10}|^2} \right) / 1000, \]
  \[ |u_{10}| \leq 3 \, \text{m/s} \quad \Rightarrow \quad C_D = C_D(3\,\text{m/s}) \]

(Yelland and Taylor, 1996)
Nonlinear terms

\[
\frac{\partial \mathbf{u}}{\partial t} = -f \mathbf{k} \times \mathbf{v} - \left[ \nabla \cdot (\mathbf{v} \otimes \mathbf{v}) \right]_h
\]

Flux form:

\[
\frac{\partial u}{\partial t} = + \left[ f + \frac{1}{h_1 h_2} \left( v \frac{\partial h_2}{\partial \xi_1} - u \frac{\partial h_1}{\xi_2} \right) \right] v - \nabla \cdot (\mathbf{v} u)
\]

\[
\frac{\partial v}{\partial t} = - \left[ f + \frac{1}{h_1 h_2} \left( v \frac{\partial h_2}{\partial \xi_1} - u \frac{\partial h_1}{\xi_2} \right) \right] u - \nabla \cdot (\mathbf{v} \mathbf{v})
\]

Vector invariant form:

\[
\frac{\partial \mathbf{u}}{\partial t} = -(\zeta + f) \mathbf{k} \times \mathbf{v} - \frac{1}{2} \nabla_h (\mathbf{u} \cdot \mathbf{u}) - w \frac{1}{h_3} \frac{\partial \mathbf{u}}{\partial \xi_3}
\]
Coriolis term

- energy conserving scheme (available: enstrophy conserving, energy+enstrophy conserving, 4-point average)
- all points not only wet points
- 3rd order Adams-Bashforth extrapolation
- noise $\Rightarrow$ divergence damping

$$\frac{\partial u}{\partial t} = \cdots + \nabla_h (\lambda \nabla_h \cdot u)$$

- vorticity dynamics unaffected (rotational motions unaffected)
- divergence equation has an additional diffusion term
Advection schemes

\[
\partial_t \theta + \nabla_h \cdot (u \theta) + \partial_z (w \theta) = \nabla \cdot F^\theta, \\
\partial_t S + \nabla_h \cdot (u S) + \partial_z (w S) = \nabla \cdot F^S,
\]
Advection schemes

- Lax-Wendroff scheme of Smith et al. (2010)
- Lax-Wendroff scheme with flux limiters
- DST3 with flux limiters
Flux limiters

- combine the best of both the 1st order and higher order methods
- smooth parts of the solution $\Rightarrow$ higher order (2nd, 3rd) advection
- jump or a sudden gradient $\Rightarrow$ switch to the 1st order
- TVD (total-variation diminishing)

$$TV(\theta) = \int_a^b \left| \frac{\partial \theta}{\partial x} \right| \, dx \rightarrow TV(\theta) = \sum_{i=1}^{N} |\theta_i - \theta_{i-1}|$$
Splitting method of Adcroft et al. (2014)

\[
\frac{\partial \theta}{\partial t} = -\frac{1}{h_1 h_2 h_3} \left[ \frac{\partial (h_2 h_3 u \theta)}{\partial \xi_1} + \frac{\partial (h_1 h_3 v \theta)}{\partial \xi_2} + \frac{\partial (h_1 h_2 w \theta)}{\partial \xi_3} \right]
\]

\[
\frac{\partial \theta}{\partial t} = -\partial_x(u \theta) + \theta \partial_x u
\]

\[
\theta^{n+1/3} = \theta^n - \Delta t \left[ \partial_x F_x(\theta^n) + \theta^n \partial_x u \right]
\]

\[
\theta^{n+2/3} = \theta^{n+1/3} - \Delta t \left[ \partial_y F_y(\theta^{n+1/3}) + \theta^n \partial_y v \right]
\]

\[
\theta^{n+3/3} = \theta^{n+2/3} - \Delta t \left[ \partial_z F_z(\theta^{n+2/3}) + \theta^n \partial_z w \right]
\]
Equation of state of sea water

- UNESCO formula (UNESCO, 1981)

\[ \rho(S, T, p) = \frac{\rho(S, T, 0)}{1 - \frac{p}{K(S, T, p)}} \]

- State equation of Jackett et al. (2006)

\[ \rho(S, \theta, p) = \frac{P_1(S, \theta, p)}{P_2(S, \theta, p)} \]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho = \rho(S, T, p)$</td>
<td></td>
<td>$\rho(S, \theta, p)$</td>
</tr>
<tr>
<td>Coefficients</td>
<td>41</td>
<td>25</td>
</tr>
<tr>
<td>Accuracy</td>
<td></td>
<td>improved</td>
</tr>
</tbody>
</table>
Conversion of $T$ to $\theta$ with Jackett et al. (2006)

\[ \sigma(S, \theta, p_r) = \sigma(S, T, p), \]

where $\sigma$ is a specific entropy

Solution:
- iterative Newton-Raphson technique
- initial guess $\theta_0$:
  \[ \theta_0(S, T, p, p_r) = T + (p - p_r)P(S, T, [p + p_r]), \]

where $P$ is a given polynomial
- pressure $p$ unknown but required $\Rightarrow p_0 =$ hydrostatic $\rightarrow$ apply repeatedly, refine $p$
What did I skip?

- barotropic system
- generalized horizontal coordinates
- tripolar grid
- partial bottom cells
- isopycnal mixing
- GM stirring
- convective adjustments
- sea ice
- values of vertical diffusivities/viscosities
- tides
Figure: Inner domain and halo region for the thread number 1 and the halo region for the thread number 0 (master thread).
Figure: Performance of the 0.25° BT tidally-driven version.
Figure: Performance of the 1° BC wind-driven version.
Figure: Performance of the $0.5^\circ$ BC wind-driven version.
Current section

1 Model description

2 Input data

3 Benchmarks and realistic runs
**Input data**

- **bathymetry data**
  - GEBCO or ETOPO1 data
  - resolution 1’

- **temperature and salinity data**
  - WOA 2013 data
  - horizontal resolution: 0.5° (1440 × 720 data points)
  - vertical resolution:

<table>
<thead>
<tr>
<th>Climatology</th>
<th>Number of depth levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>annual</td>
<td>102 (0–5500 m)</td>
</tr>
<tr>
<td>seasonal</td>
<td>102 (0–5500 m)</td>
</tr>
<tr>
<td>monthly</td>
<td>57 (0–1500 m)</td>
</tr>
</tbody>
</table>
Input data

- horizontal wind speed
  - NCEP/NCAR Reanalysis 1
    - resolution in time: 4-times daily, daily and monthly mean values for 1948/01/01 to present
    - horizontal resolution: 2.5°
  - ERA interim data
    - resolution in time: 4-times daily and monthly mean values for 1979/01/01 to present
    - horizontal resolution: 0.75°
Figure: NCEP/NCAR zonal wind [m/s], monthly mean for January.
Figure: ERA Interim zonal wind [m/s], monthly mean for January.
Figure: NCEP/NCAR zonal wind [m/s], LSOMG grid.
Figure: ERA Interim zonal wind [m/s], LSOMG grid.
Current section

1. Model description

2. Input data

3. Benchmarks and realistic runs
Munk problem

- BT geostrophic equations (homogeneous fluid, nonlinearities neglected)

\[ f k \times U + gh \nabla_h \eta = A_H \Delta_h U + \tau, \]
\[ \nabla_h \cdot U = 0 \]

- harmonic friction
- ocean bottom is flat
Barotropic stream function

\[
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0
\]

⇒ \exists \psi : U = \frac{\partial \psi}{\partial y}, \quad V = -\frac{\partial \psi}{\partial x}

More general case:

\[
\nabla_h \cdot \mathbf{U} = -\eta, t \approx 0
\]

⇒ \exists \psi : \mathbf{U} = \nabla \times (\psi \mathbf{k}) = -\mathbf{k} \times \nabla \psi
Munk problem, analytic solution

\[
\psi = \frac{\mathbf{k} \cdot (\nabla \times \tau)}{\beta \rho_0} (\psi_I + \psi_E + \psi_W)
\]

\[
\psi_I = d_B - d
\]

\[
\psi_E = d_M e^{\frac{d-d_B}{d_M}}
\]

\[
\psi_W = -d_B e^{-\frac{d}{2d_M}} \left[ \cos \left( \frac{\sqrt{3}d}{2d_M} \right) + \frac{1}{\sqrt{3}} \sin \left( \frac{\sqrt{3}d}{2d_M} \right) \right]
\]

where

\[
d_M = \left( \frac{A_H}{\beta} \right)^{1/3}
\]

\[
\beta = \frac{1}{R} \frac{\partial f}{\partial \phi} = \frac{2\Omega \cos \phi}{R}
\]
Frisius et al. (2009):

- idealized basin: $\lambda \in \langle -90^\circ, 90^\circ \rangle$, $\phi \in \langle -85^\circ, 85^\circ \rangle$
- flat bottom: $H = 5000$ m
- wind stress: $\tau_\lambda = -0.1 \cos(3\phi)$ Nm$^{-2}$
Figure: Theoretical stream function [Sv].
Figure: LSOMG stream function [Sv].
Ekman layer benchmark, theory

Simplified governing equations

\[ f v + A_V \frac{\partial^2 u}{\partial z^2} = 0, \]

\[ -fu + A_V \frac{\partial^2 v}{\partial z^2} = 0, \]

with surface boundary conditions

\[ A_V \frac{\partial u}{\partial z} = \frac{\tau_x}{\rho_0}, \]

\[ A_V \frac{\partial v}{\partial z} = \frac{\tau_y}{\rho_0}. \]
Ekman layer benchmark, theory

Solution:

\[ u = V_0 e^{az} \sin\left(\frac{\pi}{4} - az\right), \]

\[ v = V_0 e^{az} \cos\left(\frac{\pi}{4} - az\right). \]

where

\[ a = \sqrt{\frac{f}{2A_V}}. \]

Specially, \( z = 0 \):

\[ u = V_0 \sin\left(\frac{\pi}{4}\right), \]

\[ v = V_0 \cos\left(\frac{\pi}{4}\right). \]
Ekman layer benchmark, settings

- Depth of the bottom: $H = 200$ m
- Vertical resolution: 20 equally thick layers $\Rightarrow$ layer thickness is 10 m
- Vertical viscosity: $A_V = 0.05$ m$^2$/s
- Predicted thicknesses $D_E$ of the Ekman layer:

$$D_E = \frac{\pi}{a} = \pi \sqrt{\frac{2A_V}{f}}$$

$\Rightarrow D_E \approx 98$ m at $\phi = 45^\circ$
Figure: Horizontal velocities in 5 m, 25 m and 55 m (from top) depth.
Benchmark of horizontal advection of tracers

- Gaussian-shape and box distribution of tracers
- Resolution: 1°
- Horizontal velocity: $u = 1.1$ m/s
- Vertical velocity: $w = 0$ m/s
- Time step: 1800 s
- Computation time: 420 days $\Rightarrow$ 1 period
Figure: Tracer field advected by Quick schemes, Lax-Wendroff scheme of Smith et al. (2010) and Lax-Wendroff scheme with Superbee flux limiters.
Benchmark of vertical advection of tracers

- Gaussian-shape distribution of tracers
- 22 layers
- Horizontal velocity: $u = 0$
- Vertical velocity: $w = 2 \times 10^{-3}$ m/s
- Time step: 1800 s
- Computation time: 10 days
Figure: Initial tracer field.
Figure: Advected tracer fields after 10 days of computations.
Figure: Winds used by Frisius et al. (2009).
Figure: Winds in the LSOMG.
Figure: BT stream function [Sv] in BT wind-driven run of Frisius et al. (2009), contour interval 5 Sv.
Figure: BT stream function [Sv] in BT LSOMG wind-driven 1° run.
Figure: Surface elevations [m] in LSOMG wind-driven 1° run.
Figure: Surface elevations [m] in ECCO wind-driven 1° run (Wunsch, 2011).
Figure: Zonal BT transports \( [\text{m}^2/\text{s}] \) in LSOMG wind-driven 1\(^\circ\) run
Figure: Meridional BT transports [m$^2$/s] in LSOMG wind-driven 1° run
Figure: Barotropic transports $[m^2/s]$ in LSOMG wind-driven 1° run
Figure: Barotropic transports [m²/s] in OMCT wind-driven 1.875° run (Irrgang et al., 2015)
Figure: BT stream function [Sv] in ECCO wind-driven 1° run (Wunsch, 2011).
Figure: BT stream function [Sv] in LSOMG wind-driven 1° run after 1 month.
Figure: BT stream function [Sv] in LSOMG wind-driven 1° run after 1 year.
Figure: BT stream function [Sv] in LSOMG wind-driven 1° run after 20 years.
Figure: BT stream function [Sv] in CNRM-CM5.1 model.
LSOMG model has been

- coded and supplied with input data (initial conditions + forcing)
- tested in several benchmarks
  - tsunami propagation (not shown)
  - tidal benchmarks (not shown)
  - Munk problem
  - Ekman layer
  - advection of tracers
- run in realistic settings
  - BT wind-driven circulation $\Rightarrow$ acceptable BT stream function
  - BC wind-driven circulation
    - SSH comparable with the ECCO model
    - BT transports comparable with the OMCT model
    - BT stream function somewhat weak


Einspigel, D., Martinec, Z., 2015. A new derivation of the shallow water equations in geographical coordinates and their application to the global barotropic ocean model (the DEBOT model). Ocean model. 92, 85–100.


Smith, R., Jones, P., Briegleb, B., Bryan, F., Danabasoglu, G., Dennis, J.,
Dukowicz, J., Eden, C., Fox-Kemper, B., Gent, P., Hecht, M., Jayne, S.,
Jochum, M., Large, W., Lindsay, K., Maltrud, M., Norton, N., Peacock, S.,
Vertenstein, M., Yeager, S., 2010. The Parallel Ocean Program (POP)
Reference Manual Ocean Component of the Community Climate System
Model (CCSM) and Community Earth System Model (CESM). Technical
Report. URL: http://www.cesm.ucar.edu/models/cesm1.0/pop2/doc/

Timmermann, R., Danilov, S., Schröter, J., Böning, C., Sidorenko, D.,
Rollenhagen, K., 2009. Ocean circulation and sea ice distribution in a finite
element global sea ice-ocean model. Ocean Dyn. 27, 114–129.

UNESCO, 1981. Background papers and supporting data on the international
equation of state of seawater 1980. UNESCO technical papers in marine sci.
36.

Wunsch, C., 2011. The decadal mean ocean circulation and Sverdrup balance.
J. Marine Res. 69, 417–434.