Lecture course on
Sea level variations and global geodynamics

5. Greenland uplift and sea level change
(November 11, start 10:40)

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Charles University in Prague, Department of Geophysics
November 7–11, 2011
Overview:

- A few words on the “sea level rise” problem skip intro

- Contribution of Greenland to past and present sea level variations,

- Modeling the effects of melting of Greenland: the global view point and

- ... the regional view point

- Discussion, problems, directions, ETC...
The results are from:

Acknowledgements:

Louise S. Sorensen (DTU Space), Copenhagen, Denmark
she is the PI of the “The cryosphere” paper where the Greenland mass balance is published

Daniele Melini Istituto Nazionale di Geofisica e Vulcanologia (INGV) Rome, Italy
He has developed a parallelized multi-platform version of the Sea Level Equation solver (SELEN)

Karina Nielsen (DTU Space), Copenhagen, Denmark
Now in Italy thanks a COST ES0701 grant. Main objective: regional elastic rebound

Shafaqat Abbas Khan (DTU Space), Copenhagen, Denmark
GPS analysis of the Greenland data

Tom James, Jerry Mitrovica for discussion about the SLE
Bert Wouters, Matt King and Erik Ivins for providing data, suggestions, and discussions
Valentina Barletta (now at DTU Space), Copenhagen, Denmark for benchmark computations and discussion.
Overview:

- A few words on the “sea level rise” problem,

- Contribution of *Greenland* to past and present sea level variations,

- Modeling the effects of melting of Greenland: the *global* view point and

- ... the *regional* view point

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Outline map of Greenland with ice sheet depths.

GISP refers to a main site of the Greenland Ice Sheet Project, where a 3 km deep ice core was taken.

Global sea level rise of 7.2 m
Ice sheets

the LAST GLACIAL MAXIMUM (LGM)

Total ESL = 127.40 m

\[ ESL = \frac{\rho_i V}{\rho_w A_0} \]

ICE5G ESL (m)

time BP (kyrs)

today

~70m

LGM

CRE’s?

\[ \rho_i \]
Ice sheets last two glacial cycles

Sea level (m) relative to modern

-150 -120 -90 -60 -30 -20 -10 0 +20 +6 m +8 m +0.5 m

Time (thousands of years BP)

0 20 40 60 80 100 120 140 160 180 200

Modern sea level

WISCONSINAN GLACIAL PERIOD

Stage 5

Previous glacial, Illinoian

Last full interglacial, Sangamonian

Glacial, cold with low sea level

Interglacial, warm with high sea level

LGM

Friday, November 11, 2011
**Land ice contribution to sea level**

**Recent estimates for 1993-2003 (mm/yr)**

<table>
<thead>
<tr>
<th></th>
<th>Glaciers</th>
<th>Greenland</th>
<th>Antarctica</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>West</td>
<td>East</td>
</tr>
<tr>
<td>Dyugerov &amp; Meier (2005)</td>
<td>0.8 +/- 0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Krabill et al. (2004)</td>
<td>0.1-0.2</td>
<td></td>
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</tr>
<tr>
<td>Rignot &amp; Kanagaratnam (2006)</td>
<td>0.23-0.57</td>
<td></td>
<td></td>
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<tr>
<td>Velicogna &amp; Wahr (2005)</td>
<td>0.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zwally et al. (2005)</td>
<td>~0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ramillien et al. (2006)</td>
<td>0.35</td>
<td></td>
<td></td>
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<tr>
<td>Davis et al. (2005)</td>
<td>&gt;0</td>
<td>-0.12</td>
<td></td>
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<tr>
<td>Thomas et al. (2004)</td>
<td>0.16</td>
<td></td>
<td>&lt;0</td>
</tr>
<tr>
<td>Zwally et al. (2005)</td>
<td>&gt;0</td>
<td></td>
<td>0.4</td>
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<tr>
<td>Velicogna &amp; Wahr (2006)</td>
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<td></td>
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<tr>
<td>Ramillien et al. (2006)</td>
<td>0.3</td>
<td>-0.16</td>
<td></td>
</tr>
</tbody>
</table>

**IPCC’s estimate**

0.8 +/- 0.5
0.2 +/- 0.07
0.2 +/- 0.35

(Adapted from a slide of Anny Cazenave et al.)
Recent Sea-Level Contributions of the Antarctic and Greenland Ice Sheets

Andrew Shepherd and Duncan Wingham

Fig. 3. (A) Rate of elevation change of the Greenland Ice Sheet, 1992 to 2003, determined from satellite radar altimetry [from (22)], and (B) time series of elevation change of individual sectors, 2003 to 2005, determined from satellite gravimetry [from (16)]. Also shown (inset) is the ice surface geometry, highlighting areas above (gray) and below (black) 2000 m elevation. Both instruments concur that high elevation areas are growing and low elevation areas are losing mass. According to gravimetry (16) and repeat InSAR measurements of ice discharge (12), the rate of mass loss at low elevations has increased over the past decade (see Table 1).
Mass balance of Greenland since 1990 - from various sources -

Our study: 240 +/- 28 Gt/yr ICESat - based

Courtesy of Jonathan Bamber, University of Bristol UK
Mass balance of Greenland since 1990 - from various sources -

Our study: 240 +/- 28 Gt/yr

ICESat - based

Courtesy of Jonathan Bamber, University of Bristol UK
Goal: estimating the time-averaged mass variation over 2003-2008 (including density modeling...)

Mass balance of the Greenland ice sheet – a study of ICESat data, surface density and firn compaction modelling

L. S. Sørensen$^{1,2,*}$, S. B. Simonsen$^{3,4,*}$, K. Nielsen$^5$, P. Lucas-Picher$^4$, G. Spada$^6$, G. Adalgeirsdottir$^4$, R. Forsberg$^1$, and C. S. Hvidberg$^3$

$^{1}$Geodynamics Department, DTU Space, Juliane Maries vej 30, 2100 Copenhagen, Denmark
$^{2}$Planet and Geophysics, NBI, University of Copenhagen, Juliane Maries Vej 30, 2100 Copenhagen, Denmark
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we use three different methods to derive the TOPEX/Poseidon reference ellipsoid. The satellite laser tagged ice sheet surface elevation estimates, with respect to a smooth surface, which covers the entire ice sheet, to the...inclination of the satellite, the tracks are separated by approx...The Cryosphere, 5, 173–

by climate parameters from a regional climate model (RCM). The firn correction and the surface density models are forced change correction terms and a simple surface density model. In spite of the criteria used to select the ICESat campaigns...technical approach, the number is reduced by approximately 13% to...A procedure of data culling and the application of correction...of the laser campaigns in the dataset are listed in Table 4.

<table>
<thead>
<tr>
<th>ID</th>
<th>RL</th>
<th>Time span</th>
<th>N</th>
<th>M</th>
</tr>
</thead>
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<tr>
<td>L2A</td>
<td>531</td>
<td>4 Oct 2003–18 Nov 2003</td>
<td>1 095 647</td>
<td>941 052</td>
</tr>
<tr>
<td>L2C</td>
<td>531</td>
<td>18 May 2004–20 Jun 2004</td>
<td>739 672</td>
<td>680 031</td>
</tr>
<tr>
<td>L3C</td>
<td>531</td>
<td>20 May 2005–22 Jun 2005</td>
<td>800 876</td>
<td>679 827</td>
</tr>
<tr>
<td>L3E</td>
<td>531</td>
<td>22 Feb 2006–27 Mar 2006</td>
<td>883 492</td>
<td>752 123</td>
</tr>
<tr>
<td>L3H</td>
<td>531</td>
<td>12 Mar 2007–14 Apr 2007</td>
<td>838 647</td>
<td>778 350</td>
</tr>
<tr>
<td>L3J</td>
<td>531</td>
<td>17 Feb 2008–21 Mar 2008</td>
<td>375 239</td>
<td>368 148</td>
</tr>
</tbody>
</table>

Total 10 367 807 9 053 639

Table 1. ICESat data description. Shown is the laser campaign identifier (ID), data release number (RL), and time span of the campaigns. N and M are the number of measurements from the GrIS before and after the data culling, respectively.

Fig. 6. The spatial distribution of the mass change of the GrIS, given in metres of ice equivalent. The result is based on the estimate derived by M3. The pattern of coastal thinning seen in Fig. 1 is also found in the mass change of the GrIS.
Gravity anomaly map from the NASA GRACE (Gravity Recovery And Climate Change).

http://earthobservatory.nasa.gov/Features/GRACE/
1) The LAGEOS satellites are passive vehicles covered with retroreflectors designed to reflect laser beams transmitted from ground stations. By measuring the time between transmission of the beam and reception of the reflected signal from the satellite, stations can precisely measure the distance between themselves and the satellite. These distances can be used to calculate station positions to within 1-3 cm.

2) Long term data sets can be used to monitor the motion of the Earth's tectonic plates, measure the Earth's gravitational field, measure the "wobble" in the Earth's axis of rotation, and better determine the length of an Earth day.

3) Ground tracking stations are located in many countries (including the US, Mexico, France, Germany, Poland, Australia, Egypt, China, Peru, Italy, and Japan) and data from these stations is available worldwide to investigators studying crustal dynamics.

4) LAGEOS 1 also contains a message plaque addressed to human and other beings of the far distant future with maps of the Earth from 3 different eras - 268 million years in the past, present day, and 8 million years in the future (the satellite's estimated decay date).

http://msl.jpl.nasa.gov/QuickLooks/lageosQL.html
GOCE

Mapping the gravity field to advance research in *Earth-interior processes, oceanography and geodesy.*
Two targets:

-1- Updating the mass balance of the GrIS by ICESat data elaboration and firn compaction modeling, *...done by DTU*

-2- Given the “preferred” mass balance (right), the problem is providing estimates for the present-day vertical movements along the coasts of Greenland. Implications for the i) *regional future sea level rise* and ii) *geodetic observations* - *(our role - work in progress...)*

Mass balance 2003-2008 (m/yr)
Two different points of view:

1) Solving the SLE globally:

**Advantage**: this provides a self-consistent sea level variation and vertical movements.

**BUT the disadvantage is**: only the long-intermediate wavelengths of deformation can be captured (*because of our limited computing power! --- but we are making some progress*)

2) Building a regional “elastic rebound” model for Greenland:

**Advantage**: all the information derived by the inversion of ICESat data can be employed.

**Main disadvantage**: some physical ingredients are lost: effects from melt-water load, “self-gravitation” of the oceans, etc.

In both cases: Maxwell time of the mantle is ~ 300 years: an ELASTIC rheology is appropriate!
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- ... and the *regional* view point

- Discussion, problems, directions, ETC...
SELEN solves the “Sea Level Equation” [Farrell, 1976]:

\[
S = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_o S - \frac{m_i}{\rho_w A_o} - \frac{\rho_i}{\gamma} G_s \otimes_i I - \frac{\rho_w}{\gamma} G_s \otimes_o S
\]

with:

- \(S\) = sea level change
- \(m_i\) = ice mass variation
- \(\rho_i, \rho_w\) = ice and water density
- \(A_o\) = area of the oceans
- \(G_s\) = sea level Green function
- \(\otimes_i, \otimes_o\) = 3(2+1)D convolutions
- \(I\) = ice thickness variation
- \(\ldots\) = ocean average

\[
\left\{ \begin{array}{c}
U_i \\
\Phi_i
\end{array} \right\}(\omega, t) \equiv \left\{ \begin{array}{c}
G_u \\
G_\phi
\end{array} \right\} \otimes_i \rho_i I,
\text{ and }
\left\{ \begin{array}{c}
U_o \\
\Phi_o
\end{array} \right\}(\omega, t) \equiv \left\{ \begin{array}{c}
G_u \\
G_\phi
\end{array} \right\} \otimes_o \rho_w S
\]
Outline map of Greenland with ice sheet depths. GISP refers to a main site of the Greenland Ice Sheet Project, where a 3 km deep ice core was taken.

Preferred mass balance of Sorensen et al. (2011)

SLE input: Mass balance from ICEsat (M3): 240 +/- 28 Gt/yr

Outline map of Greenland with ice sheet depths. GISP refers to a main site of the Greenland Ice Sheet Project, where a 3 km deep ice core was taken.

Preferred mass balance of Sorensen et al. (2011)

global sea level rise of 7.2 m
Actual M3 mass balance

Uniform thickness

SLE input: Mass balance from ICEsat (M3): 240 +/- 28 Gt/yr

...sea level falls
“close to Greenland”

ESL = +0.67 mm/yr
(average sea level rise)

- sea level rises
“far away”

Friday, November 11, 2011
GIA, Greenland, Antarctica, & Glaciers fingerprints

- **GIA**
  - Map of sea level change
  - Units: mm/yr
  - Region: Global

- **Greenland**
  - Map of sea level change
  - Units: mm/yr
  - Region: Greenland

- **Antarctica**
  - Map of sea level change
  - Units: mm/yr
  - Region: Antarctica

- **Glaciers & caps**
  - Map of sea level change
  - Units: mm/yr
  - Region: Glaciers & caps

**Notes:**
- Maps generated using SELEN 3.2
- Date: 2011 May 18 12:51:59

**Legend:**
- Color scale: -1.0 to 1.0
- Units: mm/yr
- Region: Global

**Equation:**
- GIA + Greenland + Antarctica + Glaciers fingerprints = 100 Gt/yr
SLE input: Mass balance from ICEsat (M3): 240 +/- 28 Gt/yr

\[ \frac{dS}{dt} \]

mm/yr
GIA sealevel fingerprints for the Mediterranean Sea (ICE-5G vs RSES)

**ICE-5G**

Close to equilibrium ("low viscosity")

\[ \sim 0.3 \text{ mm/yr} \]

**ANU05**

Far from equilibrium ("high viscosity")

\[ \sim 0.5 \text{ mm/yr} \]
SLE input: Mass balance from ICESat (M3): 240 +/- 28 Gt/yr

Elastic solution of the Sea Level Equation
SLE: ICESat vs GRACE (Wouters et al., 2008)

a) dU/dt (ICESat)  
b) dU/dt (GRACE)  
c) ICESat–GRACE

(this) GRACE mass balance: 179 +/- 25 Gt/yr (Feb 2003 - Jan 2008) < ICESat
SLE input: Mass balance from ICEsat (M3): 240 +/- 28 Gt/yr

Are we missing something?

Elastic solution of the Sea Level Equation
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**Spherical non-rotating elastic isotropic (SNREI) model**

\[ \mu = \mu(r) \quad \lambda = \lambda(r) \]

Lame constants

\[ \rho = \rho(r) \]

Density

**Source** (disc)

**Source-observer geometry**
Vertical displacement due only to (un)loading:

\[ U(\theta) = 3\Delta h \left( \frac{\rho_i}{\rho_e} \right) \sum_{n=0}^{\infty} \frac{\sigma_n(\alpha)}{2n + 1} h_n P_n(\cos \theta) \]

with:
- \( \rho_i, \rho_e \) ice and Earth average densities
- \( n \) harmonic degree
- \( \alpha \) half-amplitude of the disc load
- \( \sigma_n(\alpha) \) load spectral density
- \( h_n \) vertical “Love number”
- \( P_n(\cos \theta) \) Legendre polynomial
- \( \theta \) source-observer co-latitude
- \( \Delta h \) ice thickness variation
Computing the **elastic Love numbers** $h_n$

**Two options:**

1) **Use those available from the literature.**

E.g. the “Gegout” Love numbers, based on **PREM**, to degree LMAX= 1024. Available from NASA. They are for a compressible multilayered Earth, Reference frame of the CM of the WHOLE EARTH.


2) **Compute your own Love numbers.**

E.g., using free software available from GS. ALMA works well for multi-layered models (N ~ 10^3). Advantages: LMAX can be of O(10**5), *ad hoc* regional layering, Lamé constants and density can be employed. Open source. Disadvantage: Assumes incompressible layers.

(*) [http://www.fis.uniurb.it/spada/ALMA_minipage.html](http://www.fis.uniurb.it/spada/ALMA_minipage.html).
Asymptotic Love number “$h$” is only sensitive to the outmost layers! (Farrell, 1972)

$$h_\infty = -\frac{\lambda + 2\mu}{3(\lambda + \mu)} \frac{\rho_{eGa}}{\mu} \bigg|_{PREM} \approx -6.2129$$
C6. Love numbers of Farrell ('72, GBA model) vs those of Gegout (today, PREM model)

Fig. 1. Love numbers for a unit mass load on the surface of a Gutenberg-Bullen A earth model. Selected values are listed in Table A2. At $n = 10,000$, the computed Love numbers agree with the Boussinesq approximation to within 1%. The Love numbers for the other earth models differ significantly from these Love numbers above $n = 20$ to 30.
Number of elements: \( N \approx 70 \times 10^3 \)

Size of each element: \( \alpha_k \approx 0.025^\circ \)

Thickness is variable: \( \Delta h = F(k) \)
Test displacement along the 1D grid to degree LMAX=131,072

A 0.025 deg disc decomposed to degree 130000 - Boys Love numbers rectified for degree ge 1024

elastic displacement caused by a thickening of +1 meter (units are cm)

distance from the center of the disc (degrees)
Test displacement along the 1D grid to degree LMAX=131,072

A 0.025 deg disc decomposed to degree 130000 - Boys Love numbers rectified for degree ge 1024

LMAX=128
256
512
1024
2048
4096
8192
16384
32768
65536
131072

LMAX ~ 16000: ~ convergence
Details of method

1. for all observers $\omega_k$ compute "U" to degree LMAX using previous formula. This 1D grid does not need to be equally spaced! Use a "test" unit disc;

2. Given a point P of the ICO 2D grid, find its distance from a given ice element. In P, find U, possibly by interpolation, using the 1D grid above. Rescale to compensate for the actual disc thickness variation;

3. Repeat for all discs, and sum in P the individual contributions (for all discs, the radial unit vector is the same - note that this NOT true for horizontal displacement...);

4. Repeat for all points P of the 2D ("almost" equally spaced) ICO grid.
spherical grid spacing = 25 km, LMAX=128

vertical velocity

Global

<0.55/10.08>

(ONLY local ice is accounted for)
LMAX > 1E5, varying the pixels spacing...

Vertical velocity

Ice model: GR3R  
Elasticity: Gegout  
LMAX > 1E5  OBS_SPAC = 25km

Regional, 25 km
<-1.95/23.46>

Ice model: GR3R  
Elasticity: Gegout  
LMAX > 1E5  OBS_SPAC = 10km

Regional, 10 km
<-2.21/24.17>

Ice model: GR3R  
Elasticity: Gegout  
LMAX > 1E5  OBS_SPAC = 5km

Regional, 5 km
<-2.18/25.41>
Geguot Love numbers

Vertical velocity

- Ice model: GR3R  - Elasticity: Geguot
- LMAX > 10^5  OBS_SPAC = 25 km

Regional, 25 km  <-1.95/23.46>

ALMA Love numbers

Vertical velocity

- Ice model: GR3R  - Elasticity: ALMA
- LMAX > 10^5  OBS_SPAC = 25 km

Regional, 25 km  <-0.74/11.26>
The image shows two maps of Greenland with different colors and labels indicating the direction of change in elevation and ice thickness. The left map is labeled "dU/dt" and measures changes in elevation, while the right map is labeled "dG/dt" and measures changes in ice thickness. The maps use a color gradient from blue to red to indicate the magnitude of these changes. The scale on both maps ranges from 0 to 24 mm/yr for "dU/dt" and from -4 to 0 mm/yr for "dG/dt." The maps also include grid lines and labels for geographic locations.
Scoresby Sund in Greenland - 350 km (217 mi) - the LONGEST fjord of the world -
Scoresby Sund in Greenland - 350 km (217 mi) - the LONGEST fjord of the world -
Scoresby Sund Fjord

Grid size: 5 km

vertical uplift rate

mm/yr

2010 Oct 26 13:05:54

2010 Oct 26 13:05:54
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Figure 1. Locations of the GPS sites (red dots), the th

Figure 4. Multiday averages and their assigned errors at (a) KELY, (b) THUL, (c) SCBB, (d) KULU, gauge station (red triangle), DYE3 (yellow triangle), and (e) QAQ1. The solid line shows the best fitting offsets, annual, semiannual, and secular terms. Paamiut (green triangle).
Constraints on the Greenland Ice Sheet since the Last Glacial Maximum from sea-level observations and glacial-rebound models

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\textsuperscript{a}GeoForschungsZentrum Potsdam, Department 1: Geodesy and Remote Sensing, Telegrafenberg, D-14473, Potsdam, Germany
\textsuperscript{b}Research School of Earth Sciences, Australian National University, Canberra, ACT, 0200, Australia

Received 1 August 2002; accepted 8 November 2003

Fig. 13. Comparisons of observations and scaled-predicted relative sea-level curves (Table 2, lines 18 and 19) for various sites around Northwest Greenland (Fig. 2). Dark-grey shading indicates the estimated local marine limits (Fig. 1). Symbols: triangles—lower limits, diamonds—mean sea level, inverted triangles—upper limits.

\textsuperscript{*}Corresponding author. Tel.: +49-331-288-1449.
Rate of sea level change today

- Ice model: ICE5G
- Viscosity profile: /4.0 0.4 0.4/

Rate of sea level change today

- Ice model: ANU05
- Viscosity profile: /20 0.5 0.5/

SELEN 3.0
a) ICE-5G

b) ANU05 (L)

c) ANU05 (N)

d) ANU05 (U)

mm/yr
Future projections - Uplift rate

GPS / tide gauge site

Load

ALMA

Geguot

Vertical Velocity, dU/dt (mm yr$^{-1}$)

GPS site

THUL  KELY  QAQ1  KULU  SCOR

RER

RER+(ICE−5G)

6.0 ± 3.8 mm/yr

3.7 ± 2.3 mm/yr

elastic displacement caused by a thickening of +1 m (units are cm)

distance from the center of the disc (degrees)

m/yr

LMAX > 10$^5$