Questions and comments - Dr. Erik R. Ivins

Question 1
The Preface nicely summarizes an outline of what is done in the thesis. Considering the tremendous challenges posed by the magnitude of the computational problem, would it not be appropriate to just say a few words about what is clear that cannot be done theoretically? Or perhaps those things which, if one had a wish list for those things that would get resolved in the next 10 years, what would be at the top of the list?

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Limitations of the presented formulation, what cannot be done theoretically?

- Various time and spatial scales - precise glaciological modelling is a multi-scale problem both in space and time, the way small-scale features (local ice-stream basal activation, iceberg calving, basal lubrication, hydrology evolution, local strain-induced anisotropy) affect large-scale ice flow and stability over large time spans (glacial cycles) is very unclear and very hard if not impossible to assess.

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Wish list of things that would get resolved within the next 10 years?

- Satisfactory unifying formulation capable of treating simultaneously all existing approximations - SIA in slowly flowing interior of the ice-sheet, Heinz-Blatter approximation or full-Stokes over ice streams and SSA over ice shelves
- Grounding line dynamics - dynamics of the transition between grounded and floating ice, both theoretically and numerically open problem
- Calving front treatment, calving law
- Basal sliding law
- Ice rheology, Glen's flow law really?

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Question 2
Equations 1.16. If you were to consider anisotropic flow, which might be important in the study of anisotropic fabric development, how do the equations look? And could you allow this in your development?

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$$\begin{aligned} \boldsymbol{\tau} &= -p\mathbf{I} + \boldsymbol{\sigma} \ , \\ \mathbf{D} &= \mathcal{A}(T)\sigma_{II}^{n-1}\boldsymbol{\sigma} \ . \end{aligned}$$

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Question 2
Equations 1.16. If you were to consider anisotropic flow, which might be important in the study of anisotropic fabric development, how do the equations look? And could you allow this in your development?

- There is no unity and common agreement concerning the form of anisotropic constitutive law. (Not surprisingly as even the validity of the isotropic Glen's flow law is still questioned)
- What time of anisotropy? (transversal isotropy?, orthotropy?)
- Ice Anisotropy and fabric evolve, strain dependent. How does the evolution look like? Number of models have been proposed based on various mechanisms considered such as rotational orientation of the crystal c-axis towards the compressional axis, recrystallization, grain size evolution
- microscale models, macroscale continuum mechanical models

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- Unfortunately, the numerical experiments such as (Mangeney et al, 1996) show a possible non-negligible effect of anisotropy to large-scale polar ice cap modeling (factor 1.5), anisotropy induces generally higher flow velocities
- No type of anisotropy has been implemented yet and it is not planned at the moment - a whole new universe of computational difficulties. The generalisation of the SIA-I algorithm is, however, in principle possible as long as the stress-strain rate relation remains invertible. Anisotropic SIA already derived (Mangeney, Califano, 1998)

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Question 2
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- Simple macroscopic continuum-mechanical model (Staroszczyk & Gagliardini, 1999)

$$\boldsymbol{\sigma} = \sum_{r=1}^{3} \phi_{r+3}[\mathbf{M}_r\mathbf{D} + \mathbf{D}\mathbf{M}_r - \frac{2}{3}\mathrm{tr}(\mathbf{M_r}\mathbf{D})\mathbf{I}] + \phi_{12}[\mathbf{D}\mathbf{B} + \mathbf{B}\mathbf{D} - \frac{2}{3}\mathrm{tr}(\mathbf{D}\mathbf{B})\mathbf{I}]$$

$\mathbf{M}_r = \mathbf{e}_r \otimes \mathbf{e}_r$ structure tensors, $\mathbf{e}_r$ principle strain axes, $\mathbf{B} = \mathbf{F}\mathbf{F}^\mathbf{T}$ Cauchy-Green strain tensor, ϕ response functions (depending on invariants of $\mathbf{D}, \mathbf{B}, \mathbf{M_r}$)

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Question 3

Gary Clarke has considered 'composite' rheology, in which simple addition of the component-wise form of the tensor constitutive equation is considered. He claims it allows 'faster' ice sheet evolutions. What is your opinion?

- Clarke mentions the following type of composite rheology:

$$
\begin{aligned}
\mathbf{d} &= \mathbf{d}_{diff} + \left(\frac{1}{\mathbf{d}_{basal}} + \frac{1}{\mathbf{d}_{gbs}}\right)^{-1} + \mathbf{d}_{disl} \\
\mathbf{d} &= B\frac{\boldsymbol{\sigma}^n}{d^q}\exp\left(-\frac{E+pV}{RT}\right)
\end{aligned}
$$

- diffusion creep, dislocation creep, grain boundary sliding and grain boundary-sliding accommodated basal sliding. Goldsby & Kohlstedt (2001) suggest that Glen's flow law underestimates the deformational velocities for lower differential pressures.
- Only experimental data can constrain and verify the newly proposed flow law. Most numerical approaches, however neglect part of the stress field, I would expect this effect to be of bigger importance for the final deformational rates than some fine tuning of the flow law.
- However a debate is still present about the exponent in Glen's flow law, I find that more important.

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Question 4
Regarding the boundary conditions, are bifurcating ice streams, and their development allowed? Under what conditions?

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Regarding the boundary conditions, are bifurcating ice streams, and their development allowed? Under what conditions?

- Yes they are!
- Ice-streams are fast-flowing regions, where the basal sliding usually dominates the deformational velocities.
- Ice-stream evolution may be implemented by allowing for basal sliding in regions with basal temperature above melting temperature
- Capability of our model to capture ice-stream evolution has been confirmed in the ISMIP-HEINO benchmark, simulation of Heinrich events
- Intrinsic thermo-mechanical ice-sheet instability resulting from the coupling of the the geometry and temperature evolution, and temperature-triggered rapid basal sliding.

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Question 5
End of Section 1.3. Under the set of conditions and definitions defined by the mathematical development so far, it appears that to allow ponding of liquid water at the surface (or even interior to) the ice sheet, and then allow subsequent percolation into the depths of ice sheet via percolation is impossible. Or is this so?

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Question 5
End of Section 1.3. Under the set of conditions and definitions defined by the mathematical development so far, it appears that to allow ponding of liquid water at the surface (or even interior to) the ice sheet, and then allow subsequent percolation into the depths of ice sheet via percolation is impossible. Or is this so?

- Yes that is correct.
- Ponding is not allowed in the presented formulation. The formulation would have to be properly extended to allow for such situation. The boundary conditions would be derived from transition conditions for a two-phase mixture (water+ice) and limiting the ice content to zero at one side of the boundary.

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- Mass balance

$$\begin{aligned} (1-w^-)(\vec{v}_2 - \vec{\nu})\cdot\vec{n} &= -\frac{r_2}{\rho} \\ (\vec{v}_1 - \vec{\nu})^+\cdot\vec{n} - w^-\vec{u}_1\cdot\vec{n} &= -\frac{r_2}{\rho(1-w^-)} \end{aligned} \tag{1}$$

- Linear momentum balance

$$\boldsymbol{\tau}_1^+\cdot\vec{n} = (\boldsymbol{\tau}_1^- + \boldsymbol{\tau}_2^-)\cdot\vec{n} \tag{2}$$

- Energy balance

$$0 = r_2(\varepsilon_2 - \varepsilon_1) - [\vec{v}_1\cdot\boldsymbol{\tau}_1]_-^+\cdot\vec{n} + \vec{v}_2^-\cdot\boldsymbol{\tau}_2^-\cdot n + [\vec{q}]_-^+ = 0\ . \tag{3}$$

- Entropy inequality

$$0 \le r_2 T(s_2 - s_1) + [\vec{q}]_-^+\cdot\vec{n} \tag{4}$$

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Question 6
Before Equation 1.83, define 'barycentric' (in words).

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$$\vec{v}_b := \rho_1 \vec{v}_1 + \rho_2 \vec{v}_2$$

- Velocity of the barycenter = center of mass of the mixture.

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Question 7
1st sentence of Chapter 2. Who first formulated the SIA? What fields in physics and ocean sciences utilized these concepts?

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- SIA first formulated by K. Hutter (1983)

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- Such a scaling perturbation expansion applicable everywhere where a dominant scale is present (boundary layer theories, shallow water approximation, thin membrane approximation, etc...)

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Question 8
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- The SIA approach has been already implemented in at least three coordinate systems, cartesian (Hutter, 1983), spherical (Calov, 2006), orthographic (Greve, pers. comm.)

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Question 8
It is tough to imagine the context in which the SIA in general orthogonal curvilinear coordinates would have a useful purpose. Is there one?

- The SIA approach has been already implemented in at least three coordinate systems, cartesian (Hutter, 1983), spherical (Calov, 2006), orthographic (Greve, pers. comm.)
- Curved coordinates definitely more appropriate than cartesian for a large-scale paleo-modeling (global), the derivation tedious, why do it more than once?

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Question 9
Among the 16 dimensionless parameters, which ones are new in this development? In transport theory, the Peclet number plays a role. Is there an analogy here? For ice transport? For water transport?

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$$\epsilon = \frac{[h_3][\Delta_3]}{[h_1][\Delta_1]}, \quad \mathcal{C} = \frac{g[h_3][\Delta_3]}{[c_v][T]}, \quad \mathcal{J} = \frac{[w][v_w][h_1][\Delta_1]}{[v_h][h_3][\Delta_3]}, \quad \mathcal{E} = \frac{g[h_3][\Delta_3]}{L[w]},$$

$$\mathcal{K} = \frac{[\mathcal{A}]\rho^n g^n([h_3][\Delta_3])^{2n+1}}{([h_1][\Delta_1])^n[v_h]}, \quad \mathcal{B} = \frac{\rho g([h_3][\Delta_3])^2}{[h_1][\Delta_1][v_h][\beta]^2}, \quad \mathcal{D} = \frac{[k][h_1][\Delta_1]}{\rho[c_v][v_h]([h_3][\Delta_3])^2}, \quad [w],$$

$$\mathcal{T}_0 = \frac{T_{m0}}{[T]}, \quad \mathcal{T} = \frac{c_{Cl}\rho g[h_3][\Delta_3]}{[T]}, \quad \gamma, \quad \mathcal{Q} = \frac{Q}{k_B[T]},$$

$$\mathcal{N}_1 = \frac{[\alpha_1]}{[h_3][\Delta_3]\rho[v_w]}, \quad \mathcal{N}_2 = \frac{[\alpha_2]g}{[v_w]}, \quad \mathcal{F} = \frac{[v_h]^2}{[h_1][\Delta_1]g}, \quad \mathcal{L} = \frac{p^{atm}}{[h_3][\Delta_3]\rho g}.$$

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- New in what sense? Only Greve (2000) has also formulated the SIA for temperate ice. Our formulation differs only in the treatment of water transport slightly.

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- Peclet number $P_e = \frac{\text{advection}}{\text{diffusion}} = \frac{LV}{\alpha} = \frac{LV\rho c_p}{k}$
- For ice

$$\mathcal{D} = \frac{[k][h_1][\Delta_1]}{\rho[c_v][v_h]([h_3][\Delta_3])^2} = \frac{1}{\epsilon P_e}$$

- For water

$$\mathcal{N}_1 = \frac{[\alpha_1]}{[h_3][\Delta_3]\rho[v_w]}$$

$$\mathcal{N}_2 = \frac{[\alpha_2]g}{[v_w]}$$

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Question 10
Certainly N1 and N2 have some quantifiable bounds. What are they?

$$\begin{aligned} \mathcal{N}_1 &= \frac{[\alpha_1]}{[h_3][\Delta_3]\rho[v_w]} \\ \mathcal{N}_2 &= \frac{[\alpha_2]g}{[v_w]} \end{aligned}$$

- No reference found, lack of measurements.

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Question 11
Material from Eq. 2.25 to 2.162 is elegantly developed in the dimensionless representation. But it seems a waste not to discuss, just a bit, what has been gained by looking at the equations, just at the face of them with those nice dimensionless numbers multiplying the independent variables of the SIA initial-boundary value problem. (Suggestion: just key-in on the most important things!)

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Most important features of the Shallow-Ice Approximation

- Possibility to analytically resolve the leading-order stress tensor components $(p, \boldsymbol{\sigma}_{13}, \boldsymbol{\sigma}_{23})$ from the SIA linear momentum equations.

$$
\begin{aligned}
0 &= -\frac{1}{H_1}\frac{\partial \tilde{p}}{\partial \tilde{x}_1} + \tilde{\boldsymbol{\sigma}}_{13}\left(2\frac{H_{13}}{H_1 H_3} + \frac{H_{23}}{H_2 H_3}\right) + \frac{1}{H_3}\frac{\partial \tilde{\boldsymbol{\sigma}}_{13}}{\partial \tilde{x}_3} + e_{g\,1}\ , \\
0 &= -\frac{1}{H_2}\frac{\partial \tilde{p}}{\partial \tilde{x}_2} + \tilde{\boldsymbol{\sigma}}_{23}\left(\frac{H_{13}}{H_1 H_3} + 2\frac{H_{23}}{H_2 H_3}\right) + \frac{1}{H_3}\frac{\partial \tilde{\boldsymbol{\sigma}}_{23}}{\partial \tilde{x}_3} + e_{g\,2}\ , \\
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\begin{aligned}
\frac{\partial}{\partial \tilde{x}_3}\left(\tilde{\boldsymbol{\sigma}}_{13} \ln\left(H_1^2\, H_2\right)\right) &= -\frac{H_3}{H_1}\frac{\partial \tilde{p}}{\partial \tilde{x}_1} + + H_3 e_{g\,1}\ , \\
\frac{\partial}{\partial \tilde{x}_3}\left(\tilde{\boldsymbol{\sigma}}_{23} \ln\left(H_1\, H_2^2\right)\right) &= -\frac{H_3}{H_2}\frac{\partial \tilde{p}}{\partial \tilde{x}_2} + + H_3 e_{g\,2}\ , \\
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\end{aligned}
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Most important features of the Shallow-Ice Approximation

- Possibility to resolve the velocity explicitly from the leading-order stress tensor components and equation of continuity.

$$
\begin{aligned}
\frac{1}{H_3}\frac{\partial \tilde{v}_1}{\partial \tilde{x}_3} - \frac{H_{13}}{H_1 H_3}\tilde{v}_1 &= 2\mathcal{K}\tilde{\mathcal{A}}\tilde{\boldsymbol{\sigma}}_{II}^{n-1}\tilde{\boldsymbol{\sigma}}_{13} \\
\frac{1}{H_3}\frac{\partial \tilde{v}_2}{\partial \tilde{x}_3} - \frac{H_{23}}{H_2 H_3}\tilde{v}_2 &= 2\mathcal{K}\tilde{\mathcal{A}}\tilde{\boldsymbol{\sigma}}_{II}^{n-1}\tilde{\boldsymbol{\sigma}}_{23} \\
\frac{\partial \tilde{v}_1}{\partial \tilde{x}_1} + \frac{\partial \tilde{v}_2}{\partial \tilde{x}_2} + \frac{\partial \tilde{v}_3}{\partial \tilde{x}_3} &= 0
\end{aligned}
$$

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$$
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\frac{\partial}{\partial \tilde{x}_3}\left(\frac{\tilde{v}_1}{H_1}\right) &= 2\frac{H_3}{H_1}\mathcal{K}\tilde{\mathcal{A}}\tilde{\boldsymbol{\sigma}}_{II}^{n-1}\tilde{\boldsymbol{\sigma}}_{13} \\
\frac{\partial}{\partial \tilde{x}_3}\left(\frac{\tilde{v}_2}{H_2}\right) &= 2\frac{H_3}{H_2}\mathcal{K}\tilde{\mathcal{A}}\tilde{\boldsymbol{\sigma}}_{II}^{n-1}\tilde{\boldsymbol{\sigma}}_{23} \\
\frac{\partial \tilde{v}_3}{\partial \tilde{x}_3} &= -\left(\frac{\partial \tilde{v}_1}{\partial \tilde{x}_1}+\frac{\partial \tilde{v}_2}{\partial \tilde{x}_2}\right)
\end{aligned}
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Most important features of the Shallow-Ice Approximation

- Dominance of vertical gradients in the field equations (neglected horizontal heat diffusion, water diffusion)

$$\tilde{c}_v \left(\frac{\partial \tilde{T}}{\partial \tilde{t}} + \frac{\tilde{v}_1}{H_1} \frac{\partial \tilde{T}}{\partial \tilde{x}_1} + \frac{\tilde{v}_2}{H_2} \frac{\partial \tilde{T}}{\partial \tilde{x}_2} + \frac{\tilde{v}_3}{H_3} \frac{\partial \tilde{T}}{\partial \tilde{x}_3} \right) = 2\mathcal{C} \left(\tilde{\boldsymbol{\sigma}}_{13} \tilde{\boldsymbol{d}}_{13} + \tilde{\boldsymbol{\sigma}}_{23} \tilde{\boldsymbol{d}}_{23} \right)$$
$$+ \mathcal{D} \left(\tilde{k} \frac{\partial \tilde{T}}{\partial \tilde{x}_3} \left(\frac{H_{13}}{H_1 H_3^2} + \frac{H_{23}}{H_2 H_3^2} \right) + \frac{1}{H_3} \frac{\partial}{\partial \tilde{x}_3} \left(\frac{\tilde{k}}{H_3} \frac{\partial \tilde{T}}{\partial \tilde{x}_3} \right) \right)$$

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- Dominance of vertical gradients in the field equations (neglected horizontal heat diffusion, water diffusion)

$$
\begin{aligned}
& \frac{\partial \tilde{w}}{\partial \tilde{t}} + \frac{\tilde{v}_1}{H_1}\frac{\partial \tilde{w}}{\partial \tilde{x}_1} + \frac{\tilde{v}_2}{H_2}\frac{\partial \tilde{w}}{\partial \tilde{x}_2} + \frac{\tilde{v}_3}{H_3}\frac{\partial \tilde{w}}{\partial \tilde{x}_3} + \frac{\mathcal{J}}{[w]}\left(\frac{1}{H_3}\frac{\partial \tilde{j}_3}{\partial \tilde{x}_3} + \tilde{j}_3\left(\frac{H_{13}}{H_1 H_3} + \frac{H_{23}}{H_2 H_3}\right)\right) \\
= \;& 2\mathcal{E}(\tilde{\sigma}_{13}\tilde{d}_{13} + \tilde{\sigma}_{23}\tilde{d}_{23}) + \frac{\mathcal{D}\mathcal{E}}{\mathcal{C}}\left(\frac{1}{H_3}\frac{\partial}{\partial \tilde{x}_3}\left(\frac{\tilde{k}}{H_3}\frac{\partial \tilde{T}_m}{\partial \tilde{x}_3}\right) + \tilde{k}\frac{H_{13}}{H_1 H_3^2}\frac{\partial \tilde{T}_m}{\partial \tilde{x}_3} + \tilde{k}\frac{H_{23}}{H_2 H_3^2}\frac{\partial \tilde{T}_m}{\partial \tilde{x}_3}\right) \\
- \;& \frac{\mathcal{E}}{\mathcal{C}}\tilde{c}_v\left(\frac{\partial \tilde{T}_m}{\partial \tilde{t}} + \frac{\tilde{v}_1}{H_1}\frac{\partial \tilde{T}_m}{\partial \tilde{x}_1} + \frac{\tilde{v}_2}{H_2}\frac{\partial \tilde{T}_m}{\partial \tilde{x}_2} + \frac{\tilde{v}_3}{H_3}\frac{\partial \tilde{T}_m}{\partial \tilde{x}_3}\right) .
\end{aligned}
$$

Questions and comments - Dr. Erik R. Ivins

Question 12
Eq. 2.179-81 are all the same equations. Why not just say so?

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Question 13
Under what conditions would Eq. 2.183-5 allow stick-slip behavior of the type observed for diurnal and fortnightly forcing (e.g. Anandakrishnan and Alley, Bindschadler, etc.)?

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Question 13
Under what conditions would Eq. 2.183-5 allow stick-slip behavior of the type observed for diurnal and fortnightly forcing (e.g. Anandakrishnan and Alley, Bindschadler, etc.)?

- Additional modeling of subglacial processes required to model dynamically, such as water transport along the glacial bed, subglagical till rheology,...
- Simple parametrization of the sliding coefficient would enable "kinematic" modeling of such basal activation process

Questions and comments - Dr. Erik R. Ivins

Question 14
Christian Schoof has demonstrated that much physics is lost in the classical SIA, and cannot examine the case of negative bed-slope instabilities, for example (Schoof, C. 2007. Marine ice sheet dynamics. Part I: The case of rapid sliding. J. Fluid Mech., 573, 27-55). Will the development here evolve to the point where the equivalent Schoof formulation(s) are handled - i.e., outside of the strictest sense of the SIA? This is a very important point.

Questions and comments - Dr. Erik R. Ivins

Question 14
Christian Schoof has demonstrated that much physics is lost in the classical SIA, and cannot examine the case of negative bed-slope instabilities, for example (Schoof, C. 2007. Marine ice sheet dynamics. Part I: The case of rapid sliding. J. Fluid Mech., 573, 27-55). Will the development here evolve to the point where the equivalent Schoof formulation(s) are handled - i.e., outside of the strictest sense of the SIA? This is a very important point.

- We believe, and the number of numerical benchmarks have demonstrated that the SIA-I approach substantially exceeds in applicability the SIA. We have seen in the ISMIP-HOM benchmark that higher-order physics can be captured very well up to ($\epsilon = \frac{1}{10}$). It is one of our future plans to apply the SIA-I to transition zone dynamics and test its performance compared with Schoof Blatter approximation or full-Stokes solutions.

Questions and comments - Dr. Erik R. Ivins

Question 15
In 3.30-3.34 all terms have X A S . Why not define it as a new parameter and explain its significance?

Questions and comments - Dr. Erik R. Ivins

Question 15
In 3.30-3.34 all terms have X A S . Why not define it as a new parameter and explain its significance?

$$
\begin{aligned}
\frac{\partial \tilde{v}_x}{\partial \tilde{x}} &= \epsilon^{-2} \mathcal{X} \tilde{\mathcal{A}} \tilde{\mathcal{S}} \tilde{\boldsymbol{\sigma}}_{xx} \ , \\
\frac{\partial \tilde{v}_y}{\partial \tilde{y}} &= \epsilon^{-2} \mathcal{X} \tilde{\mathcal{A}} \tilde{\mathcal{S}} \tilde{\boldsymbol{\sigma}}_{yy} \\
\frac{\partial \tilde{v}_x}{\partial \tilde{y}} + \frac{\partial \tilde{v}_y}{\partial \tilde{x}} &= 2\epsilon^{-2} \mathcal{X} \tilde{\mathcal{A}} \tilde{\mathcal{S}} \tilde{\boldsymbol{\sigma}}_{xy} \\
\frac{\partial \tilde{v}_x}{\partial \tilde{z}} + \epsilon^{2} \frac{\partial \tilde{v}_z}{\partial \tilde{x}} &= 2\epsilon^{-1} \mathcal{X} \tilde{\mathcal{A}} \tilde{\mathcal{S}} \tilde{\boldsymbol{\sigma}}_{xz} \\
\frac{\partial \tilde{v}_y}{\partial \tilde{z}} + \epsilon^{2} \frac{\partial \tilde{v}_z}{\partial \tilde{y}} &= 2\epsilon^{-1} \mathcal{X} \tilde{\mathcal{A}} \tilde{\mathcal{S}} \tilde{\boldsymbol{\sigma}}_{yz}
\end{aligned}
$$

- $\mathcal{X}$ ... dimensionless parameter
- $\tilde{\mathcal{A}}$ ... dimensionless rate function
- $\tilde{\mathcal{S}} = \tilde{\boldsymbol{\sigma}}_{xx}^2 + \tilde{\boldsymbol{\sigma}}_{yy}^2 + \tilde{\boldsymbol{\sigma}}_{xx}\tilde{\boldsymbol{\sigma}}_{yy} + \tilde{\boldsymbol{\sigma}}_{xy}^2 + \tilde{\boldsymbol{\sigma}}_{xz}^2 + \tilde{\boldsymbol{\sigma}}_{yz}^2$

Questions and comments - Dr. Erik R. Ivins

Question 16
After Eq. 3.46. Nice description!

Questions and comments - Dr. Erik R. Ivins

Question 16
After Eq. 3.46. Nice description!

- Thank you!

Questions and comments - Dr. Erik R. Ivins

Question 17
Page 55. What is experiment C?

Questions and comments - Dr. Erik R. Ivins

Question 17
Page 55. What is experiment C?

- ISMIP-HOM experiment C described in detail in Section 3.5.7. on p.67.
- Higher-order experiment forced by irreqularity in basal sliding coefficient rather than bedrock topography as in experiments A, B.

Questions and comments - Dr. Erik R. Ivins

Question 18
Bottom page 56. How stable is the nonlinear algebraic solving routine?

Questions and comments - Dr. Erik R. Ivins

Question 18
Bottom page 56. How stable is the nonlinear algebraic solving routine?

- Not really stable. We had great problems with implementing the ISMIP-HOM experiment A setting in full-Stokes. We performed several attempt with commercial FEM solvers, without success. The developed simple full-Stokes solver converged only if good initial guess was provided (from the SIA-I routine), SIA as initial wasn't close enough.

Questions and comments - Dr. Erik R. Ivins

Question 19
Page 73. "We solve a Stokes-flow problem looking for a steady-state solution.." But what evidence is there that a steady solution exists? Did Pattyn supply one? Or is this data? Please elaborate.

Questions and comments - Dr. Erik R. Ivins

Question 19

Page 73. "We solve a Stokes-flow problem looking for a steady-state solution.." But what evidence is there that a steady solution exists? Did Pattyn supply one? Or is this data? Please elaborate.

- Dronning Maud Land, Antarctica, simulation
- Steady-state character of the sought solution is given merely by the technique we use - the SIA-I algorithm designed to solve the Stokes-flow given for a fixed geometry, being in this sense steady state (time appears in the equations only as a parameter). This doesn't correspond to a steady-state of the evolving physical system.
- By existence of the steady-state we thus mean the existence of a solution to the Stokes-flow problem for the given conditions, which was hopefully proved for this type of power-law fluid.

Questions and comments - Dr. Erik R. Ivins

Question 20

Section 4.1, Introduction. So an evolution problem will be solved by updating a series of steady stokes flow problems. Hmm? There must be some history of someone else performing the evolution problem in a similar SIA way. Who? When? And how successful?

Questions and comments - Dr. Erik R. Ivins

Question 20
Section 4.1, Introduction. So an evolution problem will be solved by updating a series of steady stokes flow problems. Hmm? There must be some history of someone else performing the evolution problem in a similar SIA way. Who? When? And how successful?

- I presume that this question is a consequence of possibly incorrectly used term "steady-state" Stokes solution as already discussed in the previous question.
- Again, by steady-state in each time step, we mean, that since the material time derivative of velocity in the momentum equation has been neglected, time remains there only as a parameter and formally, we thus solve a Stokes steady-state problem for one particular value of this parameter.
- From this point of view all existing glaciological models use the same approach, including SIA models (Greve, Ritz, Huybrechts) Higher-Order (Pattyn, Hindmarsch) or FEM (Gagliardini)

Questions and comments - Dr. Erik R. Ivins

Question 21

In further development we see that the 'free' surface has time-dependence. Please summarize what is explicitly time dependent and what is not. Is the thermal equation being solved with dT/dt explicitly? And the advection of U dot Grad T ? How does the margin of the ice sheet march forward/ backward? Are you treating ice rises?

Explicit scheme for free surface evolution

$$\frac{\partial \varphi(\vec{x},t)}{\partial t} = L(\varphi(\vec{x},t)) \; ,$$
$$\varphi(\vec{x},t_0) = \varphi_0(\vec{x})$$

- Runge Kutta method for time-discretisation

$$\begin{aligned}
\hat{\varphi}^{n+1} &= \varphi^n + \Delta t \, L(\varphi^n) \\
\hat{\varphi}^{n+\frac{1}{2}} &= \varphi^n + \frac{\Delta t}{4}\Big(L(\varphi^n) + L(\hat{\varphi}^{n+1})\Big) \\
\varphi^{n+1} &= \varphi^n + \frac{\Delta t}{6}\Big(L(\varphi^n) + 4L(\hat{\varphi}^{n+\frac{1}{2}}) + L(\hat{\varphi}^{n+1})\Big)
\end{aligned}$$

Questions and comments - Dr. Erik R. Ivins

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Explicit scheme for free surface evolution

$$\frac{\partial \varphi(\vec{x}, t)}{\partial t} = L(\varphi(\vec{x}, t)) \ ,$$
$$\varphi(\vec{x}, t_0) = \varphi_0(\vec{x})$$

- Essentially non-oscillatory (ENO) schemes + upwinding used to discretize the spatial differential operator $L = -\vec{v} \cdot \mathrm{grad}\varphi$
  - $v_x \geq 0$:

$$v_x \, \frac{\partial \varphi}{\partial x}(x_i) \simeq v_x(x_i) \frac{\hat{\varphi}^-_{i+\frac{1}{2}} - \hat{\varphi}^-_{i-\frac{1}{2}}}{x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}}$$

  - $v_x < 0$:

$$v_x \, \frac{\partial \varphi}{\partial x}(x_i) \simeq v_x(x_i) \frac{\hat{\varphi}^+_{i+\frac{1}{2}} - \hat{\varphi}^+_{i-\frac{1}{2}}}{x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}}$$

Questions and comments - Dr. Erik R. Ivins

Question 21
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Semi-implicit scheme for free surface evolution SIA

$$\frac{\partial \tilde{f}_s}{\partial \tilde{t}} + \tilde{v}_x(\cdot, \tilde{f}_s(\cdot))\frac{\partial \tilde{f}_s}{\partial \tilde{x}} + \tilde{v}_y(\cdot, \tilde{f}_s(\cdot))\frac{\partial \tilde{f}_s}{\partial \tilde{y}} - \tilde{v}_z(\cdot, \tilde{f}_s(\cdot)) = \tilde{a}^s(\cdot, \tilde{f}_s(\cdot))$$

Questions and comments - Dr. Erik R. Ivins

Question 21

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Semi-implicit scheme for free surface evolution SIA

$$\frac{\partial \tilde{f}_s}{\partial \tilde{t}} + \frac{\partial}{\partial \tilde{x}} \int_{\tilde{f}_b}^{\tilde{f}_s} \tilde{v}_x(\cdot, \tilde{z}')\, d\tilde{z}' + \frac{\partial}{\partial \tilde{y}} \int_{\tilde{f}_b}^{\tilde{f}_s} \tilde{v}_y(\cdot, \tilde{z}')\, d\tilde{z}' = \tilde{a}^s \ .$$

Questions and comments - Dr. Erik R. Ivins

Question 21
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Semi-implicit scheme for free surface evolution SIA

$$\frac{\partial \tilde{f}_s}{\partial \tilde{t}} - \frac{\partial}{\partial \tilde{x}}\left(\tilde{D}\frac{\partial \tilde{f}_s}{\partial \tilde{x}}\right) - \frac{\partial}{\partial \tilde{y}}\left(\tilde{D}\frac{\partial \tilde{f}_s}{\partial \tilde{y}}\right) = \tilde{a}^s$$

$$\tilde{D}(\cdot) := 2\mathcal{K}\left(\left(\frac{\partial \tilde{f}_s}{\partial \tilde{x}}\right)^2 + \left(\frac{\partial \tilde{f}_s}{\partial \tilde{y}}\right)^2\right)\int_{\tilde{f}_b}^{\tilde{f}_s}\int_{\tilde{f}_b}^{\tilde{z}'} \tilde{\mathcal{A}}(\tilde{T})(\tilde{f}_s - \tilde{z}'')^3\, d\tilde{z}''\, d\tilde{z}'$$

Questions and comments - Dr. Erik R. Ivins

Question 21

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Semi-implicit scheme for free surface evolution SIA

$$\frac{\tilde{f}_s^{n+1} - \tilde{f}_s^n}{\Delta \tilde{t}} - \frac{\partial}{\partial \tilde{x}} \left(\tilde{D}^n \frac{\partial \tilde{f}_s^{n+1}}{\partial \tilde{x}} \right) - \frac{\partial}{\partial \tilde{y}} \left(\tilde{D}^n \frac{\partial \tilde{f}_s^{n+1}}{\partial \tilde{y}} \right) = (\tilde{a}^s)^n$$

Questions and comments - Dr. Erik R. Ivins

Question 21

In further development we see that the 'free' surface has time-dependence. Please summarize what is explicitly time dependent and what is not. Is the thermal equation being solved with dT/dt explicitly? And the advection of U dot Grad T ? How does the margin of the ice sheet march forward/ backward? Are you treating ice rises?

Semi-implicit scheme for free surface evolution SIA-I

$$\frac{\partial \tilde{f}_s}{\partial \tilde{t}} - \frac{\partial}{\partial \tilde{x}}\left(\tilde{D}_x \frac{\partial \tilde{f}_s}{\partial \tilde{x}}\right) - \frac{\partial}{\partial \tilde{y}}\left(\tilde{D}_y \frac{\partial \tilde{f}_s}{\partial \tilde{y}}\right) = \tilde{a}^s$$

$$\tilde{D}_x := \frac{\int_{\tilde{f}_b}^{\tilde{f}_s} \tilde{v}_x \, d\tilde{z}'}{\frac{\partial \tilde{f}_s}{\partial \tilde{x}}} \qquad \tilde{D}_y := \frac{\int_{\tilde{f}_b}^{\tilde{f}_s} \tilde{v}_y \, d\tilde{z}'}{\frac{\partial \tilde{f}_s}{\partial \tilde{y}}}$$

Questions and comments - Dr. Erik R. Ivins

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Semi-implicit scheme for free surface evolution

$$\frac{\tilde{f}_s^{n+1} - \tilde{f}_s^n}{\Delta \tilde{t}} - \frac{\partial}{\partial \tilde{x}} \left(\tilde{D}_x^n \frac{\partial \tilde{f}_s^{n+1}}{\partial \tilde{x}} \right) - \frac{\partial}{\partial \tilde{y}} \left(\tilde{D}_y^n \frac{\partial \tilde{f}_s^{n+1}}{\partial \tilde{y}} \right) = (\tilde{a}^s)^n$$

Questions and comments - Dr. Erik R. Ivins

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Heat-transport equation

$$
\begin{aligned}
& \hat{c}_v \left(\tilde{\mathcal{H}}^2 \frac{\partial \tilde{T}}{\partial \tilde{t}} - \tilde{\mathcal{H}} \frac{\partial \tilde{T}}{\partial \xi} \left(\tilde{v}_x a_x \tilde{\mathcal{H}} + \tilde{v}_y a_y \tilde{\mathcal{H}} - \tilde{v}_z + a_t \tilde{\mathcal{H}} \right) + \tilde{\mathcal{H}}^2 \left(\tilde{v}_x \frac{\partial \tilde{T}}{\partial \tilde{x}} + \tilde{v}_y \frac{\partial \tilde{T}}{\partial \tilde{y}} \right) \right) \\
- \quad & \mathcal{D} \frac{\partial}{\partial \xi} \left(\tilde{k} \frac{\partial \tilde{T}}{\partial \xi} \right) = 2\mathcal{C}\tilde{\mathcal{H}}^2 (\tilde{\boldsymbol{\sigma}}_{xz} \tilde{\boldsymbol{d}}_{xz} + \tilde{\boldsymbol{\sigma}}_{yz} \tilde{\boldsymbol{d}}_{yz}) \; .
\end{aligned}
$$

Questions and comments - Dr. Erik R. Ivins

Question 21
In further development we see that the 'free' surface has time-dependence. Please summarize what is explicitly time dependent and what is not. Is the thermal equation being solved with dT/dt explicitly? And the advection of U dot Grad T ? How does the margin of the ice sheet march forward/ backward? Are you treating ice rises?

Heat-transport equation

- Vertical derivative of T treated implicitly in time, rest explicitly (Greve, 1997)

$$\hat{c}_{\boldsymbol{v}} \left(\tilde{\mathcal{H}}^{\boldsymbol{n}\,\boldsymbol{2}} \frac{\tilde{T}^{\boldsymbol{n}+\boldsymbol{1}} - \tilde{T}^{\boldsymbol{n}}}{\Delta \tilde{t}} - \tilde{\mathcal{H}} \frac{\partial \tilde{T}^{\boldsymbol{n}+\boldsymbol{1}}}{\partial \xi} \left(\tilde{v}_{\boldsymbol{x}} a_{\boldsymbol{x}} \tilde{\mathcal{H}} + \tilde{v}_{\boldsymbol{y}} a_{\boldsymbol{y}} \tilde{\mathcal{H}} - \tilde{v}_{\boldsymbol{z}} + a_{\boldsymbol{t}} \tilde{\mathcal{H}} \right)^{\boldsymbol{n}} + \tilde{\mathcal{H}}^{\boldsymbol{n}\,\boldsymbol{2}} \left(\tilde{v}_{\boldsymbol{x}}^{\boldsymbol{n}} \frac{\partial \tilde{T}^{\boldsymbol{n}}}{\partial \tilde{x}} + \tilde{v}_{\boldsymbol{y}}^{\boldsymbol{n}} \frac{\partial \tilde{T}^{\boldsymbol{n}}}{\partial \tilde{y}} \right) \right)$$

$$- \quad \mathcal{D} \frac{\partial}{\partial \xi} \left(\tilde{k} \frac{\partial \tilde{T}^{\boldsymbol{n}}}{\partial \xi} \right) = 2 \mathcal{C} \tilde{\mathcal{H}}^{\boldsymbol{n}\,\boldsymbol{2}} (\tilde{\boldsymbol{\sigma}}_{\boldsymbol{xz}} \tilde{\boldsymbol{d}}_{\boldsymbol{xz}} + \tilde{\boldsymbol{\sigma}}_{\boldsymbol{yz}} \tilde{\boldsymbol{d}}_{\boldsymbol{yz}})^{\boldsymbol{n}}$$

Questions and comments - Dr. Erik R. Ivins

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Ice margin treatment

- Thin layer of ice present everywhere - no margin (explicit and implicit)
- Ice margin tracked by the level-set approach (explicit)

Questions and comments - Dr. Erik R. Ivins

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Ice rises

- Treatment of ice rises is theoretically possible within the SIA-I approach, as its limitations are given by the smallness of the aspect ratio of dominant vertical-to-horizontal scales, and by other than free-slip conditions at the base. The ISMIP-HOM experiment F (time-dependent flow over a Gaussian bump) demonstrated its good performance for this situation.

Questions and comments - Dr. Erik R. Ivins

Question 22
Page 89. Before moving on to section 4.3.2, what should the reader now be convinced of?

- In sections 4.3.1. and 4.3.2. we discuss the two implemented approaches to the solution of the kinematic equation for free surface evolution. The first , explicit scheme is discussed in 4.3.1. and is based on the use of Runge Kutta method combined with ENO schemes. Also this explicit approach allows to implement the level-set technique to track the margin of the ice sheet. On the other hand, this approach seems to fail for the large-scale modeling (Greenland Ice Sheet), where the semi-implicit approach described in section 4.3.2 was demonstrated to be of better use. This second approach mimics the standard SIA approach to free-surface evolution by a trick from Pattyn (2003) and results in a parabolic problem, compared to the original hyperbolic transport problem.

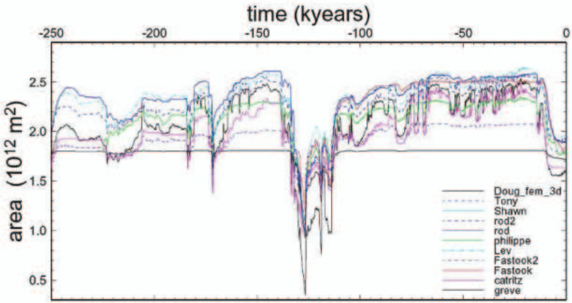
Questions and comments - Dr. Erik R. Ivins

Question 23
The remainder of Section 4.6: Nice job!

- Thank you!

Questions and comments - Dr. Erik R. Ivins

Question 24
Figure 5.4. It would be good to contact Philippe and see if he can supply the original figure. The detail here is important.



- Will be done.

Questions and comments - Dr. Erik R. Ivins

Question 25
Greenland simulations and Concluding Remarks (Section 6). First, there is nothing objectionable about material in pages 108-119. But what is new? These are great achievements, just to write the code from scratch and compete with these 'big guns of glaciology'. If nothing is really new (which can't be true! - Toot your horn man!), then summarize why the EISMINT project is unrealistic in its portrayal of ice sheet evolution (both for Antarctica and for Greenland). What's missing in the models? List them.

- This section deals with the EISMINT benchmark - Greenland Ice Sheet Models.
- The aim is to demonstrate that in the SIA regime the code is capable of reproducing the three listed scenarios, with complexity already resembling the original goal and exceeding the slightly artificial benchmarks listed before this section.

Questions and comments - Dr. Erik R. Ivins

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- What is new? The most important new feature is that the model is capable of providing both the SIA results when only the first iteration of the SIA-I algorithm is taken, but is also capable of improving the SIA solution and including the longitudinal stresses by employing further iterations of the SIA-I algorithm. Only few models exist (Larour, Seddik & Zwinger) that attempt to incorporate other than SIA approach for large-scale modeling. The important feature of the SIA-I approach is its locality, meaning that the improvement of the solution can be performed only locally in regions of interest (ice-streams), while keeping the SIA only in the rest. This corresponds to solving a higher-order approximation with SIA boundary-values.

Questions and comments - Dr. Erik R. Ivins

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- EISMINT Greenland benchmarks are unrealistic, but they don't intend to be fully realistic. This benchmark only aims at testing the outputs of SIA-based models for realistic topography and complex and nature-like climatic forcing.
- Missing physics: Glacial isostatic adjustment of the underlying lithosphere, proper treatment of the sea-level equation, subglacial processes, ice-shelves = floating ice, ice-stream = fast flowing regions with rapid sliding, calving front treatment, proper climatological forcing which should include also feedback between climate and cryosphere, so should be present a feedback between oceanological models and cryosphere and many others... The models that participated in the EISMINT-Greenland benchmark were the following ones: Philippe Huybrechts, Roderik Van De Wal, Lev Tarasov, Ralf Greve, Shawn Marshall, Tony Payne, C.Ritz & A. Fabre, James Fastook, Douglas MacAyeal

Questions and comments - Dr. Erik R. Ivins

Question 26
What is your plan in the future to fix the situation? You seem to emphasize GIA in the end. If these models can't simulate real ice sheets, how can they help with GIA? The answers are tough ones!

- There are many open issues, so my plan is to pick up one of them - grounding line migration and focus on it.
- The GIA mentioned in the end is a goal to be achieved hopefully quite soon. After preparing a spherical version of the code we intend to couple it with the GIA code of prof. Martinec and run a coupled simulation of both processes. The ice-sheet models can't simulate real ice-sheets of course, but physically consistent despite a bit simplistic modeling seems to be the only correct way of treating the problem.
- We need to assess the importance of various processes to the large-scale glaciological modeling as the ultimate goal is not an infinitely-complex nature-like model of "everything" but a sufficiently accurate simplification capturing the important leading-order effects.