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# **Dynamické napětí kinematických modelů seismického zdroje**

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Summary of Doctoral Thesis

# Dynamic Stress Field of Kinematic Earthquake Source Models

by

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# 1 Abstract

Source models such as the  $k$ -squared stochastic source model with  $k$ -dependent rise time are able to reproduce source complexity commonly observed in earthquake slip inversions. An analysis of the dynamic stress field associated with the slip history prescribed in these kinematic models can indicate possible inconsistencies with physics of faulting. A parallelized code for dynamic stress change calculations along a planar fault was developed and applied. The static stress drop, the strength excess, the breakdown stress drop, the critical slip weakening distance  $D_c$  and fracture energy distributions are determined in this study for the kinematic  $k$ -squared source model with  $k$ -dependent rise time. Several studied  $k$ -squared models are found to be consistent with the slip weakening friction law along a substantial part of the fault. A new quantity, the stress delay, is introduced to map areas where the yielding criterion of the slip weakening friction is violated. Hisada's slip velocity function is found to be more consistent with the source dynamics than boxcar, Brune's and Dirac's slip velocity functions. Constant rupture velocities close to the Rayleigh velocity are inconsistent with the  $k$ -squared model, because they break the yielding criterion of the slip weakening friction law. The bimodal character of  $D_c/D_{tot}$  frequency-magnitude distribution was found.  $D_c$  approaches the final slip  $D_{tot}$  near the edge of both the fault and asperity. Both filtering and smoothing routinely applied in slip inversions may have a strong effect on the space-time pattern of the inferred stress field, leading potentially to an oversimplified view of earthquake source dynamics.

## 2 State of the Art

An important practical goal of present seismology is to reduce losses due to earthquakes by estimating probable ground motions for future earthquakes at a given site. A natural way of the solution is to extrapolate the information on past earthquakes. One approach is based on straight extrapolation of ground motions due to previous earthquakes. However, such approach is not reliable at present because recurrence times of strong earthquakes are very long - up to several thousands of years - and first quantitative ground motion observations are scarcely one hundred years old. Moreover, seismic observations are not (even nowadays) dense enough to make a coherent picture of damaging ground motions for a area of even several square kilometers close to the causative fault. Another approach is based on 'deciphering' observed ground motion due to earthquakes employing both the physical theories and mathematical models which could fill the gaps in recent observations. The latter approach is very promising and under rapid development. This thesis belongs to the latter class and is focused on the modelling of an earthquake source. In the following paragraphs we describe the state of the art briefly.

It has been recognized soon that the observed Earth surface shaking contain the information on both seismic wave propagation through the Earth's interior and the source of these waves - earthquake source. Although the idea that the cause of an earthquake is a failure of Earth's material along the fault due to accumulated elastic strain was formulated by Reid already in 1910, the quantitative description of that phenomenon was introduced to geophysics almost fifty years later, e.g., by [Steketee \(1958\)](#).

The two fundamental approaches have developed for the earthquake source representation - the *dislocation models* and the *crack models*. The dislocation model, based on the works of [Volterra \(1907\)](#) and [Somigliana \(1914, 1915\)](#), prescribes a space-time slip (displacement jump) distribution along the fault. This slip distribution is not constrained by the dislocation model itself. In other words, for example, it not necessarily reflects the properties of the material surrounding the fault, the slip could appear on the fault instantaneously etc. It is clear that this is not the case of the earthquake source - the true slip distribution always reflects somehow the stress state along the fault. Nevertheless, the dislocation model has been widely applied in the seismology to both the forward and inverse problems. The crack model, based on the work of [Griffith \(1921\)](#), is quite different. Griffith studied the energy balance on the fracture edge and introduced the formal criterion for the fracture extension. Such approach was very general but not practical. To model the displacement jump along the fault, one had to assume physical law governing the fracturing, initial stress and elastodynamic response of the medium surrounding the fault. Despite the fact that neither initial stress nor governing constitutive relations were known for the faults, a formal solution represented also a tough mathematical problem itself with very few known an-

alytical solutions. Nonetheless, fracture mechanics represents a solid framework for the earthquake source physics.

The dislocation and crack models are not far from each other. A crack model results exactly in a specific dislocation model. It was already pointed by [Steketee \(1958\)](#) that the Griffith crack could be also considered as a type of the Somigliana dislocation. This is analogous to the two descriptions of motion of a material point. If we know the trajectory of the point, we speak about material-point kinematics. On the other hand, if we know the force acting on this point, the laws governing its motion (e.g., Newton laws) and the trajectory is to be determined, we speak about the material-point dynamics. Hence, the term *kinematic fault model* is usually used for the dislocation model and the term *dynamic fault model* for the crack model.

The kinematic and dynamic models can be simply distinguished by their fault boundary conditions for the elastodynamic equation of motion. For the kinematic model we prescribe boundary conditions

$$\begin{aligned}\tau_{mn}^+ \nu_m - \tau_{mn}^- \nu_m &= 0 \\ u_n^+ - u_n^- &= \Delta u_n(\boldsymbol{\xi}, t),\end{aligned}\tag{1}$$

along the fractured fault surface  $\Sigma^*(t)$ , which is changing with time (as rupture grows over the entire fault  $\Sigma$ ), where  $u^+$  ( $u^-$ ) are displacements on the positive (negative) side of the fault,  $\Delta u_n(\boldsymbol{\xi}, t)$  is the displacement jump (slip),  $\tau_{mn}^+$  ( $\tau_{mn}^-$ ) are the stress perturbations on the positive (negative) side of the fault and  $\boldsymbol{\xi}$  denotes the position along the fractured fault  $\Sigma^*(t)$ . Hence, one could obtain both the displacement and stress in the whole volume by solving the elastodynamic equation with the boundary condition (1). The problem is linear.

For the dynamic model we prescribe boundary conditions

$$\begin{aligned}\tau_{mn}^+ \nu_m - \tau_{mn}^- \nu_m &= 0 \\ \tau_{mn} \nu_m &= -\sigma_{mn}^0 \nu_m + \Upsilon_n(\sigma^\nu, \Delta \mathbf{u}, \Delta \dot{\mathbf{u}}, \theta, \dots, \boldsymbol{\xi}),\end{aligned}\tag{2}$$

along the fractured fault surface  $\Sigma^*(t)$ , which is changing with time (as rupture grows over the entire fault  $\Sigma$ ) where the function  $\Upsilon_n$  represents the law controlling the fracture,  $\sigma_{mn}^0$  is the initial stress state (the stress state just before the earthquake). In the earthquake dynamics the term *friction law* (or fault constitutive relation) is usually used for it.  $\Upsilon_n$  generally depends on a large number of parameters (see [Bizzarri & Cocco, 2005](#); for general discussion), we show just the most important ones - the normal stress along the fault ( $\sigma^\nu = \sigma_{mn} \nu_m \nu_n$ ), the displacement jump  $\Delta \mathbf{u}$ , the displacement jump rate  $\Delta \dot{\mathbf{u}}$ , the fault state variable  $\theta$  and the position along the fractured fault  $\boldsymbol{\xi}$ . A lot of authors present a much broader set of dependencies, however, it is known very little about the true fault friction laws, so in practice, the huge set of the proposed dependencies shrinks usually to these mentioned above. Hence, one could obtain both the displacement (including the displacement jump along the fault) and stress in the whole

volume by solving the elastodynamic equation with the boundary condition (2) along  $\Sigma^*(t)$ . The time dependence of  $\Sigma^*(t)$ , i.e., the rupture propagation, is part of the solution which is determined by some fracture criterion (e.g., Griffith criterion, Kostrov & Das, 1988). It has to be determined simultaneously with the solution of elastodynamic equation, thus the problem is non-linear and hard to solve. The term 'spontaneous rupture' is usually used in the connection with these dynamic models. Sometimes, instead, the rupture front propagation is assumed a priori to make the problem easier (e.g., Madariaga, 1976).

The clear difference between the two approaches (kinematic vs. dynamic model) can be recognized from the boundary condition (2) which depends on the solution of the elastodynamic equation itself (e.g., the displacement jump is present in the argument of friction law in (2)). Another, less apparent, but a serious difference is the time dependence of  $\Sigma^*(t)$ . In the kinematic model, the time dependence of the fractured area is immaterial, as we can prescribe some ultimate fault area (e.g.,  $\Sigma^*(t) \rightarrow \infty$ ) and control the rupture directly by the prescription of the displacement jump (zero displacement jump is prescribed at the unfractured part of the fault).

Since they were introduced to seismology, both the kinematic and dynamic earthquake source models have developed considerably (reviews by, e.g., Kostrov & Das, 1988; Madariaga & Olsen, 2002; Das, 2003). However, only kinematic models have so far found their broad direct use in interpreting seismic data. Kinematic models were applied for both forward and inverse problems. The forward kinematic problem concern calculation of ground motion due to prescribed space-time slip distribution along the fault, these find mainly use in strong ground motion simulation studies or in modelling of other earthquake induced phenomena (tsunami, landslides, etc.). Various slip distribution have been proposed from simple uniform slip models (Haskell, 1964) to stochastic  $k$ -squared models (Andrews, 1980; Herrero & Bernard, 1994; Mai & Beroza, 2002; Gallovič & Brokešová, 2007). The inverse kinematic problem, usually called kinematic source inversion or slip inversion, is formulated as a retrieval of space-time slip distribution along the fault (Spudich, 1980), fitting the observed records of both the static (GPS, InSAR) and dynamic displacements (seismograph records) on the Earth surface. It is a quite peculiar problem, which is inherently unstable (Kostrov & Das, 1988), and some a priori constraints are necessary for its solution (e.g., so called slip positivity constraint). The well-recorded 1979 Imperial Valley, California, earthquake was an impulse for developing kinematic source inversions (Olson & Apsel, 1982; Hartzell & Heaton, 1983). Since that time kinematic source inversion have been performed for a significant set of earthquakes.

Analytic dynamic source models are known only for the fractures which are semi-infinite (e.g., Kostrov, 1966), expand self-similarly forever (e.g., Kostrov, 1964; Burridge & Willis, 1969; Nielsen & Madariaga, 2003) or for steady state fracture pulses (Broberg, 1978; Freund, 1979; Rice *et al.*, 2005), and these are hardly applicable for real cases. As a computer power increased a couple of

dynamic models of recent earthquakes were also developed in 90' (e.g., [Quin \(1990\)](#) for the M6.5 1979 Imperial Valley earthquake and [Olsen \*et al.\* \(1997\)](#) for the M7.2 1992 Landers earthquake). These models were constructed by the trial-error method from the results of kinematic source inversions. Until the work of [Quin \(1990\)](#), the kinematic and dynamic source models were treated more or less separately. Quin recognized, that some features of the kinematic source model of real earthquake could be utilized for the boundary conditions in the forward dynamic modelling. The work of Quin was followed by group of Japanese scientists who extended Quin's trial and error approach with several iterative schemes ([Mikumo, 1994](#); and references therein). The common point of these first dynamic inversions was that they all utilized forward dynamic source modelling for the retrieval of frictional parameters along the fault for an a priori given friction law.

An alternative approach of the dynamic source model parameters estimation from the kinematic source model was presented by [Chen & Aki \(1996\)](#). They assumed neither any friction law nor forward dynamic modelling, they just solved the elastodynamic equation for the stress perturbation  $\tau_{mn}^{(\Delta u')}(\boldsymbol{\xi}, t)$  due to the space-time slip distribution of the kinematic model  $\Delta \mathbf{u}'(\boldsymbol{\xi}, t)$ . Now the question arises: 'What can the stress perturbation  $\tau_{mn}^{(\Delta u')}$  tell us about the possible friction on the fault?' To provide an answer, let us check the fault boundary conditions for the dynamic model. The relation between the stress perturbation and friction can be read from the equation (2):

$$\tilde{\Upsilon}_n(\sigma^{\nu'}, \Delta \mathbf{u}', \Delta \dot{\mathbf{u}}', \theta', \dots, \boldsymbol{\xi}) = \tau_{mn}^{(\Delta u')} \tilde{\nu}_m + \sigma_{mn}^{0'} \tilde{\nu}_m, \quad (3)$$

where  $\tilde{\Upsilon}$  is the unknown friction law. The dash symbol denotes the properties connected with the particular earthquake, whereas the tilde symbol denotes the properties connected with the particular fault (friction law and fault geometry). Hence, e.g., initial stress  $\sigma_{mn}^{0'}$  is the reference state just before the earthquake. Thus, if we know the initial stress  $\sigma_{mn}^{0'}(\boldsymbol{\xi})$ , we can determine resulting friction at the fault for the earthquake. Generally, performing such procedure for several earthquakes acting on the same fault would lead to the estimation of the possible friction laws holding at that particular fault. It is useful to substitute for the time dependency in the stress perturbation  $\tau_{mn}^{(\Delta u')}(\boldsymbol{\xi}, t)$  as the friction laws do not usually depend explicitly on time, but rather on the other quantities listed in the argument of the friction law  $\tilde{\Upsilon}_n(\sigma^{\nu'}, \Delta \mathbf{u}', \Delta \dot{\mathbf{u}}', \theta', \dots, \boldsymbol{\xi})$ . In other words, it is reasonable to study, e.g.,  $\tau_{mn}^{(\Delta u')}[\sigma^{\nu'}(\boldsymbol{\xi}, t), \Delta \mathbf{u}'(\boldsymbol{\xi}, t), \Delta \dot{\mathbf{u}}(\boldsymbol{\xi}, t), \boldsymbol{\xi}]$ .

An estimation of the fault constitutive relations  $\Upsilon$  outlined above is much more straightforward and easier to solve than the approach of [Mikumo \(1994\)](#), who employs forward dynamic modelling. Moreover, one does not have to assume a friction law a priori, as in the dynamic modelling. In practice, the estimation of the fault friction parameters strongly depends on the 'quality' of the utilized kinematic models, i.e., on the space-time resolution of the slip, accompanied with serious difficulties mentioned above. A very serious problem also is also

represented by the initial stress present in (3), which is generally heterogeneous but not known in the crust. The homogenous initial stress is usually assumed. Nevertheless such an assumption can clearly lead to biased friction estimations. However, the problem of the initial stress is common to the forward dynamic problem, too. A number of authors applied the methodology described above for the friction estimation along existing faults (Chen & Aki, 1996; Bouchon 1997; Ide & Takeo 1997; Day *et al.*, 1998; Dalguer *et al.*, 2002; and Tinti *et al.*, 2005b) for several kinematic source models of real earthquakes. Authors of these studies did not study quantitatively the effects of space-time filtering which is applied during the kinematic inversion, although it can affect the results considerably. Also uncertainties of the space-time slip distributions were not take into account. A partial step forward was done in the papers by Piatanesi *et al.* (2004) and Tinti *et al.* (2005a), where the authors study the influence of slip velocity function on the friction parameters estimation.

Nevertheless, opposed to studies based on slip inversions of real earthquakes, one can also investigate the stress implications of the theoretical slip distributions. There is a broad class of theoretical kinematic models, which are used mainly for the near fault ground motion modelling. The most widely applied has been the Haskell fault model (Haskell, 1964; Haskell, 1969) and more recently the  $k$ -squared source model (e.g., Andrews, 1981; Herrero & Bernard, 1994; Bernard *et al.*, 1996; Hisada, 2000; Gallovič & Brokešová, 2007). The dynamic stress field of the Haskell model was studied by Madariaga (1978), who found strong contradictions to earthquake source physics. The static stress field of  $k$ -squared model was analyzed by Andrews (1980), however,  $k$ -squared model has undergone some development since that time (e.g.,  $k$ -squared models with asperities, Gallovič & Brokešová, 2004). The dynamic stress field of  $k$ -squared models has not been studied yet. Thus it has been nearly impossible to confront the  $k$ -squared model with the earthquake dynamics, e.g., it has not been assured yet whether the  $k$ -squared model is consistent with earthquake source physics.

### 3 Objectives and Goals

Based on the previous analysis, the main goal of this thesis is formulated as to fill the existing gap between  $k$ -squared models recently developed for the strong ground motion predictions and recent findings in the dynamic source model studies. To this goal, we develop a method of dynamic stress calculations, apply the method to the  $k$ -squared model and perform the quantitative analysis of the resulting stress field. Such computation and analysis of the dynamic stress field due to  $k$ -squared model should clarify the physical basis of this model. Particularly, we want to clarify if the  $k$ -squared model is consistent with some friction laws of the earthquake dynamics. Moreover,  $k$ -squared source model has a lot free parameters, which are to be tuned a priori. A parametric study concerning the dynamic stress field analysis can help in constraining some of these parameters.

Thus the study can be useful for future strong ground motion simulations. The space-time filtering effects on the dynamic stress field can be also demonstrated for the case of  $k$ -squared model, as it includes short-wavelength and short-period variations which are smeared out in the results of kinematic inversions. Thus it is possible to discuss these effects quantitatively.

To perform the study outlined above, we have to adopt some method of dynamic stress field calculation. Particularly, the method which is capable of solving the elastodynamic equation with the boundary condition (1) including the solution at the boundary itself. We decided to adopt the boundary element method, calculating Green's functions by the discrete wavenumber method of Bouchon (1997).

## 4 Results

### 4.1 Method of Calculation of Dynamic Stress

The method of calculation adopted in this thesis, numerical tests and some basic applications are presented in Chapter 2. To determine the dynamic stress history on a fault from the space-time slip distribution prescribed by the kinematic model we have used the boundary element method (BEM) proposed by Bouchon (1997). In particular, a parallelized code (applying *OpenMP* directives) for dynamic stress change calculations along a planar fault in a homogeneous unbounded isotropic medium was developed. In this method, the shear stress change is expressed as the space-time convolution of the slip time functions and stress Green's functions which are calculated using the discrete wavenumber (DW) method, particularly by computing the Weyl-like integrals by 2D discrete Fourier series. The time convolution is performed in the spectral domain, while the space convolution in the space domain. A rectangular fault situated in an infinite isotropic homogeneous space is assumed, approximating a buried fault. The mathematical basis of the discrete wavenumber method is presented with complete derivations in Appendix A. Formulas (A.64-A.81) for the stress Green's functions are originally presented in such general form in this work.

### 4.2 Test and Demonstration of the Method

The static part of the solution obtained by BEM-DW method is tested against an analytical solution for the static circular crack. It has been shown that the drawback of the Fourier methods - the Gibb's phenomenon can be controlled in the dynamic stress calculations by a low-pass filter. A removal of high wavenumber contribution with an appropriately smooth filter is equivalent to the smoothing of the slip distribution (Fig. 2.2, Fig. 2.3).

Further, the dynamic stress field of the Haskell fault model is presented, as basic features of the transient stress within the fractured fault can be simply

described for this model.

The dynamic stress field on a planar fault due to a circular crack was also investigated. The stress field outside the crack itself was of the main interest, to demonstrate the stress radiation pattern along the fault plane (an original contribution of the thesis). Both slip-parallel and slip-perpendicular components of the stress change along the fault are presented in Fig. 2.8, 2.9. The slip-parallel component of the dynamic stress change has generally a larger magnitude than the slip-perpendicular component. The main characteristics of the slip-parallel component are that the static solution dominates up to distance of about  $5R$  ( $R$  is the crack radius) from the center and maximal transient stress changes lie on the horizontal crack axis which is parallel to the slip vector. On the other hand, the main characteristics of the slip-perpendicular component are that the static solution dominates up to distance of about  $R$  from the center and the stress radiation pattern follows the static stress change distribution. The dynamic stress changes may induce seismicity (e.g., review by Steacy *et al.*, 2005; and references therein). In particular, these results have been already applied in the study concerning dynamic stress triggering of earthquakes during Western Bohemia seismic swarms published by Fischer & Horálek (2005).

### 4.3 Dynamic Stress Field of the $k$ -squared Model

The dynamic stress field calculations and analysis of the  $k$ -squared model are presented in Chapter 3. This chapter represents the main results of the thesis. Both the methods described in Gallovič & Brokešová (2004) and Gallovič & Brokešová (2007) were applied for generating  $k$ -squared static slip distributions with asperities. The slip time histories were prescribed adopting the general formulation of so called  $k$ -dependent rise time described in Gallovič & Brokešová (2004). The full dynamic stress fields (including static solution) were calculated and analyzed thoroughly for more than twenty  $k$ -squared models of hypothetical  $M_w = 5.9$  earthquakes. The static stress field, presented for eight different  $k$ -squared models, was found to be controlled by the properties of the asperities and strongly influenced by the wavenumber filtration (Fig. 3.1). The dynamic stress field was analyzed in terms of the slip weakening friction law. In particular, strength excess, dynamic stress drop, breakdown stress drop, critical slip weakening distance  $D_c$  and fracture energy were determined.  $D_c$  was determined directly by the fitting stress-slip curves. A new parameter, the stress delay  $T_x$ , was introduced to map the fault points where the peak stress precedes the rupture onset. Attention was paid to the stress recovery (dynamic minus static stress drop) associated with the slip pulse, the slip pulse being an ingredient of the  $k$ -squared model with  $k$ -dependent rise time. Systematically higher values of stress recovery were found close to the nucleation point for narrow pulses ( $L_0 < L/5$ , where  $L$  is the length of the fault). The amount of average stress recovery was found to be inversely proportional to the pulse width (Fig. 3.8), which is consistent with

analytic results for steady state pulses (Broberg, 1978; Freund, 1979).

As the  $k$ -squared model with  $k$ -dependent rise time has a relatively high number of free model parameters (length and width of the fault, asperity slip contrast, slip roughness - represented by  $K_c$ , stochastic slip distribution, number and positions of asperities, position of nucleation point, rupture velocity, maximum rise time, coefficient  $a$ , and slip velocity function (SVF) type), it represents a large set of source models. A parametric study was performed, covering some sections of this model space to explore its projection into the space of dynamic parameters, to possibly find some restrictions on the  $k$ -squared model originating from source dynamics.

The model with Hisada’s SVF, ‘PM’ slip distribution (Fig. 3.1) with the asperity in a corner of the fault, rupture velocity  $v_r=2.6$  km/s, maximum rise time  $\tau_{max} = 0.1$ , cut-off frequency  $f_{max} = 12Hz$ , and nucleation point at the left border of the fault was chosen as a reference model. The dimensions of the fault ( $L=11$  km,  $W=8$  km), direction of the slip (rake= $0^\circ$ ), scalar seismic moment  $M_0 = 7.8 \times 10^{17}$  Nm, number of asperities (=1) and asperity slip contrast (=2) were fixed. An influence of the rupture velocity (2.3 km/s, 2.9 km/s, 3.18 km/s), maximum rise time (0.5 s, 2.0 s), nucleation point position (center and right border of the fault), slip velocity function shape (Boxcar, Brune’s SVF) and static slip distribution ( $K_c = 0.5_I, K_c = 0.5_{II}, K_c = 0.75_I, K_c = 0.75_{II}$ , ‘CA’ - asperity in the center of the fault) were studied. Each of these were studied separately, fixing the other parameters at the reference values. Only sub-Rayleigh ( $\leq 3.18$  km/s) rupture velocities were concerned.

The results of the parametric study are presented in Fig. 3.16, 3.17, 3.18. The values of the strength excess and breakdown stress drop are in tenths of MPa as for real earthquakes as found, e.g., by Bouchon (1997), Piatanesi *et al.* (2004). The average value of  $D_c/D_{tot}$  in the positive stress drop area is  $0.65 \pm 0.05$  (except for the cases of Brune’s SVF, Boxcar SVF and  $\tau_{max} = 0.5$  s) which is similar to the value 0.63 found by Zhang *et al.* (2003) for the 1999 Chi-Chi earthquake. The  $D_c/D_{tot}$  frequency-magnitude distributions present two maxima (Fig. 3.18a), it is ‘bimodal’. One maximum lays around 50% and the other at 100%.  $D_c \simeq D_{tot}$  is found at the edges of the both asperity and fault. Rupture velocities close to the Rayleigh velocity  $v^R$  lead to a worse linear slip weakening fit (65% variance reduction for  $v_r = v^R$ ) and large areas of both negative strength excess and non-zero  $T_x$ . Thus we conclude that rupture velocities  $0.9 v^R$  to  $v^R$  are not suitable for the  $k$ -squared model with  $k$ -dependent rise time. Non-zero  $T_x$  vanishes with short maximum rise time ( $\tau_{max} = 0.5$  s), however, stress recovery after a pulse passage increases and  $D_c$  becomes closer to  $D_{tot}$ . On the other hand, longer rise time ( $\tau_{max} = 2$  s) leads to lower  $D_c$ , negligible stress recovery, but to larger areas of non-zero  $T_x$ . Thus we conclude that the maximum rise time  $\tau_{max} = 1$  s is optimal for our fault dimensions and elastic parameters. It is necessary to extend the parametric study to generalize this conclusion. Mai *et al.* (2005) found more probable ruptures nucleating from regions close to asperities and

not from zero slip areas. Our results partially agree with these findings. The nucleation point in the center of the fault (non-zero slip, asperity border) leads to smallest non-zero  $T_x$  area, however, areas of non-zero  $T_x$  cover less than 5% of the fault for the two other positions of nucleation points (fault border - zero slip). Thus, we do not prefer any position of the studied nucleation points. The area of non-zero  $T_x$  is smallest for Hisada's slip velocity function. Also  $D_c$  is shortest for Hisada's SVF. Hence, employing Hisada's SVF is more consistent with applications of the slip weakening law in earthquake source dynamics than Boxcar, Brune's SVFs. We attribute it to its similarity with the Kostrov function, which is an analytical solution of the forward dynamic problem. We conclude that nine  $k$ -squared models with dynamic parameters plotted in Fig. 3.16 (a, b, g, h, k-o) can be explained by a dynamic model with the slip weakening friction law.

The effect of filtering on the dynamic stress field of a kinematic source model was analyzed in both the space and time domains. Low-pass frequency filtering decreases the values of the strength excess and increases the values of both  $D_c$  and stress delay  $T_x$ , while the low-pass wavenumber filtering decreases the values of the fracture energy (Fig. 3.18d).

## 5 Conclusions and Outlook

The main outcome of this work is the new approach for evaluation of synthetic kinematic models from the viewpoint of the earthquake dynamics. It resulted in a (thoroughly proved) finding that the  $k$ -squared slip model with  $k$ -dependent rise time (shortly  $k$ -squared model) is basically not in contradiction to earthquake source dynamics. The  $k$ -squared model was studied in terms of the slip weakening friction law - strength excess, dynamic stress drop, breakdown stress drop, critical slip weakening distance  $D_c$ , fracture energy. Another outreach of the work is the impact on the strong motion prediction. Indeed, a class of  $k$ -squared models which are most consistent with the slip weakening friction law (Hisada's slip velocity function) was found. On the other hand, the models which violate the slip weakening friction law considerably (rupture velocity close to Rayleigh speed) were found too. Thus the analysis provided constraints on the  $k$ -squared models to be used in practice.

Further, the work contributes to general methodology of fault constitutive relations estimations. A new parameter, the stress delay  $T_x$ , was introduced to map the fault points where the peak stress precedes the rupture onset. A simple but original  $D_c$  estimation method directly from the stress-slip curves was also presented. The effect of spatial filtering was shown on various dynamic source parameters. The bimodal character of  $D_c/D_{tot}$  frequency-magnitude distribution was found and discussed, but it remains for future study, whether the same holds for kinematic models of real earthquakes.

The discrete wavenumber code (parallelized with *OpenMP* directives) for dynamic stress calculations, developed in this thesis, can be utilized for a broader set

of problems. It has been already applied in the work of Fischer & Horálek (2005) who studied dynamic stress triggering of earthquakes during Western Bohemia seismic swarms. It could be also used for tuning other, more general, numerical methods. The method of the calculation could be easily extended to more general geometries, as the fundamentals of discrete wavenumber method are presented in detail in the appendix of this thesis. For example, the dynamic stress calculation could be in the future extended to account also for the presence of free surface (homogeneous half-space).

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## 8 Electronic Supplement

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Figure 2.2

[http://karel.troja.mff.cuni.cz/~burjanek/dizertace/fig\\_2.2.jpg](http://karel.troja.mff.cuni.cz/~burjanek/dizertace/fig_2.2.jpg)

Figure 2.3

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Figure 2.8

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Figure 2.9

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Figure 3.1

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Figure 3.16

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Figure 3.17

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Figure 3.18

[http://karel.troja.mff.cuni.cz/~burjanek/dizertace/fig\\_3.18.jpg](http://karel.troja.mff.cuni.cz/~burjanek/dizertace/fig_3.18.jpg)