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Angular symmetries of hotspot distributions

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The symmetries in the distribution of hotspots are investigated by an analogy of the angular autocorrelation of scalar fields. It has been applied to two lists of hotspot locations, revealing the axes as well as the angles of symmetries. The existence of angular symmetries suggests that the hotspot distribution observable on the Earth's surface is probably influenced by symmetries of deep geodynamic processes.

1. Introduction

Almost all great thermal anomalies and volcanic activity on the Earth's surface are confined to the lithospheric plate margins. However, scattered around the globe are regions of isolated thermal activity called hotspots. It was suggested [1,2] that hotspots are the result of plumes of material that rise from the deep mantle. Consequently, when lithospheric plates pass over the mantle hotspot, swells and sometimes trails of volcanoes are left behind.

The question arises as to the interaction between the plume and the lithospheric plate, the result of which is, e.g., the thinning of the lithosphere [3–6]. On the other hand, the state of the lithospheric plate may affect the likelihood of detecting such thermal anomaly at the surface of the Earth. Pollack et al. [7,8] concluded that there is greater than 99% probability that the observed hotspot distribution is not a random sample drawn from the global distribution of "vulnerability parameter" (see also [9]). This parameter is proportional to $u^{1/2}l$, where u is the velocity of the lithosphere in the hotspot reference system [10] and l is its thickness. It may be summarized that it is frequently hard to resolve which observable features of the hotspots are directly connected with deep mantle sources and which ones are influenced by the behaviour of the lithospheric plate.

Therefore, the fundamental obstacle in any hotspot analysis is that there is no complete agreement concerning the hotspot distribution. Originally, Morgan [1] had identified only a few hotspots on the Earth's surface; later Burke and Wilson [11] found as many as 122 hotspots. Vogt [12] used their slightly modified set (Fig. 1) in his discussion with Pollack et al. [7–9,12]. It is highly probable that some anomalous regions from these sets containing many hotspots need not be caused by deep sources. Crough and Jurdy [13] carried out a critical review of the hotspot distribution; they confined themselves to only 42 hotspots (Fig. 1) and, moreover, showed that after removing the effect of subduction there is a great correlation of this hotspot set with positive geoid anomalies as did Chase [14] for a different set of hotspots. This was confirmed by further analysis of Richards et al. [15] for another hotspot catalogue.

Stefanick and Jurdy [16] summarized that "hotspots have a very nonuniform distribution but of a very simple kind. There are very few hot spots over one half of the Earth's surface, and over the remaining half, their distribution seems to be uniform." On the other hand, there are also higher-order peaks in the power spectrum of the spherical harmonic representation of the hotspot distribution [13,15]. This suggests that the hotspot distribution need not be as simple as mentioned above. The aim of this study is to test whether there are or not any finer regularities. From the many sets of hotspot I chose two: Crough and Jurdy's catalogue [13] and that of Vogt [12] study-

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ing thus the cases when both the lower number and the higher number of hotspots are taken into account.

2. Angular symmetries of a surface point field

Although a spherical harmonic expansion of an investigated field can provide a certain information as to its geometrical properties without any further analysis (as did Richards et al. [15] in the case of hotspots), it cannot yield details concerning its symmetries as are, e.g., axes of the rotation symmetry. That is why the method of angular correlations is proposed—see Pěč [17], where this approach to scalar as well as to vector fields is fully discussed. This method takes full advantage of computing autocorrelation using the coefficients of spherical harmonics. The problem is that hotspots represent a point field. It is possible to obtain its spherical harmonic expansion, e.g. by representing hotspot as point sources of equal strength (delta functions) on the surface of the Earth but autocorrelation of delta functions is not well defined. Therefore I chose the following method, where only the angular distances of hotspots are computed, to construct the analogy of autocorrelation of scalar fields.

Let us assume that any set $\{x_i\}_{i=1}^{N}$ of points on the Earth's sphere is given. This means that x_i may be represented by the pair (Φ_i, Λ_i), where Φ_i denotes the geographical latitude and Λ_i is the geographical longitude. The rotation of the field $\{x_i\}_{i=1}^{N}$ can be described by means of the rotation axis $a(\phi, \lambda)$ and the angle of rotation ω; here ϕ, λ and the angular coordinates defining the direction of the rotation axis a, e.g. $a(51.5°, 0°)$ is the rotation axis passing through London. Hence, after such rotation we obtain the transformed field $\{x_i'\}_{i=1}^{N}$, represented by the couples (Φ_i', Λ_i'), $i = 1, \dots, N$. Our aim is to compare the similarity between $\{x_i\}_{i=1}^{N}$ and $\{x_i'\}_{i=1}^{N}$. The "functional of similarity" $f(\phi, \lambda, \omega)$ may be defined as:

$$f(\phi, \lambda, \omega) = (2N)^{-1} \sum_{i=1}^{N} \left[d\left(x_i, \{x_j'\}_{j=1}^{N}\right) + d\left(x_i', \{x_j\}_{j=1}^{N}\right) \right] \tag{1}$$

where $d(x, A)$ is the angular distance of a point x from a point field A defined in the usual way:

$$d(x, A) = \min_{y \leftarrow A} d_{\mathrm{p}}(x, y) \tag{2}$$

$d_{\mathrm{p}}(x, y)$ is the angle between points x and y. The functional $f(\phi, \lambda, \omega)$ thus plays a role of quantity yielding an information about the angular symmetry of the prescribed point field, when the rotation axis $a(\phi, \lambda)$ was chosen. The minima of $f(\phi, \lambda, \omega)$ (if f is taken into account as the function of three variables—ϕ, λ, ω) thus mark the axes and the angles of rotation for which the original and rotated positions of the point field are the most similar, whereas maxima reflect the dissimilarity. If a rotation axis is fixed, i.e., f is studied as the function of only variable ω defined in the interval $\langle 0, 2\pi \rangle$, there can be a periodicity of minima of this function but they need not correspond to extremes obtained in case of studying f as the function of three variables. If, moreover, this periodicity corresponds to such extremes, then it represents the strong symmetry which suggests that there may be a similar symmetry in the processes leading to the creation of the studied field.

3. Results and discussion

Functional $f(\phi, \lambda, \omega)$ defined by equations (1), (2) were computed for both lists of hotspot locations [12,13] depicted in Fig. 1, i.e., it was scanned in the (ϕ, λ, ω) space. It is of interest that there is not a great number of deep local minima if we take functional f as the function of all three variables, i.e., with no variable fixed. Moreover, the minima are near $\omega = 2\pi/n$, where n is a low positive integer; there are the deep minima if $\omega = \pi$ and $\omega = 2\pi/3$ for both sets. Since $f(\phi, \lambda, 0)$ is the global minimum and $f(\phi, \lambda, \pi + \alpha) = f(\phi, \lambda, \pi - \alpha)$, these minima reflect the symmetry of the lowest orders (180° and 120° symmetries) when the rotation axis $a(\phi, \lambda)$ is chosen. There is also one deep minimum yielded by Crough and Jurdy's list if $\phi = 30°$N, $\lambda = 140°$E and $\omega = \pi/4$. There is not, however, any periodicity when this rotation axis is chosen and so this minimum is not connected with any angular symmetry (Fig. 2). On the other hand, Vogt's list provides a 60° symmetry connected with deep minima (see Fig. 3 and Table 1). (Naturally, each 60° symmetry is a special case of 120° symmetry.)

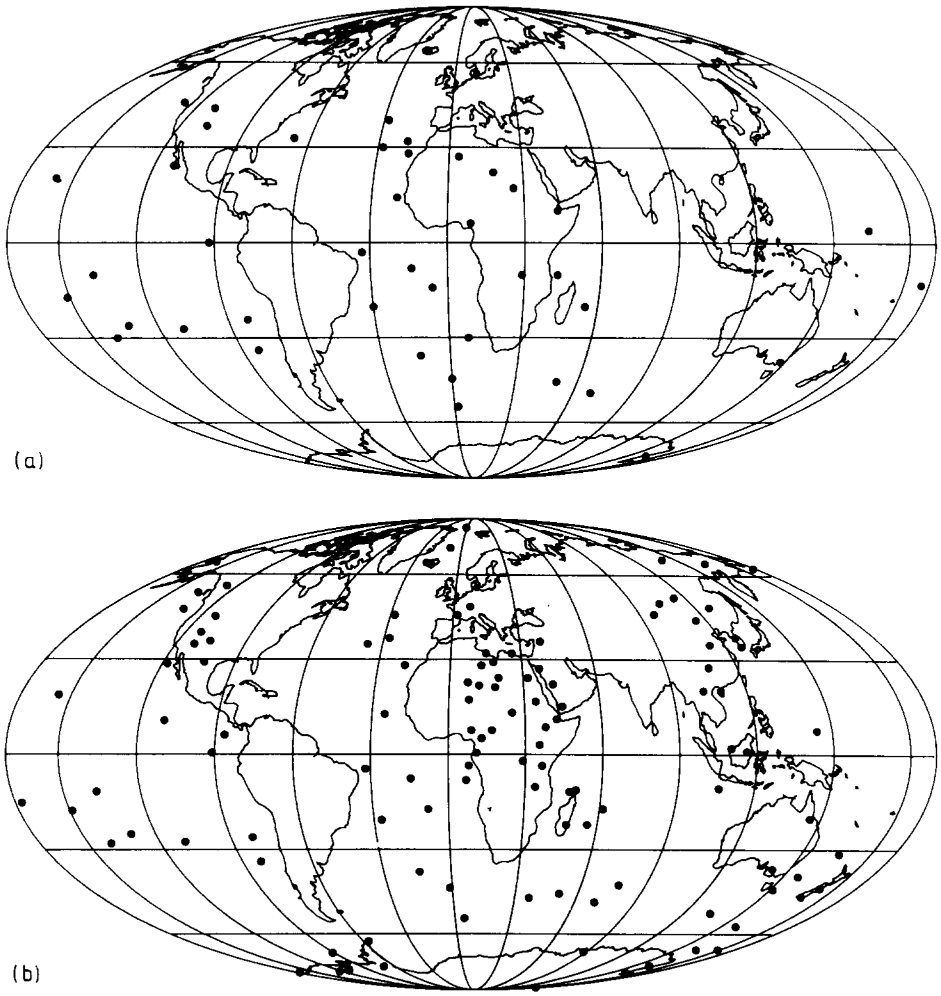

Fig. 1. (a) Crough and Jurdy's distribution of hotspots. (b) Vogt's distribution of hotspots.

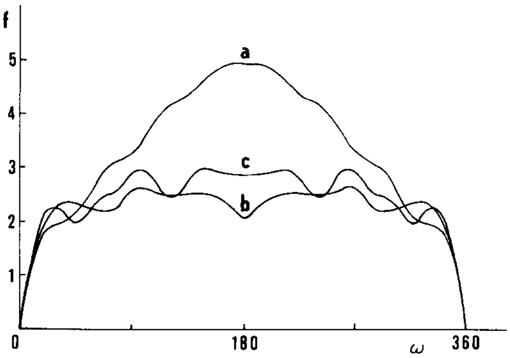

Fig. 2. Functional $f(\phi, \lambda, \omega)$ computed for Crough and Jurdy's list of hotspot locations when the following rotation axes are chosen—*a*: $\phi = 30°$ N, $\lambda = 50°$ E; *b*: $\phi = 40°$ N, $\lambda = 100°$ E; *c*: $\phi = 30°$ N, $\lambda = 140°$ E. The rotation axis *a* provides the global maximum of f. Values of f are in 10^{-1} rad and ω is in degrees.

The question arises as to the other higher-order symmetries which are not connected with the deepest local minima of f but are connected with shallower minima. I did not find any remarkable symmetries for Crough and Jurdy's set (although a higher-order periodicity, when some rotation axes are fixed, is present) but they are present if Vogt's set is taken into account—all axes chosen in Fig. 3 can be considered as axes of symmetries in the sense described in the previous section. It is clear from Fig. 3 that there is a 120° symmetry, which is not connected with the 60° symmetry. To demonstrate visually the "similarity" between the original and the rotated locations of hotspots, their positions are drawn on the same map for the two cases in Fig. 4.

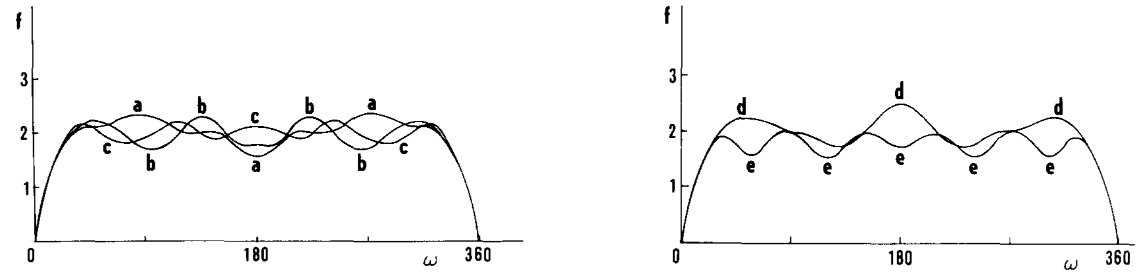

Fig. 3. Functional $f(\phi, \lambda, \omega)$ computed for Vogt's list of hotspot locations when the following rotation axes are chosen—*a*: $\phi = 60\,^{\circ}$N, $\lambda = 150\,^{\circ}$E; *b*: $\phi = 20\,^{\circ}$N, $\lambda = 300\,^{\circ}$E; *c*: $\phi = 40\,^{\circ}$N, $\lambda = 310\,^{\circ}$E; *d*: $\phi = 80\,^{\circ}$N, $\lambda = 110\,^{\circ}$E; *e*: $\phi = 30\,^{\circ}$N, $\lambda = 120\,^{\circ}$E. Values of f are in 10^{-1} rad and ω is in degrees.

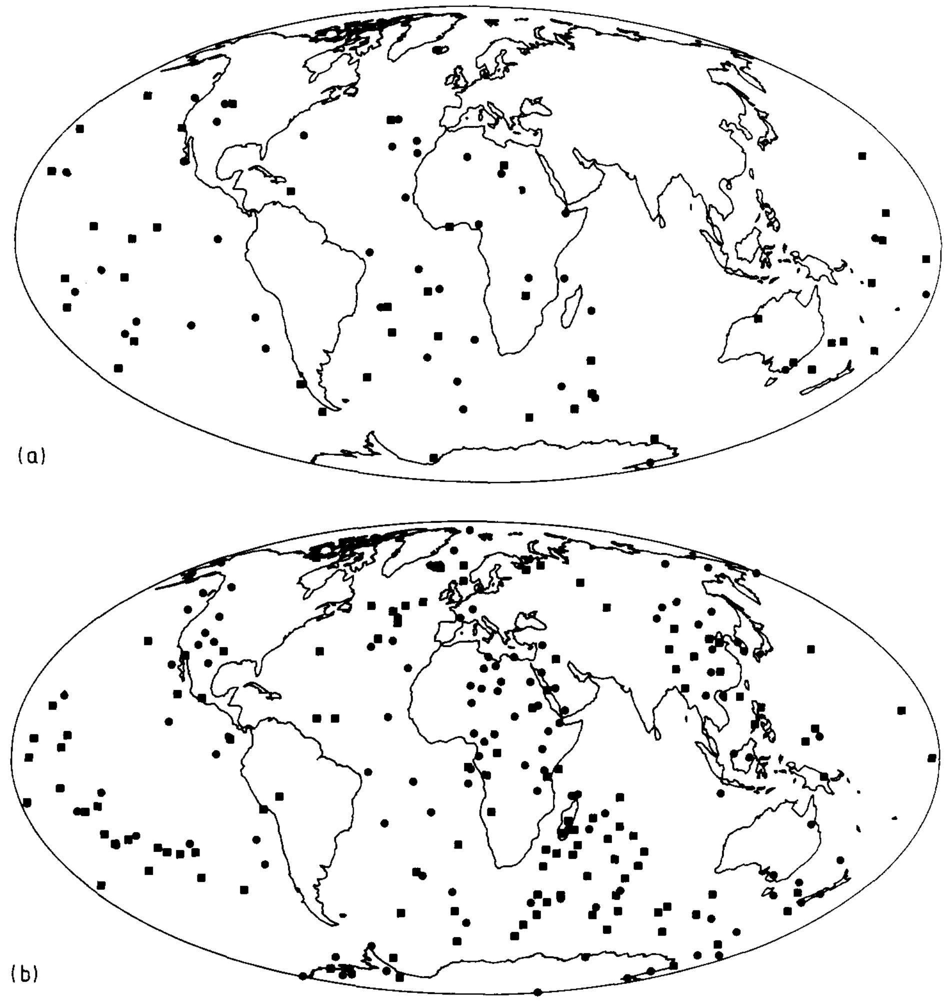

Fig. 4. The original (circles) and the rotated (squares) positions of hotspots. Figure (a) is for Crough and Jurdy's list of hotspot whereas (b) is for Vogt's list. Both figures demonstrate the situation when f achieves its minimum: (a) $\phi = 40\,^{\circ}$N, $\lambda = 100\,^{\circ}$E, $\omega = 180\,^{\circ}$; (b) $\phi = 30\,^{\circ}$N, $\lambda = 120\,^{\circ}$E, $\omega = 60\,^{\circ}$.

TABLE 1

List of the deepest local minima of functional f defined by equations (1) and (2); f_{max} denotes the global maximum of f

ϕ (°N)	λ (°E)	ω (°)	f/f_{max}
Crough and Jurdy			
40	100	180	0.42
10	120	180	0.43
40	130	120	0.46
30	140	45	0.40
Vogt			
10	10	180	0.61
10	30	180	0.61
50	50	180	0.59
0	130	180	0.61
60	150	180	0.60
40	240	180	0.59
20	120	120	0.58
30	120	60	0.61
30	140	60	0.61

In both catalogues the highest maxima are present only for $\omega = 180°$ (the location of the corresponding rotation axes can be seen in Fig. 5). This is not suprising because it reflects Stefanick and Jurdy's division [16] of the Earth's surface into a half with few hotspots and another half with many hotspots, i.e., it reflects the 360° symmetry known from this previous study. It is clear from Table 1 that the deepest minima are mostly for $\omega = 180°$, too. If we deal with $f(\phi, \lambda, \omega = \pi)$, which now represents the function of two variables ϕ and λ (Fig. 5), we can clearly recognize long deep "valleys" lying approximately on great circles in both cases. This feature can be explained by the bipolar structure of the hotspot distribution because the equator of any bipolar structure is the place of a $f(\phi, \lambda, \omega = \pi)$ minimum. The bipolar structure of Crough and Jurdy's set is evident even from Fig. 1: its poles are in Africa and in the Pacific forming thus the African and the Pacific groups of hotspots. As to Vogt's list, the bipolar structure is more hidden because the Pacific group is subdivided into two parts. It is of interest that there is a fundamental geotectonic hemisphere symmetry of the Earth as well—the Pacific plate and the African plate are homologous—as noticed by Pavoni [18]. Also other global fields exhibit such a bipolar structure as:

(1) The geoid equivalent to density anomalies inferred from the seismic wave velocities in the lower mantle [19].

(2) Residual geoid with the removed anomaly gravity signal caused by subducting slabs [13–15].

Pěč [20] carried out the angular correlation analysis of the external gravity field. The main difference from this study was that he applied the classical autocorrelation function to the gravity spherical harmonics instead of the functional f (1)-(2). His analysis revealed that 360° and 180° symmetries are also present as well as the 120°

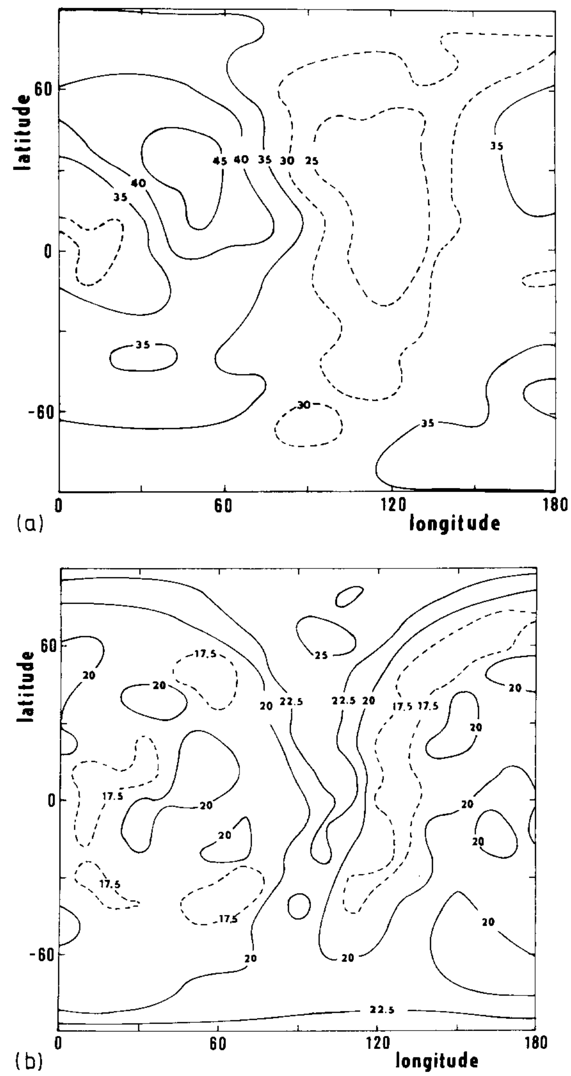

Fig. 5. Functional $f(\phi, \lambda, \omega)$ (in 10^{-2} rad) when ω is fixed to be equal to 180°: (a) Crough and Jurdy's catalogue; (b) Vogt's catalogue.

symmetry which is further evidence for the hypothesis that the hotspot distribution observable on the Earth's surface and the external gravity field anomalies can be of the same origin. An attempt to explain the similarity of both fields was done by Richards et al. [15], who concluded that their correlation can be explained by means of the image of plumes preferentially occurring in regions of large-scale background temperature positive anomalies because of the temperature-dependent rheology. They hypothesize that the cold lower mantle is due to a deeply subducted oceanic lithosphere and that hotspot plumes tend to be excluded from these cold regions of the mantle.

It is clear that the presented analysis cannot determine which hotspot catalogue is "correct" unless any physical mechanism of hotspot development, resulting in angular symmetries of their global distribution, is assumed. For example, if we assumed on the basis of some theory that there cannot be any remarkable symmetry higher than 120°, then Crough and Jurdy's set would seem better. Since the physical modelling of hotspot development [15,21–23] does not provide their global three-dimensional distribution yet, we do not have such a theory to compare model results with data set symmetries. On the other hand, the geometrical fact that there are hotspots symmetries may constrain the further modelling of their development.

4. Conclusions

Both sets of hotspots are distributed with 360°, 180° as well as 120° symmetries. Moreover, the bipolar 180° symmetry has poles in Africa and in the Pacific resembling the geotectonic hemispheric symmetry and the distribution of the residual geoid as well as that of the lower mantle velocity anomalies. Higher-order symmetries are also present if Vogt's list of hotspot locations is studied. If we assume that the hotspot sources lie near the core–mantle boundary [15,21–23], the revealed symmetries suggest that the hotspot sources and/or their development into the observable features on the Earth's surface are probably influenced by symmetries of deep geodynamic processes.

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