Appendix to the paper
"Toward understanding slip-inversion uncertainty and artifacts II: singular value analysis"
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A1 Smoothing constraint

The smoothing constraint represents an alternative regularization approach to the one presented in the main text, i.e. truncation by means of singular value decomposition. The space-time smoothing is introduced by adding new lines to matrix $G$. The lines represent linear equations $m_i - m_j = 0$, where indices $i$ and $j$ correspond to all combinations of the adjacent model parameters in the $x$-$t$ plot. All these additional lines are multiplied by factor $L_w = wML_n^{-1}/2 = w\mu ST/2M_0$, where $S$ and $T$ are the fault area and the time interval, respectively. Parameter $w > 0$ is subject to change, controlling the relative weight of the smoothing constraint; all other parameters are defined in the main text. Here we show the effects of smoothing for two methods of the least-squares inversion: i) the singular value decomposition (SVD) as in the main text (since smoothing is applied, all singular vectors in the solution expansion can be used) and ii) the non-negative least-squares (NNLS) method providing results with the positivity constraint on the slip velocity field.

A1.1 Synthetic model

Figure A1A shows the variance reduction as a function of smoothing weight $w$ for the synthetic bilateral source model using the SVD. Note the prohibitively large drop of the variance reduction for $w > 0.1$. Figure A2 displays the inversion results for selected smoothing weights (see legend). As compared with the main text without smoothing, one can see that the spurious asperity is even emphasized when the smoothing is applied; the spurious asperity exists even for weak smoothing, $w = 0.001$, with a very large variance reduction of 0.999 (the left panel in Figure A2). Figure A3 shows the same smoothing, but the inversion is performed using the NNLS method. Note that the main characteristics of the inverted model are the same as without the positivity constraint, including the false asperity in the fault center.

![Figure A1. Dependence of the variance reduction on the smoothing weight for the synthetic bilateral source model (A) and for the real Movri Mountain earthquake data (B). The SVD approach is used, employing all singular vectors (i.e. no further regularization is considered).]
Figure A2. Inverted slip velocity distribution for the synthetic bilateral source model solved by the SVD method when the smoothing constraints are applied. VR and $w$ represent the variance reduction and the smoothing weight, respectively. Note that the false asperity in the center of the fault persists and cannot be avoided by smoothing (compare with the input model in Figure 2A in the main text).

Figure A3. Same as Figure A2, but the inversion is performed using the non-negative least-squares method (NNLS). Similarly to Figure A2, the false asperity in the center of the fault is still present.

A1.2 Real data

Figure A1B shows the decrease of the variance reduction with increasing smoothing-constraint weight. Similarly to the synthetic model, we should keep $w$ below 0.1, after which the variance reduction is too low. Figure A4 displays the resulting slip velocity $x$-$t$ plots for selected smoothing weights solved by the SVD approach. Similarly, Figure A5 shows the slip velocity models obtained by the NNLS method for the same weights.
Figure A4. Same as Figure A2 (SVD approach), but for the real Movri Mountain data. Note that the minimum weight \( w = 0.01 \) (left) is larger than in Figure A2, because the smaller weights already yield an excessively distorted inverted model for real data.

Figure A5. Same as Figure A4, but the inversion is performed using the NNLS approach. Note that the result for \( w = 0.01 \) (left) is apparently more realistic than in Figure A4.

A2 Data weighting

Here we show the influence of the data weighting on the slip inversion. The weights are applied to each station component individually, being reciprocals of the L2 norm of the respective seismogram (displacement time series in the studied frequency range). This modifies matrix \( G \) and hence also the corresponding singular vectors (see Figure A6 for the whole network considered). One can see that the leading singular vectors have a different structure than those presented in the main text without weighting (Figure 4). In particular, already the first singular vectors are shaped in patches rather than in uniform strips, as expected due to down-weighting of the strongly dominant directive station (SER).

Figure A7 shows the \( \hat{c} = G^T d \) vector, assuming all stations and including the station-component weights, for both the synthetic bilateral model (A) and the real case of the Movri Mountain earthquake (B). Note that the shape of \( G^T d \) is very similar to the shape of the second leading singular vector (Figure A6). The effects of the weighting upon the inversion results are then demonstrated in Figures A8 (synthetic bilateral model) and A9 (Movri Mountain data). Note the smaller variance reductions in the case of the Movri Mountain data as compared to those presented in the main text (Figure 8). This is attributed to down-weighting the closest stations in the inversion.
Figure A6. Singular vectors for the whole station network (Figure 1); analogy to Figure 4, but applying weights to the individual station components. Starting already at \( \lambda(2) \), the singular-vector pattern includes both ‘inclinations’ typical for the individual stations (see Figure 2 in the main text). This property explains why the slip inversion results (Figures A8-A9) differ from those in the main text.

Figure A7. A) The \( G^T d \) vector for the synthetic test similar to Figure 5 in the main text, including the whole network, but assuming station-component weights. Compared to Figure 5, the SER station is down-weighted relatively to the rest of the stations, which results in a more symmetric pattern. B) Same as A, but for the real Movri Mountain data (to be compared with Figure 7 in the main text).
Figure A8. Truncated solutions of the slip inversion problem for the synthetic bilateral model (Figure 2A in the main text); station-component weighting is employed. The figure is to be compared with Figure 6 in the main text.
Figure A9. Same as Figure A8, but for the real Movri Mountain data, to be compared with Figure 8 of the main text. The numbers in the rightmost panels are the corresponding variance reductions. Note that they are lower than in the case when no weights are applied (mostly due to the worse fit of the SER station in the present case). This down-weighting of station SER resulted in a more pronounced but still very uncertain indication of the rupture propagation from 10 km to 0 km at 5 to 10 seconds.