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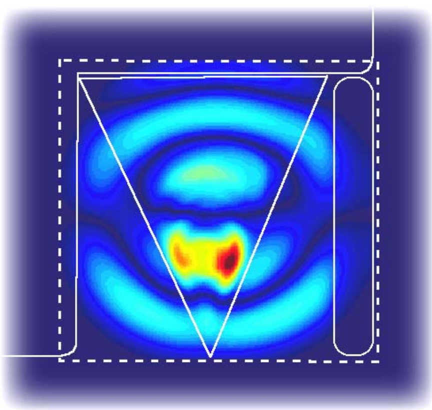
Hybrid modeling of seismic waves

Ph.D. Thesis

by Ivo Opršal



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- *The Figure on the title page – y component (East) – corresponds to the wavefield shown in panel 13d,* **P6**.

Contents

1 State of the Art

One of the highest priority problems of today's seismology is modeling and predicting earthquake ground motions. This task is necessary to mitigate and fight seismic hazard via engineering anti-seismic design, sophisticated microzonation and urban planning. A useful tool for the wave propagation studies are the finite difference (FD) methods.

More than 30 years ago, the 2D FD methods became a powerful tool for investigating the seismic waves phenomena in complex models [e.g. *Boore*, 1972]. The research formed into two main branches: homogeneous and heterogeneous FD formulation of the elastodynamic equation. The former uses various FD schemes for gridpoints in homogeneous parts of the model, and for the gridpoints at the material discontinuities, where it (in discretized form) represents appropriate boundary conditions. The latter is using just a single FD method to solve the equation of motion while the material parameters are discontinuous functions resulting in boundary conditions on material discontinuities. Thanks to algorithmic simplicity, the heterogeneous formulation is the preferred one [*Virieux*, 1986]. Using one method in the whole model, free surface included, is so-called vacuum formalism [*Tikhonov and Samarskii*, 1961] in which the Lamé's parameters (and formally also density) are zeroed above the free surface [*Boore*, 1972 (SH case); *Kelly et al.*, 1976]. For 3D schemes using vacuum formalism see *Graves* [1996], *Ohminato and Chouet* [1997], *Pitarka and Irikura* [1996], *Moczo et al.* [1999].

From the very beginning of its use, the heterogeneous formulation had not been justified theoretically until *Zahradník et al.* [1993] performed accuracy analysis for internal discontinuities (1^{st} order) and free surface of elastic medium for a number of FD methods. A surprising result of that effort showed that several heterogeneous schemes existing at that time did not fulfill the boundary condition. It resulted in a new 'PS2' method (P-SV case) with effective parameters evaluated by geometric averaging of real model parameters [*Zahradník*, 1995]. The PS2 method is traction consistent (see also *Zahradník and Priolo* [1995]) and simpler than the other methods.

Moreover, the geometric averaging is essential for correct representation of the true position of the interface passing between the gridpoints (Figure 2 of *Zahradník* [1995]), and for keeping the internal interface free of a stair-case deformation. Similar conclusions have been drawn by *Nielsen* [1994] and *Graves* [1996]. Evaluating parameters for the heterogeneous approach by geometrical averaging [e.g. *Zahradník et al.*, 1993 (P-SV case)] converts the prescribed topography of the free surface into a step-like approximation composed of elementary steps of minimum height and width given by the vertical and the horizontal grid step, respectively. Nevertheless the simplicity of the vacuum formalism was an advantage compared to other FD methods employed for topography problems, e.g. *Jastram and Tessmer* [1994], *Hestholm and Ruud* [1994], *Tessmer et al.* [1992], *Jih et al.* [1988], *Robertsson* [1996].

The FD modeling had been traditionally performed on a regular square grid. The model parameters may be in a high contrast in various parts of the models. Thus some parts of the model required fine meshing, and that meshing was the same in the whole model. It resulted in spatial oversampling in most of the model. The need of realistic models with topography led to irregular grids.

The 2D FD heterogeneous-formulation modeling on the irregular grids, and topography had been studied by *Moczo* [1989] - rectangular grid, SH case; for more general irregular grids and SH waves see *Moczo et al.* [1996], which, however, may have stability problems for the P-SV case (grids $h \times h$ neighbouring with $2h \times 2h$; [*Moczo et al.*, 1997]); *Falk et al.* [1995] - grids with locally varying time step; for staggered irregular grids, P-SV case see *Jastram and Tessmer* [1994], and *Falk et al.* [1996]; for deforming the rectangular grid into a curved grid with the topmost gridline right on the curved surface, see *Tessmer et al.* [1992], *Hestholm and Ruud* [1994]; *Jih et al.* [1988] (polygonal topography); for case with external topographic free surface condition see *Hong and Bond* [1986], and *Illan* [1977] (arbitrary polygonal surface).

The topography models investigated by other methods were recently published by *Seriani et al.* [1992] (Spectral Element Method); *Gaffet and Bouchon* [1989] (Boundary Integral Equation Method); *Nielsen* [1994] (Elimination of grid artifacts); *Kawase* [1990] (Discrete wavenumber method, hybrid method).

Advantages of the irregular grid are twofold: Staircase free-surface artifacts (due to the vacuum formalism) are reduced by the grid refinements, while oversampling of the high-velocity regions is avoided.

A way of improving the FD modeling results is going from 2D to 3D all-in-one formulations. However, including source, path and site effects into all-in-one methods leads to extreme needs of computer memory and time. An effective bypass is combining appropriate techniques for particular problems (e.g. source modeling, crustal propagation modeling, local geology and topography modeling) in hybrid methods. Hereafter, the following abbreviations are used: FD=finite difference, DW=discrete wavenumber (or FK), FE=finite element, BEM=boundary element method.

There are various approaches called 'hybrid methods' in seismic modeling. *Kamae et al.* [1998] compute strong ground motions using Green's functions obtained by FD for frequencies up to 1 Hz, and by stochastic modeling (of the same source) for frequencies above 1 Hz. The hybrid Green's function is retrieved by adding these two Green's functions for particular frequency bands. *Faeh and Suhadolc* [1994*a*] use 2D hybrid approach combining modal summation and the finite-difference technique to treat the combined effects of the source, path and the site response. *Zhang et al.* [1998] evolved 2.5D elastodynamic scattering formulation that evaluates 3D response of 2D scatterers, topography inclusive. *Wen and Helmberger* [1998] developed a 2D P-SV hybrid method to model localized structures near the core-mantle boundary, involving 2D structures. They combine FD at the heterogeneous region and generalized ray theory solutions from a seismic source. After interacting with the heterogeneous structures, the ground motions are propagated back to the Earth's surface analytically by Kirchhoff's method. Coupling of boundary element method (BEM) and FE to model 2D SH scattering of surface waves in laterally heterogeneous multi-layered media was presented by *Fujiwara* [1996]. Similarly *Moczo et al.* [1997] presented 2D computation of P-SV seismic motion at inhomogeneous viscoelastic topographic structures by a combination of methods: 1D DW (source and path), 2D FD (regional computation), and 2D FE (used for topographic surface parts of the model). Their method was extension of hybrid 2D DW-FD method of *Zahradník and Moczo* [1996] for localized structures with a flat free surface embedded in a 1D background medium. The source radiation and propagation in the background model was solved by DW method, while the propagation in the regional 2D structure was calculated by the FD method. The coupling between the two sets of calculations is performed on a rectangular excitation box surrounding the local site structure similarly as in *Alterman and Karal* [1968]. Principal inconsistency between the 3D source radiation and wave propagation on one side, and the 2D FD calculation on the other side [*Zahradnik and Moczo,* 1996], was inevitable, but small enough in some special cases as point source placed far from the site but being close to 2D-model plane. Principal inconsistency between the 3D source radiation and wave propagation on one side, and the 2D FD calculation on the other side, was inevitable, but small enough in some special cases as point source placed far from the site but being close to 2D-model plane. A 2D FD hybrid of *Robertsson and Chapman* [2000] computes various models differing in the local structure details (e.g. parameter studies). The original model is computed once and the locally altered models require calculations in the subvolume and its neighborhood only. *Goh and Schmidt* [1996] use a hybridization of FE, boundary integrals and DW to solve the Helmholtz equation for seismoacoustic modeling. *Maggio and Quateroni* [1997] developed a method combining ordinary FE for regions of high material parameters variability with spectral element methods for homogeneous or gradually changing blocks. *Faeh et al.* [1994*b*] demonstrated advantage of their 2D hybrid approach combining modal summation and the FD technique to simple 1D techniques used for seismic zonation. In *Caserta et al.* [1999] a deterministic hybrid 2D DW-FD method is added stochastic noise (perturbation) to take into account effects of small-scale heterogeneities of the crust. A hybrid approach coupling a ray tracing and 2D FD method for modeling T-wave propagation from an underwater source to an on-land seismic station is proposed by *Piserchia et al.* [1998]. *Hatayama and Fujiwara* [1998] use BEM with normal modes to evaluate the excitation of secondary surface waves in 3D basin models without waves that would be present in complete 3D computation. *Yokoi and Sánchez-Sesma* [1998] use indirect BEM for 3D topographic problems with required knowledge of analytical solution for the half-space with a flat free surface. *Olsen et al.* [2000] evaluate viscoelastic ground motion in a two stage technique: In the 1^{st} step (coarse gridded model) they save the stress tensor values for planes aside and under the site. The 2^{nd} step performs 3D computation with the stress tensor from 1^{st} step added to present stress tensor at corresponding location.

Other powerful method to be mentioned here is that with stochastic components of the wavefield describing, in a more realistic way, high frequency source radiation and propagation *Rovelli et al.* [1994]. *Boore* [1983] employs the FD modeling to the source, path, and site effects at low frequencies, while the higher frequencies are stochastically simulated. For an analogous approach, see also *Graves* [1999].

2 Objectives and Goals

The 2D PS2 method of *Zahradník* [1995] has been suited for regular square grids. The method is of heterogeneous formulation, and the effective parameters λ, μ are evaluated by geometrical averaging which converts the prescribed free surface topography into a step-like approximation. Possible diffraction caused by that may be reduced (in case of need) by refinement of the grid in the vicinity of the step-like free surface. Another need for fine meshing is the fact that in case of presence of the low velocity regions in the model, the fine meshing is necessary to keep the accuracy, according to minimum wavelength. The stability condition binds the fine gridstep with small time step. This leads to need of larger computer memory and time. An effective way to reduce the computer memory and time is the use of irregular grid so that fine meshing is applied in places with surface topography and/or the low velocity zones.

- The goal is to generalize the method for irregular grid PSi2 (based on PS2 method of *Zahradník* [1995] for regular grids, flat surface topography) and to apply it to topography problems. Theoretical justification of the free surface boundary condition is desirable, as well as numerical comparison with other methods. The purpose is to evaluate possible numerical artifacts generated by irregular gridding, and to determine whether this gridding may improve the accuracy of computation on complex topography models.

Practical seismology modeling is often connected to extreme numerical problems. Reflection seismology needs a good simulation of the wave phenomena for cases of salt domes, lignite layers, cavities or cavities filled with water. Another aspect is the earthquake seismology. It uses models containing the water basins, regions of high v_P and v_S velocity contrasts and high v_P/v_S ratio contrasts in a single model, and stochastic material perturbations.

- The aim is to test the stability and accuracy of the method using irregular grids on models with high velocity contrasts and high v_P/v_S ratio contrasts. Models with stochastic material perturbations may sustain instabilities because they are computed on a grid designed for unperturbed model. Empirical or theoretical rule for allowed perturbation that does not cause instabilities may be derived.

PS2 method for 2D FD modeling was reasonably boosted by its use as the 2^{nd} step of the hybrid modeling method [*Zahradník and Moczo*, 1996]. Because of the case-dependent principal 2D-3D inconsistency, this method was not universal in terms of variable position of the source with respect to the 2D computational model. Nevertheless, the hybrid method may be used in cases where the point (or finite-extent) source lies in the 2^{nd} step 2D model plane (or in the vicinity of the plane). To be employed in the 2^{nd} step of 3D hybrid FD modeling, straightforward extension of the 2D scheme to 3D is needed.

- The goal is to use the 2D FD hybrid method for strong-earthquake ground-motion modeling. The next step is going from 2D to 3D on irregular rectangular grid basis. This would enable full 3D FD modeling of complex structures. After that, as it was in 2D case, it is necessary to extend the hybrid modelling into 3D-3D hybrid modeling that is free of 2D-3D inconsistencies. The hybrid method should be designed to involve its 1^{st} step results (from any 3D method) into the 2^{nd} step 3D FD method performed on irregular grid. Hopefully, its profit should be possibility to compute (with relatively small demand on the computer memory and time) large regional models including the source and path effects, and then to compute (in the 2^{nd} step) local models with detailed topography and structure.

3 Results

3.1 2D FD Method on Irregular Grid (PSi2)

In paper **P1**, the PS2 method was generalized for the irregular rectangular grid and it is called PSi2 method. It provides results of the same accuracy as PS2 method but it saves the computer memory and time considerably. The PSi2 method suffers of negligible spurious numerical effects that could be possibly caused by the changes of the gridstep size between the dense and coarse parts of the grid (e.g. 20 : 1, which is more than reported in the literature, so far). Such abrupt changes of the gridstep size are possible, provided that the coarse parts are sampled by (at least) 10 gridpoints per wavelength. The non-planar topography is treated by the vacuum formalism that approximates the free surface by a stair-case boundary. The accuracy can be improved by the grid refinement close to the non-planar parts of the surface. The method was theoretically justified for free-surface boundary condition – for the cases of horizontal, vertical and 45° inclined free surface (represented by elementary staircase). Automatic zeroing of the material parameters above the free surface (vacuum formalism) may lead to unconditional instabilities. The problem appeared to be in mixing zeroed (intersected by free surface) and non-zeroed material parameters within one gridcell. A simple stabilization described in **P1** does not change the model topography properties. Thanks to employment of vacuum formalism and geometric averaging of the material parameters, this simple 2D FD heterogeneous method provides a very effective tool for computation of a wide class of 2D structures on irregular grids. The method was compared with independent methods with good results.

The nonreflecting transparent boundary conditions by *Stacey* [1988] appeared to be unstable for ratios $v_P/v_S > 2$. Therefore we attempted to use the conditions of *Emmerman and Stephen* [1983]. To improve their performance, we applied tapers of *Cerjan et al.* [1985].

As a practical example we show computation of underground cavity above a single horizontal coal seem, and the same case for the cavity filled with water.

3.2 PSi2 in Complex Media

The behaviour of the method for the case of water basin embedded in solid medium is studied in paper **P2**. The results are good from the kinematic point of view. Since the free slip is not employed in PSi2, the conversions are not modeled perfectly, however, the method is quite stable. The other case of paper **P2** is stochastic material perturbation that is an example of the scattering effects. Parameters λ, μ and ρ are perturbed independently. The computation was unstable unless low-pass filter to the parameters was applied in both directions.

A ridge topography case of paper **P2** is modelled on regular and irregular grids. The regular coarse grid solution has artifacts caused by the stair-case approximation of the even prescribed surface. The improvement was reached by refinement of the regular grid. The step from regular to irregular grid with only refinement in parts of the non-planar topography gave us the same results as for the regular fine grid, but with just 10% of computer memory and 5% of computer time needed.

3.3 PSi2 in a Realistic Model - Stabilization

Direct 2D FD modeling of seismic ground motions at sedimentary valley near Volvi Lake site is described in paper **P3**. This site was studied as an international test site EURO-SEISTEST. We used a viscoelastic linear rheology, with attenuation given by $\frac{Q}{f} = const.$ The model contained blocks with high velocity contrasts. Unconditional instabilities appeared for models with slightly inclined internal interfaces and high v_P/v_S ratios contrast between (not necessarily neighbouring) blocks. The stabilization is similar to that in **P1**. It is based on equalizing the effective interleg parameters in each computational gridcell. This trick changes the computational model but the change is always within a single gridcell.

P3 also provides a comparison to method of *Moczo et al.* [1997] for hill topography model with $v_P/v_S \rightarrow \infty$ (formally $v_S = 0.01$km/s), and with $v_P/v_S = 4$. The agreement is good, but the difference of solutions is larger for S waves, especially in cases with $v_P/v_S > 4$ (not shown here).

3.4 3D-2D Hybrid Modeling - Volvi Lake

In paper **P4** we applied 2D DW-FD hybrid method of *Zahradník and Moczo* [1996] to the Volvi sedimentary basin at EURO-SEISTEST. For the 1^{st} hybrid step, this method uses 1D discrete wavenumber (DW) technique to model the source and crustal propagation. As an alternative to that, the ray method is employed, too, and a new R-FD method is investigated. The double couple point source introduces 3D excitation features. It is located off the investigated 2D model cross-section. The 2^{nd} hybrid step uses the results from the 1^{st} step to compute the ground motion in the site of interest by PS2 FD method.

The replication experiment showed 3D-2D inconsistency in the hybrid for either DW and ray method used in the 1^{st} step. In spite of that, we performed the computation of the basin response. The reason was to compare DW-FD and R-FD hybrid methods with the same type of (unrealistic, but mutually consistent) excitations. They provided close results. An important partial conclusion was also that R-FD method gave a small difference between the simple and more complex (with more elementary rays included) ray excitation.

3.5 1995 Kobe Earthquake Hybrid Modeling

Modelling of the 1995 Kobe earthquake in **P5** is example of a practical problem with very complex finite-extent source model. It was solved by 2D EGF-FD hybrid method, where EGF stands for EGF-like method analogous to *Irikura* *and Kamae* [1994]. The computation was done for six stations at each of Kobe and Osaka profiles and the results were compared with other participants and with real data in the international Simultaneous Simulation experiment. As a result, the realistic modeling of the Kobe 1995 earthquake still remains an open question.

Inconsistency of the 3D-2D hybrid modelling in papers **P4** and **P5** was a strong stimulus for development of the 3D FD method on irregular rectangular grids, and subsequently for development of the 3D FD method.

3.6 3D FD Hybrid Method

The 3D FD displacement method for irregular rectangular grid is presented in paper **P6**. The method uses one-template FD approximation to the elastodynamic equation in the whole computational model. The free surface topography is approximated by staircase-like boundary, and it is treated by vacuum formalism. The internal boundaries are approximated by averaged material parameters. *Emmerman and Stephen's* [1983] nonreflecting boundaries have been employed. Where needed, the tapers supplement the nonreflecting boundaries [*Cerjan et al.*, 1985].

The hybrid method of **P6** takes into account the source path and site effects in two steps. The 1^{st} step may be treated by any 3D method on a model containing the source, regional structure and regional topography. A special coupling bounds the 1^{st} step results with the 2^{nd} step computation. The 2^{nd} step is performed by the 3D FD method on a local site model on irregular grid with detailed structure and topography. Thus previous problems caused by the 3D-2D inconsistency between the 3D source radiation and the 2D FD calculation are automatically avoided by the 3D generalization of the FD method.

The hybrid method reasonably saves computer memory and time, especially in cases with high frequency content, high material parameters contrasts and large scale difference between the local and regional models.

4 Conclusion and Outlook

The main outcome of our work is the contribution to existing methods of earthquake ground motion prediction for highly exposed regions. The methods can study regional phenomena, but also delineations of possibly damaged zones depending on local structure such as edge effects and large-basin structures responsible for mirroring seismic energy.

As for the new 3D FD hybrid method with irregular 3D FD grid, we hope that it will easily compete with complete all-in-one 3D FD methods, where all effects (source, path, and site) are studied in one FD model. The optimum solution would be to complement the new method in a proper way with stochastic and/or empirical technique. This would lead to more realistic modeling and prediction of disastrous earthquake ground motion, in general.

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7 Included Papers

P1: Opršal, I., Zahradník, J., Elastic finite-difference method for irregular grids, *Geophysics,* 64, 240-250, 1999.

P2: Opršal, I., Zahradník, J., Robust finite-difference modeling of complex structures, Paper No. 15 in Proc. of Int. Symposium on High Performance Computing in Seismic Modeling, June 2-4, Univ. of Zaragoza, Spain, 1997.

P3: Opršal, I., Zahradník, J., From unstable to stable seismic modeling by finite-difference method, *Phys. Chem. Earth*, 24 (part A), no. 3, 247-252, 1999.

P4: Opršal, I., Pakzad, M., Plicka, V., Zahradník, J., Ground motion simulation by hybrid methods, In: Irikura, K. et al. (eds.), The Effects of Surface Geology on Seismic Motion, Balkema, Rotterdam, pp. 955-960 (Proceedings of ESG'98, December 1-3, 1998, Yokohama, Japan), 1998.

P5: Opršal, I., Plicka, V., Zahradník, J., Kobe simulation by hybrid methods, In: Irikura, K. et al. (eds.), The Effects of Surface Geology on Seismic Motion, Balkema, Rotterdam, pp. 1451-1456 (Proceedings of ESG'98, December 1-3, 1998, Yokohama, Japan), 1999.

P6: Opršal, I., Zahradník, J., *(submitted)*, 3D finite difference method and hybrid modelling of earthquake ground motion, *Journal of Geophysical Research.*