Inversion for Focal Mechanisms Using Waveform Envelopes and Inaccurate Velocity Models: Examples from Brazil

by Juraci Carvalho, Lucas Vieira Barros, and Jiří Zahradník

Abstract One of the major challenges for the moment tensor determination is associated with the relatively low-magnitude events ($M_w \sim 4$) recorded by few regional stations at relatively large distances (300–600 km) and analyzed with standard velocity models of the region. Difficulties arise from the fact that synthetics in standard models (e.g., those routinely used in the location) cannot properly match real waveforms and favor the appearance of unmodeled time shifts and amplitude discrepancies (e.g., if VMs are constructed to minimize location residuals, they are not sensitive to uppermost shallow layers, which are insufficiently sampled by rays if shallow sources are missing). The situation is even worse when real waveforms can be matched but the retrieved focal mechanism is incorrect. This article investigates an alternative methodology that is more robust with respect to inappropriate velocity models: the inversion of waveform envelopes. The method is built on an empirical basis. It studies the effects of velocity models on synthetic waveforms and finds that the information about focal mechanism is encoded in the variation of the envelope shapes and amplitudes among the seismogram components. Besides synthetic tests, the method has been tested on real data comprising two earthquakes in Brazil: the 2010 $M_w$ 4.3 Mara Rosa (MR) and the 2017 $M_w$ 4.3 Maranhão earthquakes. When compared with solutions from previous studies, based on many polarities and ad hoc path-specific velocity models, we obtained in both cases the same mechanism with a single 1D model and a single-station polarity constraint. The envelope inversion is a promising technique that might be useful in similar sparse networks, such as the one in Brazil, where standard waveform inversion, in general, is not fully efficient.

Electronic Supplement: Figures of waveform comparisons and tables of amplitude ratio due to velocity model (ARMOD) values and velocity models.

Introduction

The initial determinations of focal mechanisms in Brazil were done by Mendiguren and Richter (1978), Assumpção and Suárez (1988), Assumpção (1998a,b, 1992), and Ferreira et al. (1998). New studies on focal mechanisms were done by Barros et al. (2009, 2015), Chimpliganond et al. (2010), Lima Neto et al. (2013), Agurto-Detzel et al. (2014), Oliveira et al. (2015), and Dias et al. (2016). Whereas the initial focal mechanism solution was constrained with the use of the first-motion polarities ($P$ phases) and/or the amplitude ratios of the body-wave phases, the more recent works also use waveform inversion. For example, Zahradník et al. (2015) revisited the previous solution of Barros et al. (2015) for the 2010 $M_w$ 4.3 Mara Rosa (MR) earthquake, which occurred in the state of Goiás, Brazil, performing waveform inversion pre-constrained by first-motion polarities according to Fojtíková and Zahradník (2014). Carvalho et al. (2016) retrieved focal mechanisms of 11 aftershocks ($M_w$ 0.8–1.4) of the MR earthquake by inverting full waveforms using temporary local stations and a local velocity model, the same as used in this work (VM2). The waveform inversion was feasible up to $\sim$10 MSW, in which MSW is the minimum shear wavelength. For example, for $V_S \sim 3.0$ km/s, and the maximum inversion frequency $\sim$2 Hz, the MSW is $\sim$1.5 km; hence, 10 MSW is about 15 km. Dias et al. (2016) demonstrated the possibility of significantly extending the feasible epicentral distance range (65 MSW) using ad hoc velocity models, specifically derived for each source–station path by inverting Love- and Rayleigh-wave dispersion curves of the same event for which the moment tensor is calculated. Polarities were used in their consistency check of the inversion results.

In all these attempts, the authors struggle with the inaccuracy of the existing velocity models of the region. Although
the models are usable in routine event location, their applicability to waveform inversion is limited because of the waveform match is deteriorating in direct relation to the epicentral distance. Synthetic seismograms differ from real ones in terms of amplitudes and time shifts. That is why in the previously cited articles, using waveform inversion, it was necessary to apply local velocity models (or path-specific models) and strong polarity constraints.

As for other attempts to deal with the limited accuracy of velocity models, it is worth highlighting the work of Hallo and Gallovič (2016) and the references therein. They proposed an efficient estimate of the covariance matrix reflecting the Green’s function uncertainty. The authors assumed that the main effect of the inaccurate velocity model is a temporal shift. Including the covariance matrix in their Bayesian formulation, they were able to calculate moment tensors together with their uncertainties. Artificial (ad hoc empirically derived) temporal shifts represent another possible approach to account for differences between the real (unknown) velocity model and the velocity model used in the inversion. This is a central idea of the cut-and-paste method of Zhao and Helmberger (1994). Optimally, the surface-wave shifts can be derived from calibration events (Zhu et al., 2006).

Here, we investigate a different approach based on waveform envelopes. The envelopes were recently proposed by Zahradník and Sokos (2018a). The authors, having obtained focal mechanisms by waveform inversion from a relatively dense network in Greece, removed near stations and investigated the resolution power of remaining relatively distant stations. Waveform inversion from the distant stations failed, but envelopes retrieved the correct mechanisms. Several features make the present article different from that of Zahradník and Sokos (2018a): (a) We are much more concentrated on explaining that the key point of envelopes is fitting the relative strengths of three components at each station (Elestudy of amplitude ratio due to velocity model [ARMOD] parameter and the test with five elementary mechanisms, available in the electronic supplement to this article). (b) Here, we clearly show that standard waveform inversion in an inappropriate model can provide a wrong focal mechanism even when fitting waveforms relatively well. (c) Zahradník and Sokos (2018a) studied only one case similar to our setup (their model A, with six stations at relatively narrow distance range of 381–609 km); we demonstrate two more difficult cases, with fewer stations and more nonuniform epicentral distances. (4) Their work was focused on Greece, where many stations are available. Our article is the first attempt to implement the envelope methodology in more difficult conditions of Brazil, and we are planning to apply the method to future events.

In the present article, our aims are to understand (1) why the envelope inversion technique (ENV) could perform better than waveforms in poor velocity models and (2) how the method performs on real data in Brazil.

In a series of synthetic tests, we analyze the effects of three velocity models on synthetic seismograms (and on their waveform inversion). We arrive at an empirical finding that when substituting waveforms by their envelopes, at least in the relatively low-frequency range, the envelope shapes (normalized at each station) still carry information about focal mechanism via variation of the envelope shapes among the three recorded components. In other words, the envelope of the seismic record drops the phase information and retains mainly the information of the shapes and relative amplitudes of the different components. As such, envelopes are less sensitive to inaccurate velocity models than waveforms.

For the first time, the envelope method is applied to two Ms 4.3 earthquakes in Brazil, the MR and Maranhão earthquakes, both including inversion of distant stations (up to 637 km). According to the Brazilian Seismic Bulletin (BSB), events of this magnitude are rare in Brazil (about every 5 yrs), and because of the country’s vast territory and the low seismic network density, each earthquake is well recorded only at a few stations. Nevertheless, their focal mechanisms are needed to calculate more precisely the present stress field. Using existing velocity models, complemented by just a single first-motion polarity for each event, we succeeded in retrieving the same focal mechanism as previously obtained with complex path-specific models and many polarities.

The envelope inversion method in this article is tested in comparison with two other techniques, the least-squares waveform inversion using the ISOLA software (Sokos and Zahradník, 2013; Zahradník and Sokos, 2018b) and a grid-search waveform inversion with free time shifts (WISH), specifically encoded for this article and explained in the Inversion of Envelopes: Synthetic Test section.

**Synthetic Tests and Method**

**Velocity Models**

In this section, we analyze the effects of velocity models on waveforms and their inversion. We use velocity models VM1, VM2, and VM3, all coming from real-world data. The VM1 (BDFB-Disp) model was derived from dispersion analysis of surface waves observed during the MR main shock at BDFB station (Dias, 2016). The VM2 (GT5) model was obtained from the MR aftershock study (Barros et al., 2015). The VM3 (NewBR) model is based on travel-time data from the BSB; it is a generic model for Brazil (Assumpção et al., 2010). The $V_p/V_s$ ratios in the VM1–VM3 models are 1.74, 1.70, and 1.72, respectively. Quite arbitrarily, we choose VM1 as a reference. The comparison of the models (Fig. 1; Table S2) shows that above Moho, the difference in velocity of VM2 with respect to VM1, reaching ~10%, is considerably greater than the difference between VM3 and VM1. The used models adopt different depths for the Moho discontinuity, 42 km in VM1 and VM3 and 36 km in VM2.

**Forward Simulation of Waveforms**

An idealized example of forward simulation and inversion is provided in the electronic supplement. Synthetic
waveforms and their envelopes are inverted in the same velocity model as used for their simulation. It is demonstrated that not only waveforms (Ⓔ Fig. S1) but also envelopes (Ⓔ Fig. S2) can retrieve the correct focal mechanism and depth.

In the present section, synthetic waveforms are forward simulated for the source–station configuration shown in Figure 2. It reflects the present configuration of the permanent seismic network in central Brazil and the source position of the 2010 MR mainshock. All stations are broadband. Few of these stations existed in 2010. Synthetics are calculated in displacement for a specific focal mechanism identical to the MR mainshock (strike/dip/rake [hereafter, s/d/r] = 253°/36°/121°), at a depth of 5 km and seismic moment $M_{0}$ = $2.04231 \times 10^{15}$ N·m.

For both models VM2 and VM3, we present comparisons of synthetic waveforms with VM1 in Figure 3. All tests are performed in the 0.05- to 0.1-Hz low-frequency range. Real data of $M_{w} \sim 4$ have a good signal-to-noise ratio in the dominant surface-wave group at the studied epicentral distances in this frequency range.

When comparing nonnormalized waveforms, we found time shifts and amplitude differences due to different velocity models (see ( @[copyright] Fig. S3). To balance the major amplitude effect, hereafter, just for plotting, we normalize synthetics in each model to the maximum value at each station.

When choosing VM1 as reference and comparing VM2 and VM3 with this reference (Fig. 3), we observe that the waveforms from VM2 and VM3 display time shifts compared with VM1. The shifts in VM2 are considerably greater, in agreement with the velocity differences in Figure 1 and Table S2. The presence of these shifts clearly indicates that the standard waveform inversion may not be suitable to retrieve the correct focal mechanism. On the other hand, we also observe significant similarities between the VM2 and VM1 synthetics (and even more similarities between the VM3 and VM1 synthetics). The similarity in Figure 3 refers to the duration of the dominant surface-wave groups and to relative amplitudes of the individual components at each station. Additional tests, proving this similarity for five double-couple (DC) elementary mechanisms (Zahradník and Sokos, 2018b), fully
Describing an arbitrary deviatoric moment tensor, are given in Figure S4. The goal of this test is to show that the similarities observed in Figure 3 are general, not related only to the specific focal mechanism of that figure.

To quantify the relative amplitude variations among the components, we introduce an auxiliary parameter called ARMOD (see Table 1). ARMOD is defined as the ratio of maximum (absolute) values of the normalized components in two models (e.g., max Z[VM1]/.0137 VM1/.0138 = max Z[VM2]/.0136 for station CAN3). To better understand the meaning of ARMOD, consider the PEXB station as an example. We see in Figure 3a that at this station, the north–south component is weak, and the east–west component is strong in both velocity models VM2 and VM1; thus, the north–south and east–west components feature ARMOD values close to 1 (0.87 and 1.0). As shown in Table 1, the average ARMOD value over all stations is also close to 1. This means that globally, for all stations, the relative amplitude variation among components is weakly affected by the model; see also Table S1.

The similarity between relative amplitudes across components in different velocity models, together with the similarity between waveform durations (Fig. 3), suggests that normalized waveform shapes (i.e., envelopes) are less dependent on velocity model than waveforms. See also Figure S3.

Table 1

<table>
<thead>
<tr>
<th></th>
<th>VM1/VM2</th>
<th>VM1/VM3</th>
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<tbody>
<tr>
<td></td>
<td>N</td>
<td>E</td>
</tr>
<tr>
<td>CAN3</td>
<td>1.00</td>
<td>1.37</td>
</tr>
<tr>
<td>PEXB</td>
<td>0.87</td>
<td>1.00</td>
</tr>
<tr>
<td>BDFB</td>
<td>1.23</td>
<td>1.25</td>
</tr>
<tr>
<td>SNDB</td>
<td>1.19</td>
<td>1.00</td>
</tr>
<tr>
<td>ARAG</td>
<td>0.73</td>
<td>0.56</td>
</tr>
<tr>
<td>LAJE</td>
<td>1.79</td>
<td>1.00</td>
</tr>
<tr>
<td>SDBA</td>
<td>1.00</td>
<td>1.41</td>
</tr>
<tr>
<td>JAN7</td>
<td>1.00</td>
<td>1.16</td>
</tr>
<tr>
<td>Average ± standard deviation</td>
<td>1.11 ± 0.26</td>
<td>1.08 ± 0.23</td>
</tr>
</tbody>
</table>

ARMOD values comparing the velocity models VM2 and VM3 with the reference model VM1 in the 0.05- to 0.1-Hz frequency range.
Thus, we assume that the envelope inversion of real data could be less affected by the inappropriate velocity model than the waveform inversion. To demonstrate this behavior, we further analyze synthetic data by performing inversion of the (normalized) envelopes. To be more realistic, white random Gaussian noise is added to the synthetics before bandpass filtering and calculating envelopes. The noise has a constant magnitude across the stations. Thus, as in real records, distant stations are more affected. We tested several noise levels, but here we present only the worst-case scenario.

Inversion of Envelopes: Method

The envelope method was recently proposed by Zahradník and Sokos (2018a). Its main features can be summarized as follows. Real, instrumentally corrected waveforms and synthetic waveforms are subjected to identical band-pass fourth-order causal filtration. Point-source synthetics for an assumed moment rate function (e.g., delta function) and for unit moment are calculated with the use of the method of discrete wavenumbers from Bouchon (1981) and Coutant (1989). The point source is situated at an assumed hypocenter position. Envelopes of the bandpass-filtered displacement are calculated by Hilbert transform. The pure-shear (100% DC) focal mechanism is grid searched in terms of the s/d/r angles. Real envelopes and synthetic envelopes for every tested s/d/r are normalized to the maximum component at each station. For every s/d/r, the real and synthetic envelope shapes are compared in terms of their L2-norm difference (misfit), calculated with a time shift maximizing their cross correlation. The envelopes are plotted with the calculated time lags providing the best match, and the lag at each station component is recorded and reported. For the best-fitting solution (s/d/r), that is, the minimum misfit, the envelopes are returned to their nonnormalized values, and scalar moment is calculated according to equation (9) of Zahradník and Gallovíč (2010). The moment is converted to moment magnitude.

Besides the best-fitting solution, the method also provides a group of s/d/r solutions fitting data within an adopted misfit threshold (e.g., 5% or 10% of the best-fit solution). The misfit threshold depends on how deep is the misfit function minimum. A shallow minimum and a large threshold yield strongly nonunique solutions (see examples later). A suitable threshold is dependent on the data set, number of stations, network geometry, frequency band filter, event magnitude, and signal-to-noise ratio (Zahradník and Sokos, 2018a), in which typical values of 3%, 5%, and 10% are proposed.

The envelopes do not change if seismograms are multiplied by −1; therefore, any solution from the envelope inversion must be always checked a posteriori against at least one polarity to avoid flipping of the P and T axes. More than a single polarity match can be required if the s/d/r solutions need to be better constrained; the solutions not agreeing with all prescribed polarities are discarded. Another possibility is to apply a polarity preconstraint; that is, the observed envelopes are inverted by grid searching in a limited group of s/d/r combinations, those that were previously derived from suitable software (e.g., FOCEM; Snake et al., 1984). The latter approach is similar to the polarity preconstrained waveform inversion in the cyclic scanning of the polarity solutions (CSPS) technique from Fojtíková and Zahradník (2014).

Inversion of Envelopes: Synthetic Test

In this section, the synthetics in the reference model VM1 play a role of data, which we invert for the focal mechanism by means of inappropriate models VM2 and VM3 (two scenarios, VM1 × VM2 and VM1 × VM3). The low-frequency range is 0.05–0.10 Hz. To avoid the flipping of P and T axes, we used one polarity, specifically at station BDFB (dilatation, D).

The result is shown in Figure 4. Despite differences of waveforms in VM1 and VM2 (shown previously in Fig. 3), the (normalized) envelope shapes can be well matched. Because the relative amplitudes of the components, controlled by focal mechanism, are well matched, the envelopes provide a good proxy of the focal mechanism. Indeed, in terms of K-angle (Kagan, 1991), its deviations from the reference focal mechanism are only 9° and 18° for VM2 and VM3, respectively. As expected and according to the differences of used velocity models, the group of solutions at the VM3 model, for which misfit is within a 3% threshold, is more compact compared with the VM2 model, as demonstrated by dispersion of nodal lines in the focal mechanism plots of Figure 4b,d. This means that variation of the envelope misfit with s/d/r angles possesses a relatively narrow global minimum in VM3. The retrieved moment magnitude is the same as that of the tested source.

It is useful to compare this result with an independent approach. In this test, we used the WISH algorithm, written to support the work in this article. WISH makes use of bandpass-filtered waveforms and inverts them into a pure DC constrained source model. The s/d/r angles are grid searched, as in ENV technique, and free time shifts, up to a prescribed time limit, are allowed to achieve the best fit between data and synthetics, hence minimum misfit. At each trial s/d/r combination, a single optimal shift is searched for each individual station component because we are interested in waveforms dominated by the surface-wave group. No envelopes are used in WISH. Figure 5 demonstrates the use of WISH at the same setup as ENV in Figure 4, that is, using the same velocity models VM2 and VM3 to invert data simulated in VM1, as well as using the same frequencies and threshold. As a result, in this frequency band, the obtained WISH solutions are not better than ENV. In Figure 5, the WISH code provided similar results as ENV in Figure 4, just slightly more deviating from the true mechanism, with K-angles of 26° and 21° for VM2 and VM3, respectively.

In summary, based on synthetic tests, we expect that if the real velocity model and the model used in the inversion
differ in line with VM1 and VM2 or VM1 and VM3, the envelope inversion in the low-frequency range can provide a good approximation of the true focal mechanism and moment.

Real-Data Applications

To validate the proposed waveform envelope inversion, we apply this new methodology to two real events for which reliable source parameters were independently obtained in previous studies. The study cases are (a) the 2010 MR mainshock and (b) the 2017 Maranhão earthquake (Table 2).

Mara Rosa Earthquake

This event occurred on 8 October 2010 (Barros et al., 2015). At that time, the number of stations in central Brazil was lower than eight. We use the four nearest available stations (121–542 km)—broadband stations BDFB and JAN7 and short-period stations CAN3 and SFA1. The inversion is performed in velocity models VM2 and VM3. The focal mechanism characterized by s/d/r = 253°/36°/121° (Dias, 2016) is adopted as a reference solution. That mechanism was obtained by a method of Dias et al. (2016) using an ad hoc set of source–station velocity models based on dispersion curves derived from the MR event. The waveform inversion of Dias et al. (2016) was constrained to fit as many as 11 polarities as possible; in that case, the 91% polarity match was achieved. Our intention is to show that we can obtain the correct mechanism without the path-specific models using envelopes. We use the lowest possible frequencies where the signal is above the noise (Table 2).

Figure 6. The envelope (ENV) inversion of synthetic data in the low-frequency range. Synthetic waveforms in model VM1 played a role of data and their envelopes (black) are inverted by means of synthetics calculated in different velocity models (gray). (a) Inversion in model VM2; (c) inversion in VM3. (b,d) The results of the envelope inversions. The shaded areas show the best-fitting solution VM2 and VM3, with K-angles of 9° and 18°, respectively, relative to the reference mechanism. The polarity from BDFB (D) was used to avoid the flipping of P and T axes.
Second, we perform the inversion of envelopes. The frequency range is the same as in the waveform inversion of Figure 6. We use grid searching of full parameter space (step 10° in s/d/r angles), and the solution is subjected to posterior polarity check at a single station, BDFB (dilatation D), just to avoid flipping of the P and T axes. The result is shown in Figure 7. In both velocity models, we observe a good match between the shape of the observed and synthetic envelopes. Because the envelopes are normalized per station (not per component), the match means that we fitted the relative amplitudes among the components related to the radiation pattern.

The best-fitting solution and the solutions within a 5% misfit threshold (i.e., matching data almost equally well) are presented in Figure 7. There are two families of P and T axes, marked 1 and 2. Family 1 is closer to the reference solution. Indeed, the minimum Kagan angles of this family are 20° in the VM2 model and 6° in VM3. Family 2 is far from the reference solution, mainly as regards the T axis; the minimum K-angles are 42° and 38° for the VM2 and VM3, respectively. The calculated magnitudes in both velocity models are the same, $M_w = 4.2$; this value differs from the reference solution ($M_w = 4.6$) but is consistent with the greatly scattered $M_w$ estimates of 4.3–4.7 for this event (Zahradník et al., 2015).

Maranhão Earthquake

The Maranhão earthquake occurred on 3 January 2017 at 12:43:47 (UTC), in the north of Brazil near the shore (Dias et al., 2017; Table 2). The event was felt in a wider area up to 250 km, with a maximum modified Mercalli intensity VI. The epicentral zone is basically aseismic. The event was detected by most stations of the Brazilian seismographic network at regional distances; the nearest station is situated at about 40 km (ROSB); the others that we use are above 470 km (TMAB, 478 km; PAL1, 562 km; TUC4, 641 km). All are broadband stations.

As for the MR event, the reference focal mechanism for the Maranhão event was obtained with a set of path-specific velocity models inferred from dispersion curves. The waveform inversion was constrained by 10 polarities, with the 79% polarity fit, and is characterized by $s/d/r = 339°/83°/−2°$ (Dias et al., 2017). Here, we show that the same mechanism can be obtained by the envelope inversion constrained by just one polarity using a simple 1D model, the same for all source–station paths.

The inversion is performed with the lowest possible frequency range, 0.05–0.1 Hz. First (Fig. 8), we show that the waveform inversion in the VM1, VM2, and VM3 models

![Figure 5](https://pubs.geoscienceworld.org/ssa/bssa/article-pdf/109/1/138/4627747/bssa-2018119.1.pdf)
Inversion for Focal Mechanisms Using Waveform Envelopes and Inaccurate Velocity Models

could not match all stations simultaneously because of the unsuitability of the velocity model at distant stations. Only the nearest station (ROSB) was fitted very well (VR > 0.7), and the waveform inversion retrieved an incorrect focal mechanism, deviating from the reference by K-angle 81°, 89°, and 69°, for VM1, VM2, and VM3, respectively. In this case, it is possible to say that the inversion is prioritizing the fit at ROSB station because of its larger amplitudes.

Second (Fig. 9), the envelope inversion was applied using the same velocity models and frequency range. Following the reference, we fixed the depth to 7 km. Polarity was constrained just at a single station, TMAB (D). The best-fitting solutions for both VM2 and VM3 are identical (340°/80°/10°), deviating from the reference by K-angle of 13° only. Except for one outlier, the nodal lines and P and T axes obtained in the 10% threshold with VM2 model have K-angle 13°–19°, but the solution with VM3 model is almost unique. The 10% threshold is used here because this event has a very narrow global minimum of the misfit function, so that in the 5% threshold (such as MR), we could observe just the best-fitting solution. Obtaining a small nodal-line scatter with a larger threshold is an indicator of a more reliable solution. The calculated magnitudes are 4.3 and 4.4 (MW), in VM2 and VM3, respectively; these values are compatible with the reference magnitude (MW 4.3).

Discussion and Conclusion

Inversion of waveforms into focal mechanisms is dependent on the quality of velocity models. Inaccurate models are responsible for temporal and amplitude discrepancies between real and synthetic data. According to our tests, we found that in most situations, the inaccurate model prevents good waveform matching. However, in some situations, the standard waveform inversion as in ISOLA, that is, without any artificial time shifts, can match real data with synthetics calculated using an inaccurate model, simply because the inversion distorts the focal mechanism. This distortion or bias is clearly demonstrated in our example of Figure 6b, in which we obtained a good waveform match for a wrong source. Another possibility studied in this article (ENV) is to add freedom in the inversion by leaving waveforms and invert for an ensemble of the focal mechanisms, which fit only the normalized envelope shapes because they are simpler than waveforms.

The inherent inaccuracy of existing velocity models becomes critical if distant regional stations are used. Three velocity models are studied in this article, all based on geophysical measurements in Brazil. Synthetic waveforms calculated in these models show significant amplitude differences and time shifts. Contrary to waveforms, their overall shapes (formally described by envelopes) are less sensitive to velocity models. We investigated an empirical method in which envelopes are inverted for focal mechanism instead of waveforms. The method is based on systematically grid searching the parameter space (s/d/r angles), aiming to match the envelope form of normalized station records, mainly to reproduce relative amplitudes of the three recorded components at each station.

The method has been tested on synthetic data, which proved its ability to retrieve correct focal mechanisms. To avoid flipping between P and T axes, the envelope inversion must be complemented by at least one (well guaranteed) first-motion polarity. If the available envelopes do not constrain the solution enough, additional polarities should be used.

The method has been applied to two MW 4+ events, the MR and Maranhão events in Brazil, both with a previously known focal mechanism, taken here as a reference. It is important to highlight that these are the only shallow events above magnitude 4 that occurred in the previous 7 yrs in Brazil according to the BSB. The data set is on the limit of possibilities in terms of number of available stations and usable frequency range. For these earthquakes, recorded mostly at distant stations (∼100–600 km), we demonstrated that in the available velocity models: (a) standard waveform inversion is not applicable, and (b) the newly proposed envelope inversion is more useful. By usefulness, we mean that a group of the solutions with similar mismatch between the observed and synthetic envelopes contains the focal mechanisms that are close to the reference solution, and that the moment magnitude MW of the solutions is correct, too.

Table 2

Summary of the Mara Rosa and Maranhão Earthquakes

<table>
<thead>
<tr>
<th>Stations</th>
<th>CAN3</th>
<th>BDFB</th>
<th>JAN7</th>
<th>SFA1</th>
<th>ROSB</th>
<th>TMAB</th>
<th>PAL1</th>
<th>TUC4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epicentral distance (km)</td>
<td>121</td>
<td>241</td>
<td>490</td>
<td>542</td>
<td>41</td>
<td>475</td>
<td>557</td>
<td>637</td>
</tr>
<tr>
<td>Frequency range (Hz)</td>
<td>0.1–0.2</td>
<td>0.05–0.1</td>
<td>0.05–0.1</td>
<td>0.08–0.13</td>
<td>0.05–0.1</td>
<td>0.05–0.1</td>
<td>0.05–0.1</td>
<td>0.05–0.1</td>
</tr>
<tr>
<td>Reference MW, s/d/r (°)</td>
<td>4.6, 253/36/121</td>
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<td></td>
<td>4.3, 339/36/2</td>
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The basic parameters of the events are taken from the Brazilian Seismic Bulletin. The reference strike/dip/rake (s/d/r) angles and magnitudes are 4.3 and 4.4 (Mw 4.3).
Figure 6. The unsuccessful waveform inversion of real data of MR earthquake. Standard waveform inversion using ISOLA in velocity models (a,b) VM1, (c,d) VM2, and (e,f) VM3, with station-dependent frequency ranges: CAN3 0.1–0.2 Hz, SFA1 0.08–0.13 Hz, and BDFB/JAN7 0.05–0.1 Hz, respectively. Black and gray traces denote observed displacement data and synthetics, respectively. (g) The map with epicenter (star) and used stations (triangles). The inset shows a broader region. The color version of this figure is available only in the electronic edition.
Intentionally, we used only a very weak polarity constraint (single station) for each earthquake. Because the tested stations were far from the epicenter and small in number, the uncertainty, or nonuniqueness of the solution (as represented by nodal lines within a given misfit threshold) was not small for the MR reverse-faulting event. On the contrary, despite the same limitations (few distant stations), the strike-slip Maranhão earthquake revealed a very narrow global minimum of the misfit function, with perfect agreement with the reference solution derived by an independent method using independent data sets. The good performance of the method in the Maranhão case probably resulted from a combination of several factors: favorable position of the stations with respect to the radiation pattern of the surface waves (which dominate the records) and the use of only broadband stations, allowing implementation of lower frequencies than in the MR case.

Importantly, the envelope inversion results for MR and Maranhão earthquakes were very similar in both tested velocity models of the region—VM2 (GT5) and VM3 (NewBR), although waveforms in these models are significantly different. This is a clear indication of the robustness of envelopes.

More than 10 polarities are available for the MR and Maranhão events. We could easily use them as a posterior

**Figure 7.** The successful envelope inversion of real data of MR earthquake. Compared are the envelopes of observed data (black) and synthetic data (gray). The synthetic data correspond to the best-fitting solution found in models (a) VM2 and (b) VM3. The obtained focal mechanisms in (c) VM2 and (d) VM3 and (e) the reference solution; the legend and hatching refer to the best-fitting solution. Nodal lines correspond to the 5% misfit threshold. Numbers 1 and 2 indicate two distinct groups of P and T axes. Number 1 is close to the reference solution (strike/dip/rake [s/d/r]: 253°/36°/121°).
Figure 8. The unsuccessful waveform inversion of Maranhão earthquake. Inversion was done with velocity models (a,b) VM1, (c,d) VM2, and (e,f) VM3. The station TUC4-EW component was removed from the inversion because of the presence of a strong instrumental disturbance (Zahradník and Plešinger, 2010). Black and gray traces are observed and synthetic waveforms; the traces are nonnormalized. (g) The map with epicenter (star) and used stations (triangles). The inset shows a broader region.
or prior constraint, thus obtaining an almost unique mechanism, very close to the reference solution. In this article, we tested the resolving power of the envelopes plus free time shift; thus, we intentionally reduced the polarity constraint to minimum (just a single polarity). Obviously, in practice, many polarities can be added as a constraint when processing real data. However, caution is needed to avoid a misleading overconstraint, for example, by prescribing opposite polarities at stations close to each other on the focal sphere. Another danger comes from near-source stations for which polarities may have inaccurate take-off angles (e.g., 60° instead of 90°) if velocity models include formal shallow discontinuities (Zahradník et al., 2015).

Although the envelope method solves some problems not resolvable with waveforms, it also has limitations. Because the method is new, not all limitations are yet well known; however, a possibly poor performance in the absence of broadband stations is one of them. Using only short-period stations (and hence working at higher frequencies) could easily make it impossible to match real envelopes by synthetics. Some of these problems are fortunately detectable—for example, when synthetics appear to have significantly shorter duration than

![Figure 9](https://example.com/figure9.png)

**Figure 9.** The successful envelope inversion of real data of Maranhão earthquake. Black and gray traces are observed data and synthetics. TUC4-EW was removed from the inversion because of an instrumental disturbance. The velocity models VM2 and VM3 were used: (a,b) for envelopes, (c,d) for focal mechanism plots, (e) the reference solution.
real data, that is, indicating the presence of unmodeled high-frequency waveform features. Other problems may be less easily recognizable such as in the case in which envelopes are seemingly well explained (because they are simpler than waveforms), but the solution is nonunique, and some of the focal mechanisms within the adopted ensemble are incorrect. It is possible that future applications will provide some simple criteria to discard unreliable solutions. Until that we can only recommend (1) to discard stations with instrumental disturbances, (2) to use as many stations with good signal-to-noise ratio as possible, (3) to use as low frequency as possible, (4) to include reasonable polarity constraints, (5) to repeatedly use several established velocity models of the studied region, and (6) to use a reliable location of the hypocenter.

As for (6), we also tested possible grid searches for the source depth, but in the context of distant stations and low frequencies, we did not find a sufficient depth resolution. For example, in synthetic test of Figure S2 with a true source at 5 km, we ran the inversion from 1 to 10 km, in steps of 1 km, and we found a small preference to the correct depth, but the depth resolution was too small to encourage the depth search in real-data applications.

The studied source–station distance can be expressed in terms of MSW. Although standard waveform inversions almost always fail at distances > 10 MSW, in the MR example, we were able to invert envelopes into focal mechanism up to 18 MSW and in the Maranhão example up to 21 MSW; similar results were obtained by Zahradník and Sokos (2018a). It seems to be a small improvement when comparing with the 65 MSW in path-specific models (Dias, 2016), but it should be noted that our inversion is based on simple standard 1D models, same for all stations. It is also worth to mention that the envelope inversion can be applied when missing one station component; this is the case in which it is generally not possible to obtain a path-specific model.

In this study, we proposed a new method, which may extend current possibilities of the focal mechanism and M_w retrieval, especially if only simple velocity models and few stations at relatively large regional distances are available. Robustness with respect to imperfect velocity models and the simplicity of the method may be useful in real-time applications, where the derivation of path-specific velocity models is not trivial.

Data and Resources

The majority of waveform data used in this article belong to the Brazilian Seismograph Network (BSN), and they are freely available at www.obsis.unb.br and www.sismo.iag.usp.br (last accessed April 2018). University of São Paulo (USP) provides federated (FDSNWS) webservices for downloading event waveforms at http://siseirequest.iag.usp.br (last accessed September 2018). The plots were made with the use of the code Generic Mapping Tools v.4.2.1 (http://www.soest.hawaii.edu/gmt, last accessed April 2018). The waveform inversion was done with the software ISOLA and can be downloaded from http://geo.mff.cuni.cz/~jz/for_Costa_Rica (last accessed April 2018). The information on the Brazilian seismicity is from the Brazilian Seismic Bulletin (BSB) available at http://moho.iag.usp.br/eq/bulletin (last accessed June 2018).

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