Resolvability of Isotropic Component in Regional Seismic Moment Tensor Inversion
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Abstract We propose a new approach to resolve the isotropic component of the seismic moment tensor and its uncertainty. In linearized inversion problems, where the earthquake or explosive-source location and origin time are fixed (e.g., assumed to be known), the uncertainty of the moment tensor can be studied through the eigenvalues and the eigenvectors of the design matrix, which allows the representation of the theoretical misfit by means of a 6D error ellipsoid. Because the design matrix depends only on the structural model and receiver source geometry, the analysis can be performed using recorded seismic waveforms, or without. In the nonlinear inversion problems, where the free parameters are eight (e.g., the six elements of the moment tensor, depth, and origin time), we propose a waveform-inversion scheme in which the trace of the moment tensor varies systematically and the remaining seven free parameters are optimized for each specific value of the trace. In this way, a 1D experimental probability density function of the moment tensor trace is constructed. To demonstrate the applicability of the method, we apply it to two shallow earthquakes ($M_w$ 4.9 and 4.7) with epicenters close to the Columbo volcano, located 20 km northeast of the island of Santorini, Aegean Sea, Greece. We use 15 near-regional (60–310 km) records at frequencies below 0.1 Hz and two alternative crustal models. We conclude that the main uncertainties are attributed to the crustal model and to the trade-off between the isotropic component and the source depth.

Introduction

The resolvability of the seismic moment tensor (MT) and its uncertainty are among the topical issues of modern seismology. The full MT can be decomposed into its deviatoric (DEV) and isotropic (ISO) parts. An equivalent name for the isotropic part is volumetric. This decomposition is mathematically unique. It is the deviatoric part that can be further decomposed following a variety of schemes (e.g., Jost and Herrmann, 1989; Julian et al., 1998); for example, the most common approach is the decomposition of the deviatoric part into the largest possible double couple (DC) that has a remainder component, the so-called compensated linear vector dipole (CLVD). The definition of the relative size of the components (i.e., the so-called ISO, DC, and CLVD percentages) may have several forms (Vavryčuk, 2001).

The moment tensor can be used in a purely formal way to relate the seismic wave field and the elastic response of the Earth (Green’s function). Some authors attempt to give a physical interpretation of the moment tensor. One of possible physical models of the full-MT source, including the ISO, DC, and CLVD components, is the tensile earthquake model, in which the slip vector is generally nonparallel with the fault plane (Vavryčuk, 2011). Dufumier and Rivera (1997) combined the full moment tensor with an additional, so-called nontectonic isotropic component.

Compared to the DC component, CLVD and ISO are the most unstable parameters in the moment tensor inversions. They often trade off with other source parameters, such as depth, source time function, source multiplicity, as well as with the structural parameters (heterogeneity, anisotropy). A number of attempts to resolve these issues have been proposed. Vasco (1990) used the method of extremal models to rigorously estimate the bounds of the MT trace. A number of approaches are more experimental, as for example, to rerun the inversions using data contaminated by artificial noise or to artificially perturb the crustal models used (e.g., Šílený and Hofstetter, 2002; Wéber, 2006; Vavryčuk, 2007). The graphic approach of Riedesel and Jordan (1989) provides the tools to visualize the moment tensor uncertainties and the deviations of the MT from a pure DC model. For the inversion schemes that provide families of acceptable solutions (in addition to the best-fitting solution), the family itself is used to experimentally construct the confidence intervals of the source parameters. A representative example is the work of Šílený (1998), who used genetic algorithms to construct probabilistic estimates of the model parameters, including
the posterior probability density function of the model parameters. Ford et al. (2010) proposed the network-sensitivity solution. They grid searched the parameter space in terms of the MT invariants, and, using the source-type plots of Hudson et al. (1989), were able to effectively identify the non-DC sources in practice. Panza and Saraò (2000) emphasized the role of synthetic tests in evaluating the reliability of the non-DC components.

Nakano et al. (2008) discussed the trade-off between the source position and the non-DC components and recommended to use the DC assumption when determining the source position. In this line of work, Zahradník and Sokos (2011) had to use a DC-constrained solution to determine the centroid position of the 2011 $M_w$ 7.2 Van earthquake, Turkey, from near-regional accelerograms. Dufumier and Rivera (1997), within the frame of the linear theory, broadly analyzed the condition numbers of the MT inverse problem and applied regularizations. Thus, they were able to theoretically interpret the previously observed trade-offs between the isotropic-source component, hypocentral depth, and source time function. Zahradník, Sokos, et al. (2008) emphasized the trade-off between the non-DC components and centroid time.

Significant isotropic components may occur, for example, during man-made explosions, volcanic events, seismic events related to migration of fluids, and gas or rupture on nonplanar faults. As certain types of these events may have a very long duration, the inversion of the MT temporal variation is also important (Auger et al., 2006; Yang and Bonner, 2009). The temporal variations of the isotropic and shear components of the source may be different (e.g., Davi et al., 2010). Vavryčuk and Kuhn (2012) developed a new method to retrieve the time function and analyze the stability of the isotropic component as a function of a random noise in waveform amplitudes and temporal shifts.

From the published research so far, it is indicated that moment tensors calculated for volcanic events do not necessarily involve an isotropic component. Tkalčić et al. (2009), who also provide a thorough literature review on non-DC earthquakes, used a sensitivity test in which they systematically decreased the number of stations down to a single one. Their test revealed no isotropic change for an $M_w$ 5 volcanic earthquake. Dreger et al. (2000) found significant isotropic components for two $M_w$ 4.6 and 4.9 earthquakes in the Long Valley caldera, possibly related to hydrothermal or magmatic processes; however, a comprehensive stability testing of the MT inversion for 33 events with $M_w > 3.5$ in the same volcanic region, made by Templeton and Dreger (2006), showed that 28 of them are best characterized by a pure double-couple model.

From the above brief review of the state-of-the-art methods, it follows that the treatment of the isotropic component in moment tensor inversions and its uncertainty is a significant issue in theoretical seismology, with a variety of approaches proposed to remedy the problem. However, new and robust techniques applicable in the routine seismological analysis are necessary, as for example, in the cases of volcano monitoring or in nuclear-test monitoring, where to resolve the uncertainty of the isotropic component is significant. In an effort to contribute to this broad goal, the specific objective of this paper is to propose a new simple numerical method for constructing a 1D experimental probability density function (PDF) related to the isotropic component. The paper is structured as follows. It starts with the description of the MT inversion and continues with the methodical details of the uncertainty estimate. First, we deal with the linear MT inversion with six parameters (a fixed centroid position and time) and present a theoretical 1D PDF allowing for the simplest estimate of the ISO uncertainty in the 6D parameter space (Zahradník and Custódio, 2012). Then we propose an extension into the nonlinear MT inversion in the 8D parameter space (i.e., the six-component MT, centroid depth, and time). The innovation is a simple way of constructing an experimental 1D PDF of the isotropic component. In the remaining sections, we apply both the traditional and the new methods to two moderate-size earthquakes with epicenters close to an active submarine volcano in the Aegean Sea and discuss the uncertainty of the obtained isotropic component in the seismic moment tensor.

Prior to the application using observed data, we performed a number of tests to validate the new codes. Extensive synthetic tests were not conducted for several reasons: (1) Synthetic tests are mainly important in multiparameter and/or strongly nonlinear inverse problems, where the inversion may provide solutions qualitatively different from the true model and/or when the solution is highly nonunique. This is not the case of the MT inversion. (2) A number of synthetic tests have been already published (see the above-cited papers) that effectively indicate how spurious non-DC components may arise in an MT inversion due to imperfections in modeling; for example, seismic noise, mislocation, mismodeling of the wave propagation (inaccurate Green’s functions), to mention a few. (3) The synthetic tests have a very limited practical significance. For example, they may predict the MT uncertainties for a given variation of the crustal model, say within the 5% or 10% of the assumed model, but this is of no practical value if we do not know how far from reality our model is.

Method

Forward and Inverse Problem

We consider a point source of seismic waves of a given position and origin time, and express displacement $u$ by means of moment tensor $\mathbf{M}$ and spatial derivatives of Green’s tensor $\mathbf{G}$ (Aki and Richards, 2002):

$$u_i(t) = \sum_{p=1}^{3} \sum_{q=1}^{3} M_{pq} \ast G_{ip,q},$$

where $\ast$ stands for temporal convolution and $p, q$ denote three Cartesian coordinates. The moment tensor can be expressed in
the form of a linear combination of six elementary (dimensionless) tensors $M^i$:

$$M_{pq} = \sum_{i=1}^{6} a_i M^i_{pq}. \quad (2)$$

It represents a convenient parametrization because in this way the source is characterized by six scalar coefficients $a_i$.

We use the elementary tensors implemented in the discrete-wavenumber code AXITRA (Bouchon, 1981, Courtant, 1989):

$$M^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$M^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}, \quad M^4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$M^5 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M^6 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

The $M^1$ to $M^5$ tensors represent five DC focal mechanisms, while $M^6$ is a purely isotropic source.

We must note that the six elementary tensors used here to aid the MT inversion should not be confused with various tensors used in the literature to decompose the MT for purposes of its physical interpretation, for example, to decompose the MT into the isotropic part and three DC tensors (e.g., fig. 3 of Julian, 1998, or p. 42 of Jost and Herrmann, 1989).

The interpretation of the MT used in this paper is defined in Decomposition of MT.

Combining (2), (3) we arrive at

$$M = \begin{pmatrix} -a_5 + a_6 & a_1 & a_2 \\ a_1 & -a_5 + a_6 & -a_3 \\ a_2 & -a_3 & a_4 + a_5 + a_6 \end{pmatrix}, \quad (4)$$

where $a_i$ (with the dimension of moment) are coefficients of the linear combination in equation (2). Note that the trace of the moment tensor is $\text{tr}(M) = 3a_6$. The scalar seismic moment is defined as the Euclidian norm of the MT (Silver and Jordan, 1982),

$$M_0 = \sqrt{\frac{\sum_{p=1}^{3} \sum_{q=1}^{3} (M_{pq})^2}{2}}. \quad (5)$$

Combining (1) and (2) yields

$$u_i(t) = \sum_{p} \sum_{q} \left( \sum_{j=1}^{6} a_j M^j_{pq} \right) * G_{ip,q}. \quad (6)$$

and then

$$u_i(t) = \sum_{j=1}^{6} a_j \left( \sum_{p} \sum_{q} M^j_{pq} * G_{ip,q} \right) = \sum_{j=1}^{6} a_j E^j_i(t). \quad (7)$$

where $E^j$ denotes the $j$th elementary seismogram corresponding to the $j$th elementary moment tensor. Here we assume that the moment temporal function is known, and it is assumed to have the form of a step function, which is a good approximation at frequencies below the corner frequency of the event. In matrix notation,

$$u = Ea. \quad (8a)$$

The overdetermined linear inverse problem (8a) for $a$ can be solved by the least-squares method

$$a_{\text{opt}} = (E^T E)^{-1} E^T u. \quad (8b)$$

where superscripts T and $-$ stand for matrix transposition and inversion. This least-squares formulation is standard (e.g., Kikuchi and Kanamori, 1991), but in our case the elementary moment tensors (3) differ from those of the referenced paper. The difference is formal and has no effect on the solution. Technically, the processing of the complete observed seismograms $u$ and the calculation of the elementary seismograms $E$ for a given time function, as well as the inversion of $a_{\text{opt}}$, are all performed using ISOLA software (Sokos and Zahradník, 2008). No station-dependent temporal adjustment to improve the fit between the observed and synthetic seismograms is introduced (Zahradník, Janský, et al., 2008).

In case of an unknown source position and time, which are related to the displacement in a nonlinear way, we seek these additional parameters (centroid position and time, hidden in $E$) by grid search. In other words, we still solve linear problem (8a) for $a$ but do so repeatedly with different $E$.

The grid search maximizes the correlation between the observed ($u$) and synthetic ($s$) seismograms

$$\text{Corr} = \frac{\int us}{\sqrt{\int u^2 s^2}}, \quad (9)$$

where $\int us = \sum_i \int u_i(t) s_i(t) dt$ and summation is over components and stations.

The match between real and best-fitting seismograms is measured by the L2-norm misfit

$$\text{misfit} = \int (u - s)^2$$

and/or by means of the global variance reduction (VR)

$$\text{VR} = 1 - \frac{\text{misfit}}{\int u^2}. \quad (11)$$

If synthetics $s$ are found by least-squares misfit minimization, that is,
where $a_{\text{opt}}$ is given by (8b), then

$$\int u \cdot s = \int s \cdot s,$$  \hspace{1cm} (12b)

and the correlation is related with the variance reduction through the simple formula

$$\text{Corr}^2 = \text{VR}.$$  \hspace{1cm} (13)

**Decomposition of MT**

As previously mentioned there are a number of schemes to interpret the resulting MT (i.e., to decompose it into components with a simple meaning). We follow the most common decomposition $M = M_{\text{ISO}} + M_{\text{DEV}}$, where $M_{\text{ISO}}$ and $M_{\text{DEV}}$ are the isotropic and deviatoric parts, respectively. Furthermore, $M_{\text{DEV}} = M_{\text{DC}} + M_{\text{CLVD}}$ (e.g., Julian, 1998, p. 530). The eigenvalues of $M_{\text{DEV}}$ define commonly used parameter $e$, ranging from 0 (pure DC) to $\pm 0.5$ (pure CLVD); $e = -e_1/\text{abs}(e_2)$, where $e_1$ and $e_2$ are the eigenvalues of $M_{\text{DEV}}$ with the minimum and maximum absolute values, respectively. The relative size of the individual components is expressed in percentages. Their definition has not been standardized in the literature; in this paper we use the percentages defined in equation (8) of Vavryčuk (2001): $\text{ISO} = 100[\text{tr}(M)/3]/\text{abs}(e^*)$, where $e^*$ is the eigenvalue of the full moment tensor $M$, which has the maximum absolute value. CLVD = 2e[100 - abs(ISO)]; DC = 100 - abs(ISO) - abs(CLVD). Alternatively, in this paper we measure the isotropic part of MT also directly by means of the $a_6$ coefficient because, according to equation (4), $a_6 = \text{tr}(M)/3$.

**Isotropic Component—Uncertainty in the Linear Case**

First we assume that the centroid depth $H$ and time $O$ are known (fixed), the MT inverse problem has six parameters and is linear, and thus the uncertainty analysis is straightforward. Because $\text{tr}(M)/3 = a_6$ is one of the model parameters, we can easily calculate its standard deviation $\sigma_{a_6}$.

For theoretical reasons, we have to introduce a standard deviation $\sigma_u$ of the data. Its squared value is the data variance. We assume the simplest possible case that $\sigma_u$ has the same value for all the data components and is independent of time. As extensively discussed in Zahradník and Custódio (2012), it is not easy to estimate the true value of $\sigma_u$; however, in problems such as the one solved in this paper, where we investigate the uncertainty in a relative sense only, we just prescribe a reasonable value of $\sigma_u$ and keep it constant in all the compared models. Here we use $\sigma_u$ of the same order of magnitude as the peak-to-peak amplitude of the displacement data in the studied frequency range at the most distant station (i.e., $\sigma_u = 1 \times 10^{-5}$ m).

Normalizing $u$ and $E$ of equation (8a) by the standard deviation, we obtain

$$\mathbf{u} = \frac{u}{\sigma_u}, \quad \mathbf{E} = \frac{E}{\sigma_u}, \quad \text{and} \quad \mathbf{u} = \mathbf{E} a,$$  \hspace{1cm} (14)

where $E$ is the design matrix (Press et al., 1997). The design matrix depends on the position of the source and stations, the crustal model, and the considered frequency range, but does not depend on the waveforms. We can assess the theoretical parameter uncertainty even without recorded seismograms, and in the following, we demonstrate how. Any single parameter $a_i$ then has a 1D Gaussian PDF. For example, if $a_1$ to $a_5$ take their optimal values, for $a_6$ we have

$$\text{PDF}(a_6) = \frac{1}{\sigma_{a_6} \sqrt{2\pi}} e^{-\frac{(a_6 - a_{opt})^2}{2 \sigma_{a_6}^2}},$$  \hspace{1cm} (15)

which is independent of a particular value of $a_{opt}$, and the standard deviation $\sigma_{a_6}$ is given by the explicit formula (Press et al., 1997, section 15.4)

$$\sigma_{a_6}^2 = \sum_{i=1}^{6} \left( \frac{V_{a_6}}{w_i} \right)^2,$$  \hspace{1cm} (16)

Here $V_{a_i}$ is the 6th component of the $i$th singular vector of the design matrix $E$, and $w_i$ is its $i$th singular value. In practice, we do not need the singular decomposition of matrix $E$, as the singular vectors $V$ of $E$ are simply eigenvectors of matrix $E^TE$, and the singular values of $E$ can be calculated from the eigenvalues $\lambda_i$ of $E^TE$:

$$w_i = \sqrt{\frac{\lambda_i}{\sigma^2}}, \quad i = 1, 2, \ldots, 6.$$  \hspace{1cm} (17)

The condition number (CN), is defined by

$$\text{CN} = \max_{i=1, \ldots, 6} \left( \frac{w_i}{\min_{i=1, \ldots, 6} w_i} \right).$$  \hspace{1cm} (18)

CN is useful in judging, at least in a relative sense, how well or ill posed is the inverse problem; small singular values (large CN) indicate an unstable solution.

Now consider $\Delta \chi^2$ (i.e., the theoretical misfit between data and synthetics, normalized by the data variance). The surfaces of constant theoretical misfit $\Delta \chi^2$ (a 6D ellipsoid) are given by (Press et al., 1997, section 15.6.)

$$\Delta \chi^2 = w_1^2 (V_1 \cdot \delta a)^2 + \ldots + w_6^2 (V_6 \cdot \delta a)^2,$$  \hspace{1cm} (19)

where $\delta a$ is the radius vector connecting the center of the ellipsoid and a point in the parameter space. It enables us to numerically construct a 1D probability density function of the $a_6$ parameter. The points can be found by numerically grid searching the 6D parameter space, and the grid limits are given by the standard deviations of the individual parameters.
For algorithmic details, see the appendix of Zahradník and Custódio (2012). We discretize \( a_6 \), and for each value of \( a_6 \) we extract the points inside the ellipsoid \((a_1, a_2, \ldots, a_5)\); here \( a_6 \) denotes a fixed value of \( a_6 \). Each point is characterized by the theoretical misfit \( \Delta \chi^2(a_6) \leq 1 \), and we determine its minimum value \( \Delta \chi^2 \) over all points \((a_1, a_2, \ldots, a_5)\). Thus, we obtain a numerical approximation of the theoretical Gaussian 1D probability density function (15).

\[
\text{TheorPDF}(a_6; H = \text{fixed}; O = \text{fixed}) = e^{-\frac{1}{2} \text{min theor misfit}(a_1, \ldots, a_5)}/a_6.
\]

Here the minimum theoretical misfit \( \Delta \chi^2 \) was denoted \( \text{min theor misfit} \); note that \( a_6 \) is a free parameter that is varied, not computed by the inversion.

The theoretical justification of this approach comes from section 15.6., theorem D of Press et al. (1997). A simpler approach has been applied by Molnar and Lyon-Caen (1989) in the teleseismic waveform inversion; they determine the uncertainty of a parameter by fixing that parameter at a series of values and inverting for the other parameters. Although the numerical approximation equation (20) of the theoretical Gaussian distribution equation (15) is quite obvious and has no practical value, it serves as a good hint as to how to proceed in the nonlinear case.

### Isotropic Component—Uncertainty in the Nonlinear Case

In this case, the inverse problem has eight parameters: \( a_1, \ldots, a_6, H \), and \( O \). The nonlinearity is due to the effect of the centroid depth \( H \) and centroid time \( O \). Contrary to the preceding section, the theoretical misfit function is no longer available. Thus, we use waveforms and evaluate the real misfit between the data and synthetic seismograms (i.e., misfit equation (10) normalized by the data variance). In analogy to equation (20), the so-called experimental probability density function can be evaluated:

\[
\text{ExperPDF}(a_6; H = \text{free}; O = \text{free}) = \text{const} \times e^{-\frac{1}{2} \text{min real misfit}(a_1, \ldots, a_5; H, O)}/a_6.
\]

Here, the minimal real misfit is denoted \( \text{min real misfit} \). The meaning of equation (21) is as follows: A value of \( a_6 \) is chosen, and the real misfit is minimized by the least-squares method in \( a_1, \ldots, a_5 \) and by a grid search in \( H \) and \( O \). Repeating this for a set of discrete \( a_6 \) values, we obtain a 1D PDF\((a_6)\) reflecting the linear effect of \( a_1, \ldots, a_5 \) and nonlinear effects of \( H \) and \( O \). The value of \( \text{const} \) normalizes the integral of PDF\((a_6)\) to unity. The 1D experimental PDF in equation (21) is the main new tool proposed in this study.

### New Algorithm

The only technical issue related to equation (21) that requires caution is the minimization of the misfit for each fixed value of \( a_6 \). For each \( a_6 \) we must find the optimal centroid depth \( H \) and time \( O \) common to all \( a_1, \ldots, a_6 \). The algorithm is the following:

1. We choose a discrete value of \( a_6 \), (close to the previously computed optimal value, but not equal to this value) and a given trial value of \( H \) and \( O \). We minimize the misfit between real data \( u \) and synthetics \( s \), thus obtaining \( a_1, \ldots, a_5 \). Combining these inverted coefficients with the chosen coefficient \( a_6 \) we obtain \( a_{opt} \).
2. The correlation between \( u \) and \( s \) = \( E(a_{opt}) \) is calculated using equation (9).
3. The procedure is repeated for each trial \( O \) and \( H \) (still fixing the same \( a_6 \)), and the \( H \) and \( O \) with maximum correlation are found for the chosen value of \( a_6 \). The corresponding misfit equation (10) is recorded for the given \( a_6 \), too.
4. The whole procedure is repeated for each value of \( a_6 \). As a result, we obtain the best-fitting parameters \( (a_1, \ldots, a_5, H, O) \), as well as the minimum misfit value (i.e., the \( \min \text{real misfit} \) value), all as a function of \( a_6 \). Thus we construct the desired experimental PDF\((a_6)\) according to equation (21).

Although the proposed method yields a 1D PDF\((a_6)\), it correctly takes into account the 8D nature of the problem, as well as its nonlinearity. On the other hand, the uncertainty of the crustal structure model is not included. It must be solved by repeating the analysis using several models that are available.

### Data and Conventional Analysis

A moderate earthquake swarm started on 26 June 2009 northeast of the Santorini (Thira) island, close to Mt. Columbo, an active submarine volcano in the Cyclades, Aegean Sea. The swarm occurred at the western boundary of the Santorini–Amorgos zone, a major structural unit in the Hellenic volcanic arc, where the strongest instrumentally recorded event occurred on 9 July 1956 (reported \( M_w \) ranging from 7.5 to 7.8), producing a great tsunami (e.g., Ambraseys, 1960; Galanopoulos, 1960; Papazachos and Papazachou, 2003; Okal et al., 2009; Konstantinou, 2010). The region around the Columbo volcano features strong temporal variations of shallow (<10 km) seismic activity on a high background level, interpreted as due to magma and fluid migrations (Bohnhoff et al., 2006). Dimitriadis et al. (2009) complemented the analysis of the Columbo volcano activity by joint relocation and inversion of the upper crustal structure; focal mechanisms of 20 small events were reported, proving a prevailing normal-faulting pattern with a northwest–southeast extension at shallow depths (6–9 km).

Here we focus on the two largest events (26 June \( M_w > 4 \)) of the 2009 Mt. Columbo swarm. Table I gives the routine location parameters, determined by the Aristotle University of Thessaloniki (AUTH). The earthquakes were recorded with a good azimuthal coverage by the broadband
stations of the Hellenic Unified Seismic Network (HUSN), spanning the epicentral distances approximately from 60 to 310 km (Fig. 1a). We use the records from 15 and 10 stations of events 1 and 2, respectively, which provided a good signal-to-noise ratio even at relatively low frequencies (0.02–0.1 Hz), which are feasible for modeling. By “feasible” we mean that the available crustal models are adequate to model the wave propagation effects and provide satisfactory waveform match up to frequency of 0.1 Hz in the studied epicentral distances. Two 1D models are employed: (1) the model by Novotný et al. (2001), hereafter “model N,” obtained from the regional surface-wave dispersion, in which $L_g$ waves dominate, and it is routinely used for MT inversions at AUTH; and (2) the model by Dimitriadis et al. (2010), hereafter “model D,” sufficiently predicting local first-arrival times (Table 2 and Fig. 1b,c). The attenuation quality factors reported in the footer of Table 2 represent rough estimates; the waveform inversion is almost insensitive to their particular values in the studied range of epicentral distances and frequencies.

The MT inversion was performed using ISOLA software (Sokos and Zahradník, 2008). ISOLA (from ISOLated Asperities) is a program package based on the multiple point-source iterative deconvolution of complete regional waveforms. Green’s functions are calculated by the discrete-wavenumber method (Bouchon, 1981; AXITRA code of Coutant, 1989). The moment tensor is solved by the least-squares method, and the origin time and 3D position of the point source (centroid) are both grid searched, the latter in the vicinity of the (independently) located hypocenter. In the preliminary stage, we considered variations of the centroid position both in the horizontal direction and depth. In the following discussions, for simplicity, we concentrate only on grid searching the centroid depth. The method is routinely used in the Seismological Laboratory of the University of Patras to calculate moment tensors in western Greece.

![Figure 1](image)

**Figure 1.** (a) Broadband stations (triangles) whose records were used to study the two earthquakes, marked as event 1 and 2 (stars) in the inset, with epicenters close to Mt. Columbo volcano (circle in the inset). Marked with encircled triangles are the stations available for both event 1 and 2. (b) Crustal model N (Novotný et al., 2001). (c) Crustal model D (Dimitriadis et al., 2010); see also Table 2. The color version of this figure is available only in the electronic edition.
15-station cases provide somewhat different results. Therefore, for event 1 we used these 10 stations, but also the two cases because for event 2 we have only 10 stations. We compare these for the solutions using 15 and 10 stations. We believe that in the 10-station case (event 1), the depth and ISO in model D are less well constrained than in model N, as suggested by the condition numbers (16.27 and 4.24 in D and N, respectively). The 15-station case (event 1) has a weaker variation; for example, ISO is positive in both models N and D. The DC parameters are less affected by the assumed crustal model than the non-DC parameters. All these resolvability issues are analyzed in more detail in the next sections.

The stability of the MT solution was further examined by jackknifing the data (i.e., systematically removing one station from the inverted data set). The results are summarized in Figure 4. Event 1 has a relatively large ISO percentage in model N. Event 2 is characterized by the opposite sign of ISO in model D. As shown in Figure 2 and Table 3, the most remarkable difference between the full-MT solutions of event 1 is due to the crustal models used, especially in the case of the 10 stations, where the explosion-like mechanism in model N changes to implosion in model D. The latter has a greater variance reduction. Another important feature is the different centroid depth of the deviatoric and full-MT solutions, while their variance reductions are almost the same. It indicates the trade-off between the depth and the non-DC part of the MT.

As shown in Figure 2 and Table 3, the most remarkable difference between the full-MT solutions of event 1 is due to the crustal models used, especially in the case of the 10 stations, where the explosion-like mechanism in model N changes to implosion in model D. The latter has a greater variance reduction. Another important feature is the different centroid depth of the deviatoric and full-MT solutions, while their variance reductions are almost the same. It indicates the trade-off between the depth and the non-DC part of the MT.

We will discuss further the two cases in Table 3 (event 1) for the solutions using 15 and 10 stations. We compare these two cases because for event 2 we have only 10 stations. Therefore, for event 1 we used these 10 stations, but also the whole 15-station set. The comparison shows that the 10- and 15-station cases provide somewhat different results. In particular, in the 10-station case (event 1) we find a relatively large difference between the centroid depths in model N and model D (5 and 1.5 km, respectively), and also the variation of the ISO percentage is large, even comprising the opposite sign (+32 percent in model N and −33 percent in model D). We believe that in the 10-station case (event 1), the depth and ISO in model D are less well constrained than in model N, as suggested by the condition numbers (16.27 and 4.24 in D and N, respectively). The 15-station case (event 1) has a weaker variation; for example, ISO is positive in both models N and D. The DC parameters are less affected by the assumed crustal model than the non-DC parameters. All these resolvability issues are analyzed in more detail in the next sections.

The stability of the MT solution was further examined by jackknifing the data (i.e., systematically removing one station from the inverted data set). The results are summarized in Figure 4. Event 1 has a relatively large ISO percentage in model N. Event 2 is characterized by the opposite sign of ISO in the two applied crustal models.

New Analysis of Isotropic Component

Figure 5a demonstrates the waveform correlation as a function of the trial depth, grid searched from 2.5 to 15 km, in steps of 0.5 km, event 1 (15 stations), and model N; the correlation is defined in equation (9). Plots such as that shown in Figure 5a represent a traditional inversion tool, used to define the optimal depth. Nevertheless, we also employ a less widely used approach: at each trial depth we employ a less widely used approach: at each trial depth we calculate the corresponding value of the ISO percentage. Although the independent variable is only depth, while the misfit and isotropic component are depth dependent, we can analyze the correlation (and depth) as a function of ISO (Fig. 5b). This plot is analogous to the “correlation versus DC plots” introduced by Zahradník, Sokos, et al. (2008). It clearly demonstrates the trade-off between the depth and ISO percentage.

| Table 2 |
|------------------|--|------------------|--|
| The Two 1D Crustal Models Used |
| Model N | Model D |
| Layer Top (km) | $V_p$ (km/s) | Layer Top (km) | $V_p$ (km/s) |
| 0.0 | 2.30 | 0.0 | 4.85 |
| 1.0 | 4.30 | 1.0 | 5.03 |
| 2.0 | 5.50 | 3.0 | 5.52 |
| 5.0 | 6.20 | 5.0 | 5.69 |
| 16.0 | 6.40 | 7.0 | 6.31 |
| 33.0 | 8.30 | 9.0 | 6.16 |
| 11.0 | 6.23 |
| 13.0 | 6.27 |
| 17.0 | 6.17 |
| 19.0 | 6.32 |
| 21.0 | 7.02 |
| 23.0 | 7.46 |
| 25.0 | 7.52 |
| 30.0 | 7.56 |

Density (g/cm$^3$) = 1.7 + 0.2V$_p$(km/s)

$^*$ $V_p/V_s = 1.78, Q_p = 300, Q_s = 300.$

$^t$ $V_p/V_s = 1.77, Q_p = 300, Q_s = 150.$

Table 2. The Two 1D Crustal Models Used

![Figure 2. MT solutions for event 1 and event 2 as obtained in this study for each crustal model. The MT inversion for event 1 was run twice: using the total 15 available stations (top) and using 10 stations (middle) common with event 2 (bottom). For details, see Table 3. The computed depth (km) is shown to the right of the beachballs. The color version of this figure is available only in the electronic edition.](image-url)
<table>
<thead>
<tr>
<th>Event 1: 15 Stations Used</th>
<th>Model</th>
<th>Inversion Mode</th>
<th>Strike (°)</th>
<th>Dip (°)</th>
<th>Rake (°)</th>
<th>Depth (km)</th>
<th>$M_w$</th>
<th>$M_0$ (Nm)</th>
<th>DC*</th>
<th>CLVD†</th>
<th>ISO‡</th>
<th>VR§</th>
<th>CN¶</th>
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<tbody>
<tr>
<td>Model N</td>
<td>DC-constrained MT</td>
<td>241</td>
<td>65</td>
<td>−80</td>
<td>3.5</td>
<td>4.6</td>
<td>9.51 $\times 10^{15}$</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
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<td>3.75</td>
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<td>59</td>
<td>−74</td>
<td>4.0</td>
<td>4.7</td>
<td>1.06 $\times 10^{16}$</td>
<td>59.2</td>
<td>40.8</td>
<td>0.0</td>
<td>0.64</td>
<td>3.20</td>
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<tr>
<td>Full MT</td>
<td>255</td>
<td>69</td>
<td>−57</td>
<td>6.5</td>
<td>4.7</td>
<td>1.12 $\times 10^{16}$</td>
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<td>0.64</td>
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<td>DC-constrained MT</td>
<td>247</td>
<td>70</td>
<td>−78</td>
<td>2.5</td>
<td>4.7</td>
<td>1.20 $\times 10^{16}$</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.67</td>
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<tr>
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<td>1.33 $\times 10^{16}$</td>
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<tr>
<td>Full MT</td>
<td>253</td>
<td>65</td>
<td>−67</td>
<td>4.0</td>
<td>4.7</td>
<td>1.17 $\times 10^{16}$</td>
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<td>Event 1: 10 Stations Used</td>
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<td>253</td>
<td>71</td>
<td>−79</td>
<td>3.5</td>
<td>4.6</td>
<td>1.01 $\times 10^{16}$</td>
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<td>0.0</td>
<td>0.67</td>
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<tr>
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<td>70</td>
<td>−78</td>
<td>3.5</td>
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<td>1.02 $\times 10^{16}$</td>
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<tr>
<td>Full MT</td>
<td>251</td>
<td>66</td>
<td>−74</td>
<td>5.0</td>
<td>4.6</td>
<td>8.49 $\times 10^{15}$</td>
<td>57.1</td>
<td>10.8</td>
<td>+32.1</td>
<td>0.68</td>
<td>4.24</td>
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<tr>
<td>Model D</td>
<td>DC-constrained MT</td>
<td>256</td>
<td>73</td>
<td>−77</td>
<td>2.5</td>
<td>4.7</td>
<td>1.25 $\times 10^{16}$</td>
<td>100.0</td>
<td>0.0</td>
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<tr>
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<td>68</td>
<td>−75</td>
<td>3.0</td>
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<td>−81</td>
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<td>4.9</td>
<td>2.71 $\times 10^{16}$</td>
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<td>33.1</td>
<td>−33.0</td>
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<td>Event 2</td>
<td>Model N</td>
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<td>47</td>
<td>−81</td>
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<td>5.37 $\times 10^{15}$</td>
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<td>−75</td>
<td>4.5</td>
<td>4.4</td>
<td>4.34 $\times 10^{15}$</td>
<td>69.4</td>
<td>30.6</td>
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<td>6.0</td>
<td>4.4</td>
<td>4.48 $\times 10^{15}$</td>
<td>35.9</td>
<td>38.8</td>
<td>+25.3</td>
<td>0.65</td>
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<td>Model D</td>
<td>DC-constrained MT</td>
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<td>−83</td>
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<td>4.5</td>
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<td>46</td>
<td>−82</td>
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<tr>
<td>Full MT</td>
<td>236</td>
<td>47</td>
<td>−83</td>
<td>3.5</td>
<td>4.5</td>
<td>7.00 $\times 10^{15}$</td>
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<td>−24.1</td>
<td>0.70</td>
<td>6.38</td>
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</tbody>
</table>

*DC, percentage of double couple.
†CLVD, percentage of compensated linear vector dipole.
‡ISO, percentage of isotropic part.
§VR, variance reduction (equation 11).
¶CN, condition number (equation 18).
The ISO values in Figure 5b have been complemented by error bars, using a method suggested by Zahradník and Custódio (2012). It is a simple method applicable to the linear case ($H$ and $O$ assumed to be known; $H =$ hypocenter position, $O =$ origin time). In brief, the idea is as follows: a set of points inside the 6D theoretical error ellipsoid is found numerically and converted into histograms of the strike, dip, rake, and ISO, thus providing also an estimate of their bounds. The minimum and maximum values of ISO, found from the histograms, define the error bars plotted in Figure 5b. Note the considerably larger uncertainty of small ISO percentages at the shallow source depths; see, for example, the large error bar of the ISO/0.0136 at the depth of 3.5 km (correlation = 0.833).

In Figure 5c we replot the content of Figure 5b in such a way that the isotropic component on the horizontal axis, so far described by ISO, is now described by $\sigma_{a_6}$ values of equation (16) to plot the $a_6$ error bars.

The error bars introduced in Figure 5b and 5c represent important additional insight to the information carried by the misfit values. Indeed, even the solutions that have high correlation (low misfit) might be irrelevant if the parameter error is large. Large parameter errors indicate poor constraint of the studied parameter by the data. For example, as seen in Figure 5b, although the correlation takes similar values (0.84) at depths of 4.5 and 9 km (where the ISO percentage is about 30 and 65, respectively), the corresponding uncertainty of ISO is not the same; the error bar is larger for the depth of 4.5 km.

A new, deeper insight into the uncertainty of the isotropic component, fully reflecting the nonlinearity of the joint inversion of MT and the centroid depth and time, can be gained by means of the experimental PDF defined in equation (21). The example for event 1 in Figure 6 is calculated in the same source–station setup as the one used to construct Figure 5. As the numerical analysis has provided the combination of ($a_1, \ldots, a_5, H, O$) for each value of the independent variable $a_6$, we can demonstrate in the same figure also the variation $H = H(a_6)$, $H$ being the source depth. We also demonstrate the variation of the focal mechanisms. The variation of the centroid time $O(a_6)$ is weak, and thus not shown in Figure 6. (For the depth range from 2.5 to 15.0 km, the centroid time varied within 0.15–0.30 seconds before the origin time reported in Table 1.)
Besides the PDF constructed according to equation (21), shown in Figure 6 as the large dots, plotted in the same figure as small dots are 15 fixed-depth PDF’s. They represent a modification of equation (21) in which only $a_1, \ldots, a_5$ and $O$ are subjected to optimization without the depth $H$. The procedure is repeated for a set of chosen depth values. All depths have the same normalization constant, in this example equal to the value const of equation (21). The probability density functions at the individual depths (fixed $H$, variable $O$) are close to the theoretical Gaussian functions of the linear case (fixed $H$ and $O$). The PDF function of the fully non-linear case (variable $H$ and $O$), shown as the large dots, is their envelope. The envelope is also close to the Gaussian in the studied example, but this is not necessarily always the case, as demonstrated later in Figure 7. The envelope indicates the same optimal $a_6$ as in the individual fixed-depth cases. However, its width is much greater than that of the individual fixed-depth functions. It demonstrates how the

![Figure 4](image-url)

**Figure 4.** Stability tests regarding the percentage of DC, CLVD, and ISO as obtained by jackknifing the data, for the two structural models used (top: model N; bottom: model D). Each symbol in the plots corresponds to a single-station removal from the inversion. Numbers 1 and 2 at the top of the columns refer to event 1 and event 2, respectively. Event 1 (15 stations) and event 2 (10 stations) were inverted in the deviatoric and full-MT modes. The deviatoric solutions for events 1 and 2 are shown by the dots and upward pointing triangles, respectively. The full-MT solutions for events 1 and 2 are shown as diamonds and downward pointing triangles. Some symbols overlap due to rounding off the percentages to integer values. The CLVD and ISO percentages are shown in absolute values. The ellipse marks the case of the negative ISO. The color version of this figure is available only in the electronic edition.

![Figure 5](image-url)

**Figure 5.** Uncertainty of event 1 as revealed by grid-searching the centroid depth using the entire set of 15 stations and crustal model N. (a) Waveform correlation between observed and synthetic waveforms as a function of the trial-source depth. The full-MT solutions (beachballs) are shown for selected depths. (b) Correlation (dots) and depth (triangles) as a function of ISO percentage. (c) Correlation and depth as a function of the $a_6$ model parameter of this paper, $a_6 = \text{tr}(\text{M})/3$. The color version of this figure is available only in the electronic edition.
uncertainty of \( a_6 \) increases if the depth is an unknown parameter.

In the nonlinear analysis presented above, both the optimal and nonoptimal solutions were derived for a fixed crustal model. Because the crustal model is also uncertain, we have to take it into consideration, too. Because the crustal model parameters do not belong to the model parameters of our inverse problem, the effect of the model should be studied by comparing alternative crustal models (the following Fig. 7).

Finally, the method of the experimental 1D PDF(\( a_6 \)), proposed in equation (21), is applied to the two investigated \( M_w > 4 \) earthquakes, using the two available crustal models (Fig. 1b, c; Table 2). The results are summarized in Figure 7. Each function was normalized to a unit integral using (its own) normalization constant \( \text{const} \) in equation (21). Event 1 (Fig. 7a) is processed with 15 and 10 stations. The difference between the 15- and 10-station cases is evident, but it is smaller than the difference due to the crustal model used. While event 1 is most likely characterized by \( a_6 > 0 \) in model N, the probability density function is considerably broader in model D, making almost no preference between \( a_6 > 0 \) (explosion) and \( a_6 < 0 \) (implosion). Event 2 (Fig. 7b) was processed only with the 10 available stations. The effect of the crustal model is the same as for event 1. Comparing the two events in model N, the isotropic component of event 2 is closer to zero, but it is also less well constrained than event 1, as indicated by the wider probability distribution in Figure 7b.

It is out of scope of this paper to study geological reliability of the two velocity models. We simply want to stress how strongly the assumed crustal structure can affect the uncertainty of the isotropic component, even though the considered frequencies were as low as \(< 0.1 \) Hz. Observing the large uncertainty of \( a_6 \) in model D (i.e., its wide PDF), we also recall that model D was characterized by a smaller misfit than model N (i.e., the larger VR in Table 3). This illustrates the limited meaning of the waveform misfit itself; indeed, the solution of a small misfit might yield an uncertain \( a_6 \) due to specific features of the assumed crustal model. Note that the poor resolvability of the isotropic component in model D
corresponds to the poor resolvability of the shallow source depth. The main differences between the two considered crustal models are in their shallow part and different MOHO depth. As a whole, model D is faster. The inferior resolvability of the MT in faster models was also theoretically predicted by Zahradník and Custódio (2012). In other words, if the real earth crust is like that in model D, the joint inversion of the full MT and the depth is problematic; an independent (location) constraint of the depth would be desirable, if accurate enough. If the real crust is like that in model N, the source depth (of about 6 km) is greater than in model D, and the trade-off between the isotropic component and the depth is weaker. Therefore, in this particular case, the joint inversion of the full MT and the depth is more feasible, and the necessity of an independent depth constraint is less crucial.

Conclusions

The isotropic component of the MT measures any volume change in an earthquake, and its study is significant, especially in volcanic and geothermal regions. To relate the isotropic component to a physical model of the shear dislocation, crack opening, and fluids is not straightforward, and it is beyond the scope of the present work.

In this methodic article, we focus on how to obtain the isotropic component and define its uncertainty using near-regional, low-frequency broadband waveforms (epicentral distances 60–310 km, $f < 0.1$ Hz). This task is more difficult than a routine deviatoric moment tensor inversion because the isotropic component is more sensitive to any uncertainties related to the hypocenter position and the crustal structure, as well as to any natural and instrumental noise. The isotropic component of a moment tensor $\mathbf{M}$ is parameterized in this study by the ISO percentage (independent of the source size) and also directly by $\text{tr} \left( \mathbf{M} \right) / 3$ (dependent on the source size). The advantage of the latter is that it represents one of the model parameters in our formulation of the inverse problem ($a_6 = \text{tr} \left( \mathbf{M} \right) / 3$).

The scope of this work is to develop a new simple method in order to construct a 1D probability density function of $a_6$, PDF$(a_6)$. Towards this goal, PDF$(a_6)$ is constructed using the experimental method of equation (21): the parameter $a_6$ is fixed at a series of values (in the vicinity of the optimal $a_6$), and for each trial $a_6$ value we seek the minimum misfit between the observed and synthetic seismograms. The minimization is made by the least-squares method for parameters $a_1, \ldots, a_5$ of the MT and by a grid search over the centroid depth and time. The results of this new technique are compared to those of the conventional analysis. To be more specific, by conventional analysis we mean repeated calculations of MT (and its isotropic component) for a series of trial-source depths, accompanied by a stability check (jackknifing by repeatedly removing one station). The experimental PDF$(a_6)$ and the conventional method were applied to two shallow $M_w 4+$ events (event 1 and event 2, respectively) in a volcanic region of the Aegean Sea, Greece, using two different crustal models.

The innovative part of this work is the experimental PDF$(a_6)$ and its application as summarized in Figure 7. Event 1 was analyzed twice, using 15 and 10 stations, respectively. The latter case was considered to facilitate comparison with event 2, for which data from only 10 stations were available. Minor differences between the 15- and 10-station results were found. Most prominent is the effect of the crustal models used: model N (Novotný et al., 2001) and model D (Dimitriadis et al., 2010). Both structural models provide satisfactory waveform match of observed and synthetic seismograms, as evident from the variance reduction, $VR \sim 0.7$ listed in Table 3. However, if the true structure of the Earth is closer to model D, then the isotropic component is almost irresolvable, as documented by its wide PDF. If the true crust can be approximated by model N, the PDF is narrower and yields a better resolution, in particular for event 1 (a positive isotropic percentage of about 30 to 50 at depths of 4–6 km). These results have an important implication, which stems from the fact that both velocity models provide sufficient fit between the observed and synthetic waveforms. We are unable to prefer any of the two models, thus we cannot say whether the resolution of the isotropic component is relatively good (in model N) or poor (in model D). In general, if the knowledge of the velocity model is poor, it is useful to test very different velocity models (providing a reasonable waveform match, like models N and D), not only to apply a single available model and perform its statistical perturbation.

A notable feature illustrated in Figure 7 is the strong trade-off between the isotropic component and source depth. It is stronger than the trade-off of the isotropic component with the seismic moment, source angles (strike, dip, and rake), and origin time. We can even state that the isotropic component mainly trades off with depth. Therefore, in model D, waveforms require shallower depths (1.5–4 km for event 1) than in model N (4 to 6 km), and the shallow depths tend to provide the negative isotropic component in the studied case. The source depth is a factor considerably affecting the resolvability of the isotropic component, which is also demonstrated by the (nonmonotonic) variability of the error bars in Figure 5, calculated in the approximation of a repeatedly fixed depth. Note also the poor MT resolution at the shallow depth of 1.5 km signaled by the condition number CN as large as 16.27 in the case of event 1, 10 stations in model D (Table 3).

Although the effect of the structural model is quite obvious in Figure 7, it is true that at a given source depth the two models N and D provide a similar estimate of the $a_6$ parameter. This observation seems to suggest the MT inversion at the location (hypocenter) depth, especially for moderate-size earthquakes, whose hypocenter and centroid are close to each other. However, it has been shown that for shallow crustal events the depth determination is always difficult (Zahradník, Janský, et al., 2008; Janský et al., 2009, 2012; Sokos et al., 2012), hence the MT inversion at the hypocenter
depth cannot be generally recommended, unless dense local networks and precise crustal models are available. It might also seem that a compromising solution is to perform an MT inversion only for the deviatoric part (constrain ISO component to zero), determine the centroid depth, and then, keeping the depth value fixed, rerun the inversion using the full MT decomposition. This approach, which is apparently very attractive, especially when the correlation–depth dependence in the deviatoric regime has a sharper maximum than in the full MT regime, is generally not viable. This stems from the fact that a MT deviatoric inversion for an earthquake which has a significant ISO component may result in an incorrect depth. The latter has been unambiguously proven in a simple synthetic test, even for the ideal case of noiseless data and exact Green’s functions.

Finally, we would like to bring to the reader’s attention one observation that is significant for those who dislike the idea of installing, learning, and running new codes for the calculation of PDF($\alpha_0$) as suggested in equation (21). This observation refers to the strong similarity of the isotropic component uncertainty estimates obtained from the conventional analysis and from our new approach. This similarity becomes apparent when comparing the behavior of the ISO percentages in Figure 4 (conventional) and Figure 7 (new). Both figures illustrate the strong effect of the assumed crustal model. The isotropic component is positive, relatively large, and better resolved for event 1, under the assumption that the relevant crustal model is model N. It implies that, with the conventional methods, if the full MT inversion is run for a series of trial centroid depths and times, and if careful jack-knifing of the data is performed, then a reasonable estimate of the uncertainty of the isotropic component can be obtained. This uncertainty would considerably decrease if the crustal model is very well known and the location is highly accurate; the location-constrained depth then would reduce the trade-off between the isotropic component and the source depth.

Data and Resources

Broadband waveforms were retrieved from the permanent stations of the Hellenic Unified Seismic Network (HUSN), operated jointly by the National Observatory of Athens (NOA), the Aristotle University of Thessaloniki (AUTH), the University of Patras (UPSL), and the University of Athens (UOA). The records from one station of the National Seismic Network of Turkey (DDA) were also used. Some UPSL stations are co-operated by the Charles University in Prague. Software ISOLA (Sokos and Zahradník, 2008) was used to calculate the moment tensors. Green’s functions in ISOLA were computed using the AXITRA code of Coutant (1989). The Generic Mapping Tools (GMT; Paul Wessel and Walter H. F. Smith, http://gmt.soest.hawaii.edu/, last accessed April 2013) and MATLAB (http://www.mathworks.com/products/matlab/, last accessed April 2013) were also used.

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References


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