Density Jumps in the Transition Zone and Their Role in Viscoelastic Responses

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Abstract.
We have employed a recently developed initial-value method to study the small-amplitude viscoelastic deformations for a self-gravitating, radially stratified, elastically compressible model. We have focussed here on the problem of viscoelastic responses with internal boundaries in the transition zone. Density jumps in the mantle can be interpreted as being due to chemical differences or to phase transitions. For the two cases the boundary conditions at any boundary with a density jump are different. The differences in the vertical responses between these two boundary conditions may exceed up to 10% for angular orders up to 10. The maximum differences are reached at the surface and decrease with depth. The influence of the boundary at 400 km depth is dominant for angular orders greater than 15, while the deeper boundary is more influential for longer wavelengths. Inclusion of the phase boundary at 520 km yields an additional contribution comparable to that at 400 km.

1. Introduction
Most studies on postglacial rebound have been focussed on the deeper discontinuity at 670 km and not much attention has been paid on the effects caused by the shallower transition at 400 km, which is likely to be of phase transitional nature [Bina and Wood, 1987]. This may also be influenced by the kinetic nature of the transition process [Daessler and Yuen, 1993; Solomatov and Stevenson, 1994; Rubie and Ross, 1994]. There exists also the possibility that a change in chemical composition may also occur at the 670 km discontinuity [e.g., Anderson, 1979]. Depending on the nature of these internal boundaries, there are some subtle issues dealing with their treatment in viscoelastic responses [Cathles, 1975; Peltier, 1974; Yuen and Sabadini, 1984].

Most of the previous studies on viscoelastic responses have made use of the correspondence principle [e.g., Wu and Peltier, 1982]. Recently there has been a growing interest in the use of initial-value techniques in viscoelasticity [Hanyk et al., 1995, 1996]. The initial-value method leaves far fewer ambiguities in the treatment of the boundary conditions at these internal density jumps. In this work we will lay the theoretical groundwork of treating both the chemical and phase-transitional boundary conditions and compare their differences within the framework of a model with the 400, 520 and 670 km discontinuities included.

2. Mathematical formulation
We have recently developed the initial-value technique [Hanyk et al., 1995, 1996] for computation of viscoelastic responses that is based on time-integration of the constitutive relation of a viscoelastic Maxwell continuum by the classical Euler method. If \( t^{i+1} \) denotes time in the \((i+1)\)-th level and \( \Delta t^i \) is the time difference between the \((i+1)\)-th and \(i\)-th levels, we may write

\[
\tau^{M,i+1} = \tau^{el,i+1} + \tau^{M,i} - \frac{\mu \Delta t^i}{\eta} (\tau^{M,i} K \nabla \cdot \mathbf{u} I),
\]

(1)

and

\[
\nabla \cdot \tau^{M,i+1} = \nabla \cdot \tau^{el,i+1} - \mathbf{h}(t^i),
\]

(3)

where \( \mathbf{h}(t^i) \) is the contribution from the past history to the time \( t^i \). Here \( \tau^M \) is the incremental Cauchy stress tensor of the Maxwell model, \( \tau^{el} \) is the elastic stress tensor and \( \mathbf{u} \) is the small-amplitude displacement vector. The Maxwell rheology is determined by the shear and bulk moduli \( \mu(r) \), \( K(r) \), and the viscosity \( \eta(r) \), where \( r \) denotes the radial distance.

Since (3) yields the divergence of the Maxwell stress tensor, we may put it into the momentum equation, which, together with the Poisson equation, describes the viscoelastic responses to a surface load. After each time step we solve for each angular order a sixth-order system of ordinary differential equations (ODE) with a forcing term, which consists of the memory of the viscoelastic responses. We have employed around 600 unevenly located grid points in the ODE’s with 2 km resolution in the lithosphere and 10 km in the lower mantle. For additional details, the reader is referred to [Hanyk et al., 1996].

The existence of internal boundaries with large density jumps [Dziewonski and Anderson, 1981] requires one to treat with care these boundary conditions in the solution vector of the ODE system. In elastic problems, it is assumed that each interior mantle boundary moves together with boundary particles, i.e., the boundaries are “frozen” into the continuum. If a boundary is caused by a phase transition, it can potentially move with respect to surrounding particles due to changes of the physical conditions. In other words, there can be a mass flux through the boundary. However, the time-scale of elastic problems is very short in comparison with the time-scale of phase transition kinetics, which ranges on the order between \( 10^7 \) and \( 10^8 \) years [Daessler and Yuen, 1996] and thus such a flux is negligible. The viscoelastic responses are usually computed in Laplacian domain, i.e., the
problems are formally treated as elastic with the shear and bulk moduli dependent on the Laplace-transform variable \( s \). This is probably the reason for treating the internal boundaries in the same way as in the elastic problems, although the time-scales of viscoelastic response are comparable with that of phase transition kinetics (e.g., [Daessler and Yuen, 1996]). To distinguish between these two end-member cases, we will call the boundary without a mass flux “chemical” and the one with a mass flux “phase” boundary.

In the Lagrangian description of linearized boundary conditions a chemical boundary is described by the continuity of both the displacement vector and the force acting on the boundary. In the case of the phase boundary the conservation of mass yields the condition

\[
\rho (\dot{\mathbf{u}} - \mathbf{v})^+ \cdot \mathbf{n} = 0 ,
\]

where \( \mathbf{v} \) is the velocity of the boundary, \( \rho \) is the density, \( \mathbf{n} \) is its normal and \( [ \ ]^+ \) denotes a jump of a corresponding quantity across the boundary. In the general case, \( \dot{\mathbf{u}} - \mathbf{v} \) is a function of time, where the velocity of the boundary \( \mathbf{v} \) depends on the history of the viscoelastic response and microphysical processes, such as nucleation phenomena in phase kinetics [Daessler and Yuen, 1993]. This is the reason why such a general case cannot be performed in the Laplace-transformed domain. Note here that the trace of the stress tensor \( K \nabla \cdot \mathbf{u} \) cannot be defined on the boundary and its limit, computed by approaching the boundary downward in an upper part, may be different from that computed below the boundary. Thus thermodynamic pressure needed to model phase transition kinetics may be only estimated from these two limits.

To study the sensitivity of viscoelastic responses to changes of boundary conditions, we will here restrict ourselves to the simplest limiting case, when \( \mathbf{v} = 0 \), which means that the boundary topography is immediately reaching its isotropic equilibrium position. The magnitude of a jump in a radial component of velocity is then controlled only by the magnitude of the density jumps. The coupled problem of viscoelastic reaction with realistic phase boundary kinetics should be a matter for future studies.

Likewise, the conservation of momentum yields

\[
[\rho \dot{\mathbf{u}} (\dot{\mathbf{u}} - \mathbf{v}) - \tau^M]^+ \cdot \mathbf{n} = 0 .
\]

This is, however, the non-linear version. We have used the usual linearized form which is the same for both kinds of boundaries,

\[
[\tau^M]^+ \cdot \mathbf{n} = 0 .
\]

3. Results

We have employed the PREM model as a starting point for implementing the physical parameters bulk modulus \( K(r) \), shear modulus \( \mu(r) \) and the density \( \rho(r) \). Three viscosity profiles have been considered. The first one is an isoviscous model (L1) with a viscosity of \( 10^{21} \) Pa s; the second (L2) has a viscosity jump by a factor of 10 across the 670 km discontinuity and the same value of upper-mantle viscosity as in the L1 model; and the third model (L3) has a low viscosity channel of \( 10^{19} \) Pa s lying between 120 and 220 km depth and, otherwise, the same kind of viscosity structure as for L2. In all models we have employed a fully compressible elastic lithosphere with a thickness of 120 km. Internal boundaries at 400, 520 and 670 km have been taken, where the 520 km discontinuity [Shearer, 1996] has been simulated by assuming the same density jump as in the 400 km discontinuity.

Fig. 1 shows the relative changes in the vertical Love number at a time \( t = 20 \) Kyr and the surface after the application of a Heaviside load. We have compared up to angular order \( l = 60 \). We consider the relative difference in the surface responses between models with phase transition boundary conditions and those with chemical boundary conditions. The base level represents a model in which the chemical boundary conditions have been imposed on all three discontinuities. The solid, dashed, dash-dotted and dotted curves represent respectively the responses from all three discontinuities, the 670 km, 520 km and the 400 km discontinuities. We can clearly observe that the changes in the vertical Love number at this time can exceed 15% for the low angular orders. The shallow discontinuities also dominate over the deeper 670 km discontinuities at higher angular order, at least from \( l = 15 \) on. We note that the relative changes of the Love numbers do not depend too much on variations of the viscosity profiles. The presence of the low viscosity zone (L3) enhances the tendency for the 400 km discontinuity to be dominant at even lower angular orders than for the isoviscous model (L1).

In Fig. 2 we compare the evolution of the vertical Love number at the surface for three discontinuities, L1 model and \( l = 2 \). It takes a long time, around 10 Myr, for the two responses to be indistinguishable, as the two models approach hydrostatic equilibrium.

In Fig. 3 we plot the depth variations of the spherical harmonic components of the stress tensor for \( l = 2 \) and the same model as in Fig. 2 at the time \( t = 20 \) Kyr. The maximum values of the radial stress are reached in the transition zone with a smooth decay to the core-mantle boundary. The tangential stresses have much smaller values and also have maxima at the 670 km discontinuity.

4. Concluding remarks

Density jumps in the transition zone are due to phase changes and/or compositional differences. Within the framework of linear viscoelasticity we have considered the effects of the different boundary conditions associated with the two end-members, which either hinder completely or allow for material passage through the boundary. These results show that for high angular orders greater than 20 the differences are rather small. Only at low angular orders, less than 10, we can expect to see differences on the order of 10% in the surface vertical displacement. Greater differences in the induced stress components are found in the upper mantle.

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References

Figure 1. Percentage difference in the vertical Love number at the surface as a function of angular order. The difference is taken between the predictions from models with 3 chemical boundary conditions (baseline case) and those with 3 phase transitions (solid curve), with one at 670 km (dashed curve), at 520 km (dash-dotted curve) and at 400 km (dotted curve). Time is taken at $t = 20$ Kyr and three viscosity models, L1, L2 and L3 (described in the text) have been employed.

Figure 2. Vertical Love number at the surface as a function of time. Solid and dashed curves denote respectively the phase-transition and chemical boundary cases with three discontinuities at 400, 520 and 670 km depth. Angular order is for $l = 2$ and the L1 viscosity model has been used.

Figure 3. Depth variations of the radial ($T_{rr}$) and tangential ($T_{r\vartheta}$) spherical harmonic components of the stress tensor at the time $t = 20$ Kyr. Angular order and the viscosity model are the same as in Fig. 2. The stress quantities have been non-dimensionalized.


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Fig.2
Fig. 3