

Estimation of High-Frequency Empirical Green's Tensor Spatial Derivatives from Observed Seismic Records

Sayumi Yoshida¹, Masanori Horike^{*1}, Nelson Pulido², and Kotoyo Tsuchida¹

ABSTRACT

We investigated the feasibility of estimating high-frequency empirical Green's tensor spatial derivatives (EGTDs) using aftershocks of the 2016 Kumamoto earthquake, Japan. Before the EGTD estimation, we accurately determined two source parameters (seismic moment and duration of the source time function [STF]) and three fault parameters (strike, dip angle, and rake angle), which are necessary for the EGTD estimation. Further, we investigated the extent to which high-frequency EGTDs can be estimated using five groups consisting of aftershocks in the Japan Meteorological Agency magnitude scale range over $1.5 < M_j < 3.5$, within zones of 1 and 2 km diameter. We found that high-frequency EGTDs with cutoff frequencies between 8 and 16 Hz could be estimated from the aftershocks within the zone of 1 km diameter using optimum values of the source and fault parameters. In contrast, it is difficult to stably estimate the high-frequency EGTDs from the aftershocks within the zone of 2 km diameter, even for the cutoff frequency of 4 Hz. Furthermore, we found that the accuracy of the EGTDs is evaluated by the reproducibility of earthquake motion records as well as the STF. We examined whether there is a difference in the reproducibility of recorded earthquake motions by earthquake motions synthesized using EGTDs with two different STFs. The two types of EGTDs, which were estimated by adopting the two STFs, reproduced the earthquake motion records with equal accuracy but were quite different in waveform and amplitude from each other, indicating that we should adopt the STF that is based on or consistent with the physics of fault fracture when estimating the high-frequency EGTDs. Thus, it is certainly feasible to estimate the high-frequency EGTDs from earthquake motion records radiated from a 1 km diameter zone using an appropriate STF.

KEY POINTS

- A method for estimating the high-frequency empirical Green's tensor spatial derivatives (EGTDs) is developed.
- The EGTD with a cutoff frequency above 8 Hz is extracted from records of aftershocks in a small zone.
- The EGTD allows us to study the properties of the propagating media and seismic source in a high-frequency band.

INTRODUCTION

Earthquake ground motions due to a point earthquake source are expressed by the convolution of the moment tensor and Green's tensor spatial derivatives (Aki and Richards, 1980). Thus, studies on earthquake sources and properties of propagating media require the deconvolution of these terms. For this purpose, one or other of them needs to be known before the deconvolution. Conventionally, the spatial derivatives of Green's functions have been obtained by numerical methods. To that purpose, numerical methods

such as the frequency–wavenumber integration method, finite-difference method, finite-element method, and, so on, have been typically employed. However, these methods require a priori knowledge of the *P*- and *S*-wave velocity structure models of the propagating media. Notably, inhomogeneous velocity structure models with short wavelengths are necessary for synthesizing high-frequency earthquake motions. However, it is difficult to survey the short-wavelength inhomogeneity with existing expedition methods. Accordingly, these numerical methods have been impossible in synthesizing high-frequency earthquake ground motions above 1 Hz.

1. Hanshin Consultants Company, Ltd., Osaka, Japan,  <https://orcid.org/0009-0008-7698-3003> (SY); 2. National Research for Earth Science and Disaster Resilience, Tsukuba, Japan

*Corresponding author: horike@hanshin-consul.co.jp

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Plicka and Zahradnik (1998) proposed a method to overcome the drawback of the numerical methods. This method extracts the empirical Green's tensor spatial derivatives (EGTDs) from earthquake motion records radiated from multiple aftershocks in a small zone that is regarded as a point source. An advantage over the conventional methods is that it does not require the velocity structure models. Accordingly, the EGTDs are valid up to a high frequency. Another advantage is that because the EGTDs consist of *P* and *S* waves and surface waves that are affected by attenuation and scattering along the propagation, there is no necessity for correction for these effects in the synthesis of earthquake ground motions, which in practice is difficult by conventional methods, especially in high-frequency ground motions. These properties of the EGTDs are important in that we can synthesize precise high-frequency earthquake motions radiated from an earthquake with an arbitrary focal mechanism and source time function (STF), and, as a result, the synthetic high-frequency earthquake motions can be applied to various research topics, such as (1) determination of the properties of propagating media such as attenuation and scattering and (2) estimation of the high-frequency earthquake motion radiating areas on fault surfaces. In particular, the latter remains a largely unresolved aspect of the earthquake source mechanics.

Furthermore, it becomes possible to determine the parameters of fault ruptures. The fault rupture parameters, such as the stress drop, strength excess, and critical distance (*D_c*), have been obtained from the evolution of the traction computed by solving the elastodynamic equation using fault displacements determined by kinematic source inversion as a boundary condition (e.g., Mikumo, 1994; Ide and Takeo, 1997), as well as the scaled energy obtained from fault rupture process of earthquakes (Pulido and Irikura, 2000). However, the accuracy of *D_c* is particularly low due to the lack of high-frequency fault displacements (Tinti, Spudich, and Cocco, 2005). In contrast, fault displacements determined by kinematic inversion of earthquake motions synthesized using high-frequency EGTDs include high-frequency displacement, enabling an accurate *D_c* to be obtained. Hence, high-frequency EGTD may become a new promising tool applicable to various research topics such as the estimation of the properties of propagating media, the detection of high-frequency radiating areas on earthquake fault surfaces, and the determination of dynamic fault rupture parameters.

However, studies on the estimation of high-frequency EGTD from earthquake motion records have been very few (e.g., Ito *et al.*, 2001; Pulido *et al.*, 2006; Ohori and Hisada, 2011; Vavryčuk and Adamová, 2020). The study by Plicka and Zahradnik (1998) was limited to numerical tests, whereas Ito *et al.* (2001) used EGTDs to estimate the focal mechanism of the aftershocks of an *M_w* 5.0 earthquake occurring on 15 September 1988, Sendai city, Japan, and Pulido *et al.* (2006) used EGTDs to simulate the strong ground motions of the 2000 Tottori-ken Seibu earthquake. Vavryčuk and Adamová (2020) applied the EGTDs to non-double-couple moment tensor

inversion. Ohori and Hisada (2011) also applied EGTDs to synthesize the strong ground motions of the Hyogo-ken Hokubu earthquake and showed that the synthetic strong ground motions reproduced recorded strong motions more accurately than those by the empirical Green's function method. This is because synthetic earthquake motions using EGTDs do not require correction for the effects of attenuation, scattering, and travel distance; moreover, the effects of the radiation pattern are incorporated more easily than with the empirical Green's function method. However, the EGTDs of Ohori and Hisada were limited to 1 Hz or less because they obtained the STF by a numerical method that required the velocity structure model.

The estimation of the high-frequency EGTD requires solving the following three research topics: (1) precise determination of the hypocenter locations and occurrence times, the fault parameter (strike, dip, and rake angle), and the source parameters (seismic moment and a time parameter used to represent the duration of the STF, hereinafter abbreviated as the duration), (2) cutoff frequency of the EGTD about aftershock zone size, and (3) selection of a functional form of the STF. Solving these topics will enable the estimation of reliable high-frequency EGTDs. We will discuss these research topics in some detail subsequently.

Let us describe the first research topic. Before the EGTD estimation, we must determine the occurrence times and hypocenter locations of aftershocks. The occurrence times are necessary for setting the origin time for the aftershock record. The hypocenter locations are necessary for determining whether aftershocks are located within a certain sized zone. It is desirable to obtain as many earthquake motion records as possible for accurate values of the parameters. We thereby conducted a temporary observation of aftershocks for 53 days after 20 days of the 2016 Kumamoto earthquake, Japan, and created a unified dataset of Kumamoto earthquake aftershocks, integrating the earthquake motion records of the temporary observation with those of permanent observations run by the National Research Institute for Earth Science and Disaster Resilience (NIED) and the Japan Meteorological Agency (JMA). Using the dataset, we accurately determine the hypocenter locations and occurrence times of the aftershocks.

The coefficient matrix in the simultaneous equations to determine the EGTDs requires the three fault parameters and two source parameters. These parameters contain errors due to noise in earthquake motion records and assumptions in the methods used to determine them. For this reason, we use a genetic algorithm (GA) to find their optimal values and adopt them to estimate the high-frequency EGTDs.

We next describe the second research topic. The cutoff frequency (*f_{ch}*) of EGTD on the high-frequency side is practically important, and it is constrained by the limited high frequency (*f_{lh}*) determined by the equation that is expressed as

$$f_{lh} = VR/2L^2, \quad (1)$$

which is derived from the point-source condition, in which V is the propagation velocity, R is the focal distance, and L is the maximum length of the aftershock zone (Aki and Richards, 1980). The cutoff frequency of the EGTS is constrained by the inequality $f_{ch} < f_{lh}$, but it does not determine the cutoff frequency. Therefore, we investigate the cutoff frequency especially in relation to aftershock zone sizes, using two different-sized aftershock zones with diameters of 1 and 2 km.

Finally, we will describe the third topic. Equation (3), which appears later, can determine the EGTD for any causal STF adopted. This means that the extracted EGTDs depend on the adopted STF, which, however, is inconsistent with the EGTDs being influenced only by properties of the propagating media. As a result, earthquake motions synthesized using the EGTDs are also dependent on the STF, and research findings obtained using the synthetic earthquake motions may be inappropriate. To avoid this dependency, we should employ an STF based on the physics of fault rupture but no such function exists. Thus, we attempt a second-best approach. Specifically, among the currently available STFs, we select the function that minimizes the difference between the synthesized earthquake motions using the EGTD and the recorded earthquake motions. We examine the possibility of selecting the STF giving the best reproducibility by this approach.

The EGTD can be used for two purposes: one is a study on attenuation, amplification, and scattering along the propagation path, and the other is for earthquake ground-motion prediction and kinematic seismic source inversion studies. In the former case, it is enough to confirm that the earthquake motions synthesized using the EGTDs can reproduce earthquake motion records. Thereby, we call the former the EGTDs for reproduction. In contrast, in the latter case, it is necessary to confirm that the earthquake motions synthesized using the EGTDs can predict earthquake motions of future earthquakes. Thereby, we call the latter as the EGTD for prediction. We deal with the EGTD for reproduction in this study, and the EGTD for prediction will be the subject of a future study. Henceforth, we will simply refer to EGTD for reproduction as EGTD in this study unless otherwise necessary.

This study aims to investigate the possibility of estimating the high-frequency EGTD by examining three research topics. For this purpose, we use earthquake motion records of aftershocks of the 2016 Kumamoto earthquake observed at the Hi-net sites.

In [Source and Fault Parameters and Aftershock Grouping](#) section, we describe the results of the first research topic and the grouping of aftershocks based on the relative distances from a reference aftershock. In [Estimation Method for the EGTDs](#) section, we describe the process for the estimation of the EGTDs. In [Estimation of the EGTDs](#) section, adopting the smoothed step function (SSF) as the STF, we investigate the upper-frequency limit of the EGTDs that can be estimated from earthquake motion records of aftershock groups within a zone of 1 and 2 km diameter. In [Selection of the STF](#) section, we examine the possibility of selecting the STF giving the best reproducibility

among currently available functions. Specifically, we compare the reproducibility of earthquake motion records by synthetic earthquake motions between two STFs: the SSF and triangular function (TAF). In addition, we compare the estimated EGTDs for two STFs to promote the understanding of the dependency of the EGTDs on the STF. Finally, we discuss improvements for the EGTD estimation and conclude.

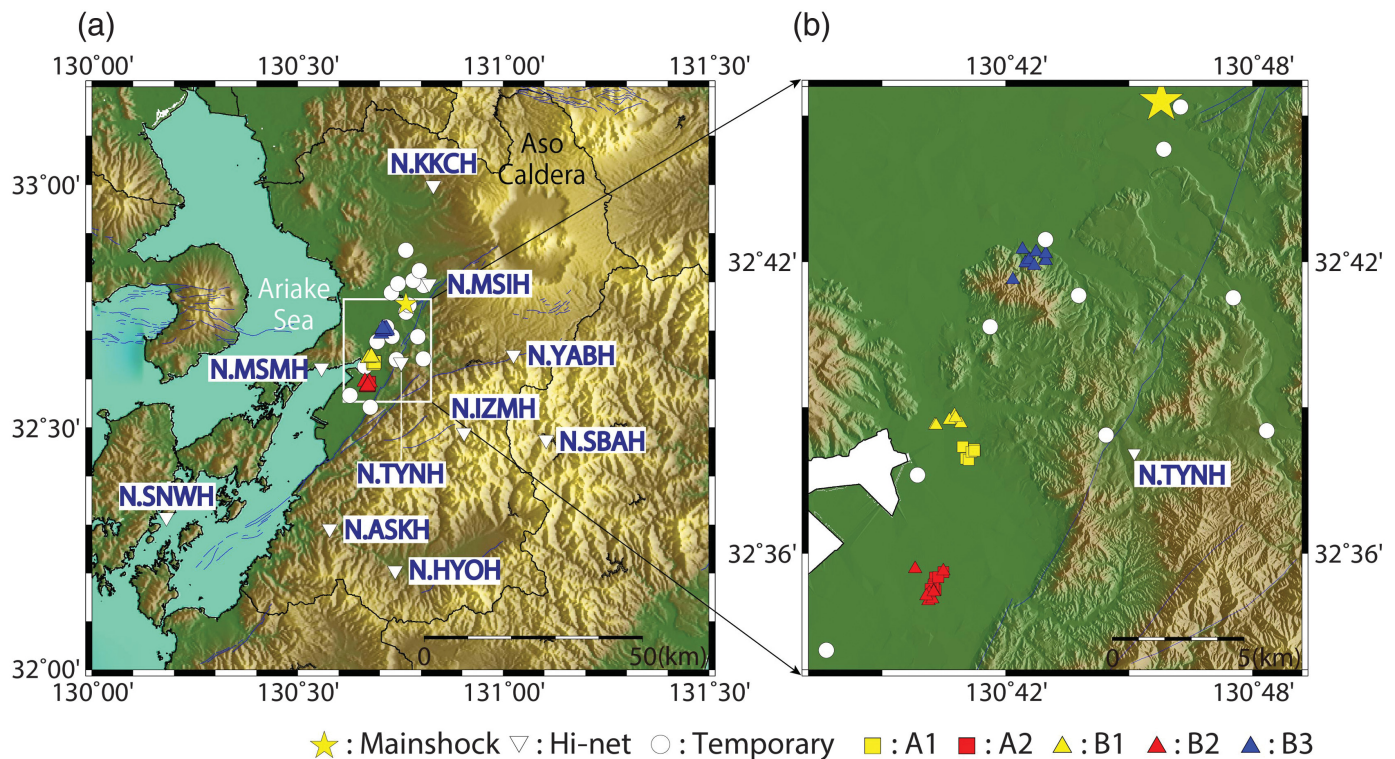
SOURCE AND FAULT PARAMETERS AND AFTERSHOCK GROUPING

As already mentioned, EGTD extraction requires parameters related to earthquake sources and fault mechanisms. Specifically, they are the hypocenter location, occurrence time, fault parameters (strike, dip angle, and rake angle), and source parameters (the seismic moment and the duration). We describe the determination of these parameters.

Prior to the determination of the parameters, we created an earthquake motion dataset. It is desirable to collect earthquake motion records from as many sites as possible to accurately determine the hypocenter locations and occurrence times of earthquakes. Thus, we deployed a temporary aftershock observation network of the Kumamoto earthquake consisting of 17 observation sites, as shown in Figure 1, from 6 May to 27 June 2016. We created the dataset by integrating the earthquake motion records obtained from this temporary observation with those from the permanent observation networks run by NIED (Hi-net) and JMA. The aftershocks included in the dataset are only those listed in the JMA Unified Earthquake Catalog. The sampling interval and duration of the earthquake motion records are 0.01 and 40.96 s, respectively. Using this dataset, we determined the hypocenter location, occurrence time, and fault parameters.

We determined the hypocenter location and occurrence time as follows. First, we read the P - and S -wave arrival times and the polarity of the P -wave onset on waveform data. Second, we constructed a 1D P -wave velocity structure model (Fig. 2) with special reference to [The Research Group for the 2003 Hinagu Fault Seismic Expedition \(2008\)](#) and then constructed the S -wave velocity structure based on an assumption that the ratio (V_P/V_S) of P -wave velocity (V_P) to S -wave velocity (V_S) is $\sqrt{3}$. Next, we obtained the hypocenter locations using the nonlinear maximum-likelihood method (Hirata and Matsu'ura, 1987) from the travel times of the P and S waves using the 1D velocity structure model. We again applied the maximum-likelihood method to obtain the hypocenter location and occurrence time more accurately by way of minimizing the residual difference between the observed and calculated travel times. Finally, we used the double-difference (DD) method (Waldhauser and Ellsworth, 2000) to obtain the hypocenter locations with higher accuracy.

We mention the determination of the fault parameters. We targeted aftershocks with more than 10 observation sites in which the polarity of the P -wave onset was detected clearly.



We obtained well-converged fault parameters of 640 aftershocks using the method described by Hardebeck and Shearer (2002). They were the aftershocks that received well-converged qualities A, B, and C out of the six evaluation qualities listed by Hardebeck and Shearer (2008). The focal mechanisms of these aftershocks are depicted at the epicenter locations in Figure 3. As shown in Figure 3, the normal faults are overwhelming among the three fault types, which were also confirmed in a ternary graph (Frohlich, 1992) of the focal mechanism.

Among the 640 aftershocks in the JMA magnitude scale range over $1.5 < M_j < 3.5$, we found 50 aftershocks possible for grouping in use of the already-obtained hypocenter locations. Aftershock groups consist of more than six aftershocks distributed within zones of 1 or 2 km in diameter. Because the seismic moments M_0 and corner frequencies f_c are extracted from the radiated source spectra, we derived them by the inversion of the earthquake ground-motion spectra of these aftershocks (e.g., Andrews, 1986; Hartzell, 1992). As a restraint condition for the inversion, we added the condition of the spectral ratios between borehole top (ground surface) and borehole bottom at the Hi-net observation site N-ASKH (Fig. 1), in which a sensor at the borehole bottom was installed in hard rock (S -wave velocity of 3 km/s). On an assumption that radiated source spectra of the aftershocks meet the ω^{-2} model, the seismic moment and corner frequency were obtained by fitting them to the curve of the ω^{-2} model (Fig. 4). The frequency band targeted for the fitting was set from 1 to 20 Hz, considering that the Hi-net sensors were velocity sensors with a natural frequency of 1 Hz. The radiated source

Figure 1. (a) The location map of the Hi-net permanent sites (white down triangles) and the temporary sites (white circles). Hi-net site names are attached to the symbols of the Hi-net sites. The star symbol denotes the epicenter of the 2016 Kumamoto earthquake, Japan. (b) The enlarged area of the inset rectangle in (a) is a location map of aftershocks of five groups: aftershocks of groups A1 and A2 with the 1 km diameter zone are shown by the yellow and red squares, respectively, and aftershocks of groups B1–B3 with the 2 km diameter zone are shown by the yellow, red, and blue triangles, respectively.

spectra fitted the ω^{-2} model well, and we think that M_0 and f_c were also determined appropriately. The duration of the STF is different from the reciprocal of the corner frequency itself. It is a reciprocal of the estimated f_c multiplied by an appropriate factor depending on the STF.

We examined the reliability of the M_0 and f_c . Figure 5a shows the relationship between the moment magnitude scale M_w converted from the estimated M_0 and the JMA magnitude scale M_j . We find that our relationship between them is similar to the relationship by Uchide and Imanishi (2018) that can be applied over a wide range of $0.5 \leq M_j \leq 7.2$. Despite our relationship being obtained using data and methods different from theirs, the similarity of the two relationships indicates that our estimated M_0 is valid. We also notice an interesting feature in both relationships that the smaller M_j is the larger the divergence from M_w . Uchide and Imanishi attributed it to the effects of attenuation and sampling. The estimated M_0 and f_c reveal that the Brune-type stress drop $\Delta\sigma$ (Brune, 1970) derived from the two estimates is almost constant in the range of $0.1 \text{ MPa} < \Delta\sigma < 2.5 \text{ MPa}$ (Fig. 5b), which shows the self-similarity indicates that the estimated values of f_c are also appropriate.

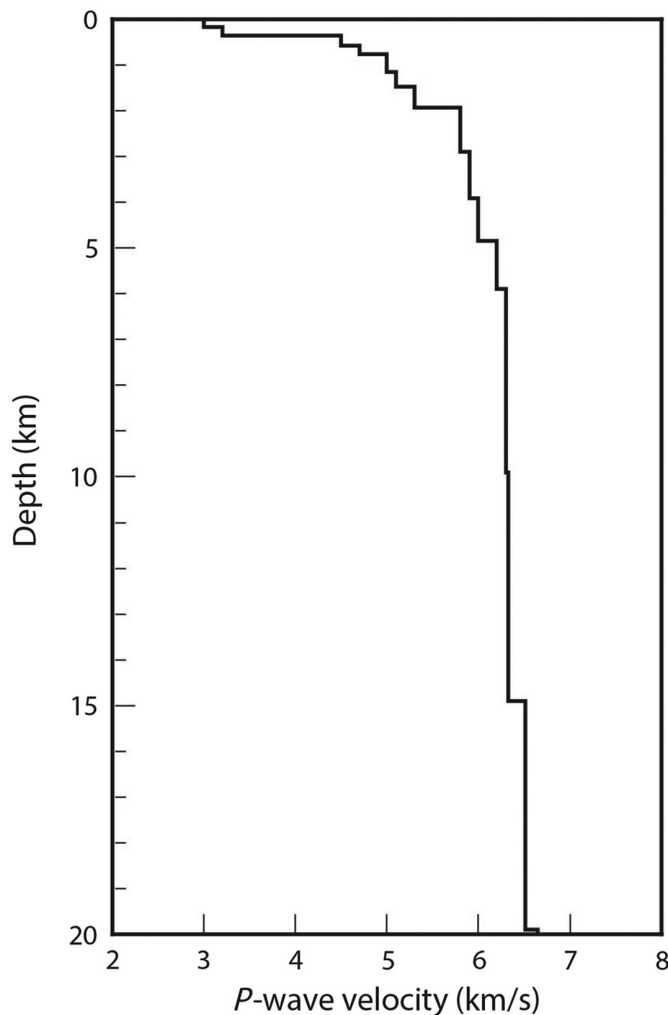


Figure 2. One-dimensional P -wave velocity structure model to determine the hypocenters of the aftershocks of the 2016 Kumamoto earthquake.

We use common aftershock groups for the estimation of the EGTDs for reproduction and prediction. Thus, we created aftershock groups that consisted of six or more aftershocks. In the estimation of the EGTD for reproduction, all aftershocks in groups are used to estimate the EGTD. Meanwhile, in the estimation of the EGTD for prediction, one aftershock is used as a virtual future earthquake, and the remaining aftershocks are used to estimate the EGTD for prediction. Hence, in this study, all aftershocks are used to estimate the EGTD.

We obtained two groups (A1 and A2) with a 1 km diameter zone and three groups (B1– B3) with a 2 km diameter zone. Their locations are shown in Figure 1. We call the areas of groups A1 and A2 as zones A1 and A2 and the areas of groups B1–B3 as zones B1–B3.

Here, we discuss the reliability of the five aftershock groups. If the accuracy of the hypocenter locations of the 39 aftershocks, which belong to the five groups, is not sufficient, there may be aftershocks far away from the zones, making the results

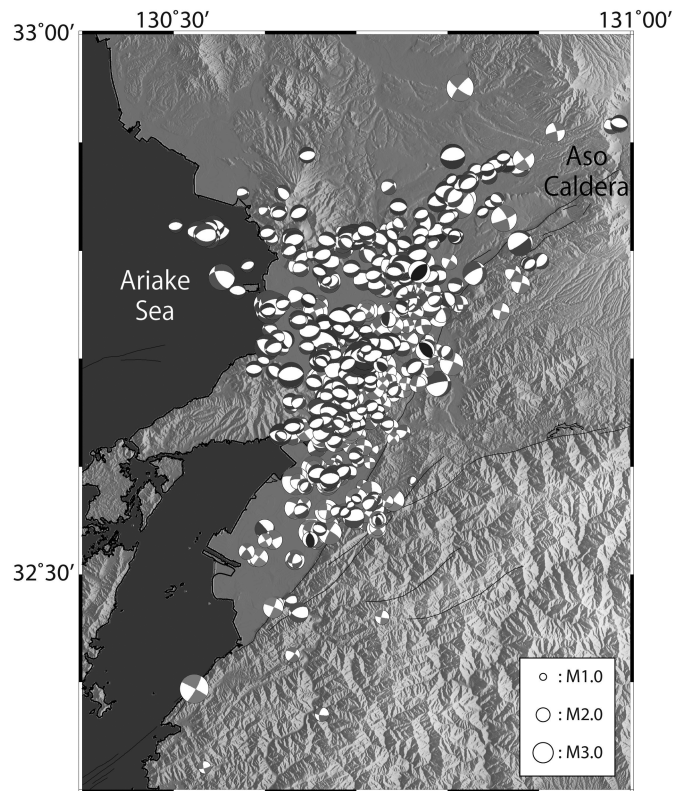


Figure 3. Location map of 640 aftershocks for which focal mechanisms are well determined. They are plotted at the individual epicenters together with the Japan Meteorological Agency (JMA) magnitude scale M_j .

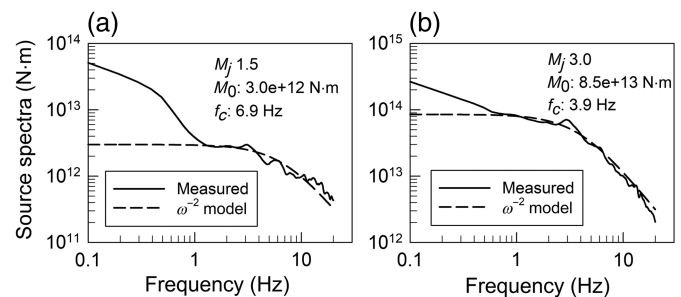


Figure 4. Examples of estimating the seismic moment and the corner frequency by fitting the measured source spectra to the ω^{-2} model in the frequency range between 1 and 20 Hz. The Brune type is employed as the ω^{-2} model. Aftershock with (a) M_j 1.5 and (b) M_j 3.0.

of subsequent research meaningless. Thus, we mention the errors of the hypocenter locations obtained by the DD method. The errors of the 39 aftershocks range from 100 to 130 m in both horizontal directions and the errors range from 100 to 150 m in the depth direction. Accordingly, some aftershocks may be outside the zones but are certainly quite close to them. Therefore, we believe an assumption that the 39 aftershocks are located within each zone is a good approximation, and the results obtained from the subsequent analysis will be reliable.

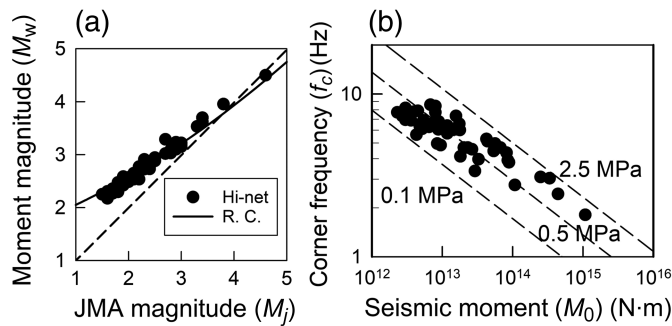


Figure 5. (a) The relationship between M_j and M_w (the moment magnitude scale) for the aftershocks included in the five aftershock groups used for the extraction of the empirical Green's tensor spatial derivatives (EGTDs). The solid circles are the M_j – M_w relationship obtained from Hi-net data of 50 aftershocks of the 2016 Kumamoto earthquake and the solid curve is a regressive formula inferred by Uchide and Imanishi (2018). (b) The relationship between M_0 (the seismic moment) and f_c (the corner frequency). The solid circles are the M_0 – f_c relationship obtained from the same Hi-net data. The broken lines are the Brune-type stress drops (Brune, 1970) of 0.1, 0.5, and 2.5 MPa from below.

The determined values of the seven parameters relevant to earthquake sources contain errors due to assumptions about the methods for determining them and noise in the earthquake motion records. For example, hypocenter determination assumes a 1D velocity structure ignoring lateral velocity inhomogeneities. In addition, the polarity of the P wave may be disturbed by noise in the records. Furthermore, the inversion assumes a homogeneous crustal velocity structure. Because of these assumptions and noise, the determined values also contain errors. Thus, it would have been difficult to estimate the EGTDs accurately if the determined values of the parameters were used as they were. In this study, we estimated the EGTDs using optimum values of the fault parameters and source parameters found by a GA, but the hypocenter locations and occurrence times were fixed to maintain the aftershock groups.

ESTIMATION METHOD FOR THE EGTDs

The estimation procedure for the EGTDs begins with setting the search ranges of the source and fault parameters, as shown in the flow chart that consists of four steps (Fig. 6). We determined the search ranges of the source parameters, that is, the seismic moment and duration, from an inequality of $|V - V_{\text{ref}}|/V_{\text{ref}} < \alpha$, in which α is a controlling parameter of the search range. Here, V_{ref} , either of the two source parameters obtained in Source and Fault Parameters and Aftershock Grouping section 2, is a reference value of the search range, and V represents either of the two source parameters. We set α to 0.3 for each of the two search ranges. Because the accuracy of the fault parameters, the strike, dip angle, and rake angle, was assessed at three qualities, a different search range for each quality was given by the inequality of $|V - V_{\text{ref}}|/\beta < 1$, in which parameter β controls the search range: $\beta = 0.03$ rad

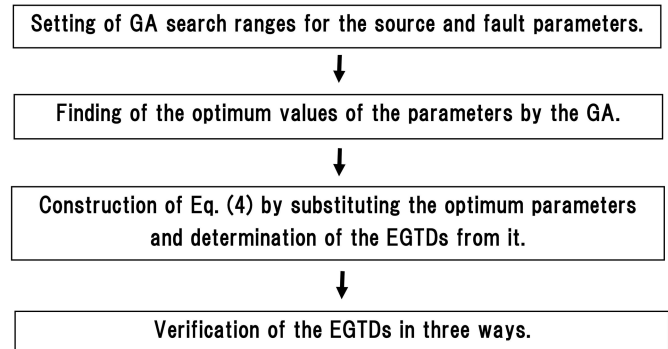


Figure 6. Flow chart of the process of the EGTD estimation.

for quality A, $\beta = 0.08$ rad for quality B, and $\beta = 0.15$ rad for quality C. In addition, we tried the GA, setting the parameter β to 0.5 times and 2 times, but it did not affect the conclusion. We set the chromosome number to 500 and the generation number to 1000.

We next describe the second step to obtain the optimum values of the source and fault parameters. They are determined with the GA, as described earlier. Consider the case in which there are multiple observation sites for a single aftershock group, as shown in Figure 7. The optimum parameters of all aftershocks are obtained from three-component earthquake motion records, which were radiated from all aftershocks in the aftershock group, at all observation sites. Thus, fitness function (F) for a chromosome in a certain generation of GA is expressed as

$$F(M_0^1, t_0^1, \phi^1, \delta^1, \lambda^1, \dots, M_0^m, t_0^m, \phi^m, \delta^m, \lambda^m) = \sum_{j=1}^n \sum_{k=1}^m \sum_{i=1}^3 \left(\frac{\sum_t (\hat{o}_i^{jk}(t) - \tilde{c}_i^{jk}(t))^2}{\sum_t (\hat{o}_i^{jk}(t))^2} \right). \quad (2)$$

This function is a sum of normalized squared residuals between the recorded earthquake motions $\hat{o}_i^{jk}(t)$ and the calculated earthquake motions $\tilde{c}_i^{jk}(t)$ over components, sites, and aftershocks, in which i , j , and k , denote the component, site, and aftershock, respectively. The optimum values of the source and fault parameters are determined by this fitness function being at the minimum. Because squared residuals are normalized, earthquake motion records of all aftershocks equally contribute to the fitness function regardless of their amplitudes.

As shown in equation (2), the calculated earthquake motion $\tilde{c}_i^{jk}(t)$ is necessary to determine the fitness functions of all chromosomes. Thus, we describe how the calculated earthquake motions are simulated. Earthquake motions in the frequency domain radiated from a point source are written as (Plicka and Zahradnik, 1998)

$$u_i^j(f)/(R(f)M_0^j S(f)) = \sum_{k=1}^5 A_k^j(\phi, \delta, \lambda) G_{ik}(f), (i = 1, 2, 3), \quad (3)$$

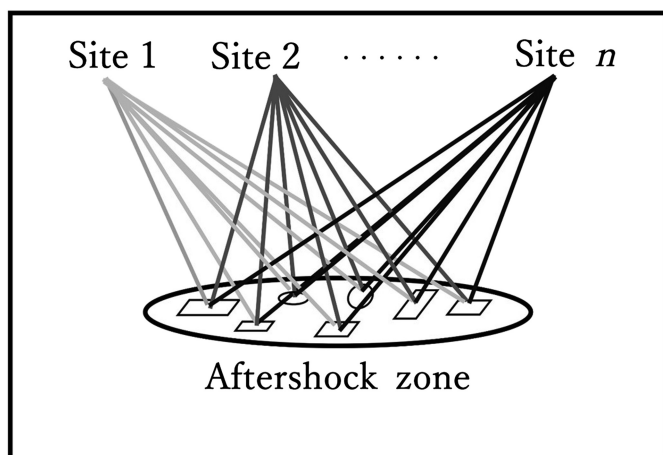


Figure 7. Schematic diagrams for the extraction of the EGTDs from single-component earthquake motion records at multiple (n) sites radiated from aftershocks located in a small zone. The EGTDs between each site and an aftershock zone are estimated simultaneously. The ellipse denotes the aftershock zone. The open patches in it are the aftershocks to be used for the estimation of the EGTDs. The solid lines denote the radiation of earthquake motions.

in which $\dot{u}_i^j(f)$ represents the Fourier spectrum at frequency f of the i th component velocity earthquake motion records at a site due to the j th aftershock (or event) in an aftershock group. Here, $R(f)$ represents the frequency response of the observation system; M_0^j and $S^j(f)$ represent the seismic moment and the Fourier spectra of the STF of the j th aftershock, respectively; $A_k^j(\phi, \delta, \lambda)$ is the radiation coefficient as a function of strike ϕ , dip angle δ , and rake angle λ , and its specific form is written in equation (1) in box 4.4 in [Aki and Richards \(1980\)](#). Here, $G_{ik}(f)$ represents the k th EGTD for the i th component. We understand from this equation that the EGTD with five elements is obtained at an observation site from each of the three components of earthquake motions radiated from five or more aftershocks in an aftershock group.

In the case of an aftershock number $m \geq 5$, replacing the left side in equation (3) with $\dot{U}_i^j(f)$, the simultaneous linear equations in the frequency domain to estimate the EGTDs are written as

$$\begin{Bmatrix} \dot{U}_i^1(f) \\ \dot{U}_i^2(f) \\ \vdots \\ \dot{U}_i^m(f) \end{Bmatrix} = \begin{bmatrix} A_1^1 & A_2^1 & A_3^1 & A_4^1 & A_5^1 \\ A_1^2 & A_2^2 & A_3^2 & A_4^2 & A_5^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_1^m & A_2^m & A_3^m & A_4^m & A_5^m \end{bmatrix} \begin{Bmatrix} G_{i1}(f) \\ G_{i2}(f) \\ G_{i3}(f) \\ G_{i4}(f) \\ G_{i5}(f) \end{Bmatrix}, (i = 1, 2, 3). \quad (4)$$

At a chromosome of a certain generation of the GA, substituting the fault parameters generated by the GA into each element of the coefficient matrix in equation (4) and substituting the source parameters generated also by the GA into each element of the left-hand vector, we construct the simultaneous

linear equations. Solving them by the least-squares method, we obtain tentative EGTDs. Putting the EGTDs into the right side of equation (3) or (4), we obtain the calculated earthquake motions of all aftershocks. Thus, putting them into equation (2), we obtain the fitness function of the chromosome. Repeating this process for the other chromosomes, we obtain the fitness functions for all chromosomes of the generation. Using these fitness functions, generations of the GA advance one by one up to a specified generation (1000 generations in this study), and optimum values of the fault and source parameters are determined by finding the minimum fitness function.

In the third step, we determine the EGTDs for reproduction between each site and an aftershock zone using the optimum parameters. We again use equation (4) in this step. Substituting the optimum fault parameters into each element of the coefficient matrix in equation (4) and substituting the optimum source parameters into each element of the left-hand vector, we construct the simultaneous linear equations. Solving the equations, we determine the EGTDs for reproduction.

At the end of the EGTD estimation process, we verify the EGTDs for reproduction in three ways described in detail in the next section.

ESTIMATION OF THE EGTDS

Group A1

We attempted high-frequency EGTD estimation from earthquake motions that radiated from aftershocks of group A1 listed in Table 1 and were recorded at Hi-net sites listed in the left half of Table 2. Table 1 exhibits the source parameters and fault parameters of the eight aftershocks. The quality column in Table 1 shows the estimation accuracy of the three fault parameters. In addition, the dist. column shows the distances between the reference aftershocks (event number 3) and the other aftershocks.

The left half of Table 2 exhibits the Hi-net sites in which all the aftershocks (events) in Table 1 were observed and the average focal distances between individual Hi-net sites and all aftershocks in group A1.

We adopted the SSF as the STF in this section. The SSF is the sum of the step function $H(t)$ and the hyperbolic tangent function $\tanh(t)$, which is expressed as $\text{SSF}(t) = \frac{1}{2}H(t)(1 + \tanh(\frac{t}{t_0}))$. The derivative of this function decays rapidly as the duration t_0 approaches zero and more slowly as it increases. Henceforth, the derivative of the SSF is called SSF for simplicity.

Prior to the EGTD estimation, the frequency band used for estimation was determined based on the point-source condition and signal-to-noise ratios (SNRs) of earthquake ground-motion records. As described previously, the limit high frequency (f_{lh}) is determined in equation (1). The shortest distance R between group A1 and the observation sites is about 8 km (Table 2), and the maximum length L of the aftershock zone is 1 km. Hence, the frequency that satisfies

TABLE 1

Source and Fault Parameters for the Aftershocks of Group A1

Event Number	M_j	M_0 (N·m)	f_c (Hz)	Latitude (°)	Longitude (°)	Depth (km)	ϕ (°)	Δ (°)	λ (°)	Quality	Dist. (km)
1	3.0	8.54×10^{13}	3.9	32.633	130.684	5.5	44	76	170	B	0.5
2	2.7	1.08×10^{14}	2.8	32.632	130.685	5.6	38	89	178	B	0.4
3	2.1	1.77×10^{13}	4.0	32.635	130.686	5.9	78	49	103	B	0
4	2.2	1.81×10^{13}	4.1	32.637	130.683	5.8	46	67	158	B	0.4
5	3.0	8.20×10^{13}	4.4	32.635	130.686	6.0	34	67	150	B	0.2
6	3.0	6.71×10^{13}	4.7	32.636	130.687	6.3	45	61	148	B	0.5
7	1.7	4.30×10^{12}	5.6	32.635	130.688	6.0	67	46	123	B	0.2
8	2.5	3.28×10^{13}	4.0	32.635	130.687	6.1	37	70	143	B	0.3

Dist. column shows the distance between each event (aftershock) and the reference event (event 3).

TABLE 2

Average Distances between the Individual Sites and Aftershocks of Zones A1 and A2

Group A1		Group A2	
Hi-net Site Name	Average Distance (km)	Hi-net Site Name	Average Distance (km)
N.TYNH	8.1	N.TYNH	11.9
N.MSMH	9.6	N.MSMH	12.2
N.MSIH	20.6	N.IZMH	20.5
N.IZMH	21.7	N.MSIH	26.4
N.YABH	21.4	N.YABH	24.4
N.ASKH	39.1	N.ASKH	34.9
N.SBAH	39.5	N.SBAH	37.6
N.KKCH	41.8	N.HYOH	44.0
N.SNWH	46.9		

The left half is for group A1, and the right half is for group A2. The first column in the left half of this table exhibits the Hi-net sites in which all the aftershocks (events) in Table 1 were observed. The first column in the right half of this table exhibits the Hi-net sites in which all the aftershocks (events) in Table 3 were observed.

the point source condition is $f_{lhp} = 15.2$ Hz for S waves and $f_{lhp} = 26.4$ Hz for P waves, assuming that the S -wave velocity is 3.8 km/s and the P -wave velocity is 6.6 km/s.

We next investigated the frequency band in which earthquake motions have sufficient SNRs. Figure 8 shows the vector synthesis of the Fourier spectra of three-component earthquake motions for eight aftershocks at observation site N.TYNH. In the high-frequency range above 25 Hz, the spectra increase for all aftershocks, which suggests that the influence of noise becomes dominant above this frequency. Thus, we determined that the frequency band below 20 Hz had a sufficient SNR. Based on the limit high frequency (f_{lh}) and the SNRs, we set three frequency bands for the estimation of the EGTs, with cutoff frequencies of 4, 8, and 16 Hz. Thus, the high-frequency EGTs in this study have cutoff frequencies between 4 and 16 Hz.

Figure 9 shows examples of the EGTs with a cutoff frequency of 16 Hz. They were extracted from the east–west–

component earthquake motions that were radiated from eight aftershocks of group A1 and recorded at two observation sites. The EGTs at the closest site N.TYNH are exhibited in Figure 9a, and those at the farthest site N.SNWH are exhibited in Figure 9b. Because P and S phases in the less noisy waveforms of the EGTs are clearly identified at both sites, we think that they were extracted with high accuracy. To verify accurate extraction, we attempted to reproduce the earthquake motion records by the earthquake motions synthesized using these EGTs. The synthetic earthquake motions were obtained by substituting the optimum parameters, such as the strike, dip angle, rake angle, seismic moment, and the optimum duration into equation (4) together with the estimated EGTs. Here, we refer to the STF into which the optimum duration is substituted as the optimum STF.

As a first way to verify the extracted EGT, we make a visual comparison of the east–west-component earthquake motions synthesized using the extracted EGT with the east–west-component earthquake motion records at two sites for all the aftershocks of group A1 (Fig. 10). The comparisons at the observation sites N.TYNH and N.SNWH are exhibited in Figure 10a,b, respectively. The earthquake motion records were low-pass-filtered below 16 Hz. Because we depicted the earthquake motion records with black lines and later the synthesized earthquake motions with red lines, stronger red lines indicate better agreement between the two. We see that the synthesized earthquake motions at both sites reproduce the earthquake motion records of all the aftershocks very well. Thus, we think that the EGTs between each of the two sites and zone A1 are estimated with high accuracy.

Instead of a visual comparison to verify the EGTs, we adopted the cross-correlation coefficients (CCCs) as an objective index of agreement between the synthesized and recorded earthquake motions. The CCC is a second way for verification. We took the CCCs from the earthquake motion records with a time length of 40.96 s. Considering the error for the epicenter location and occurrence time, we shifted earthquake motion records in the positive and negative time directions by the maximum time of 0.1 s, and the maximum CCC in this interval

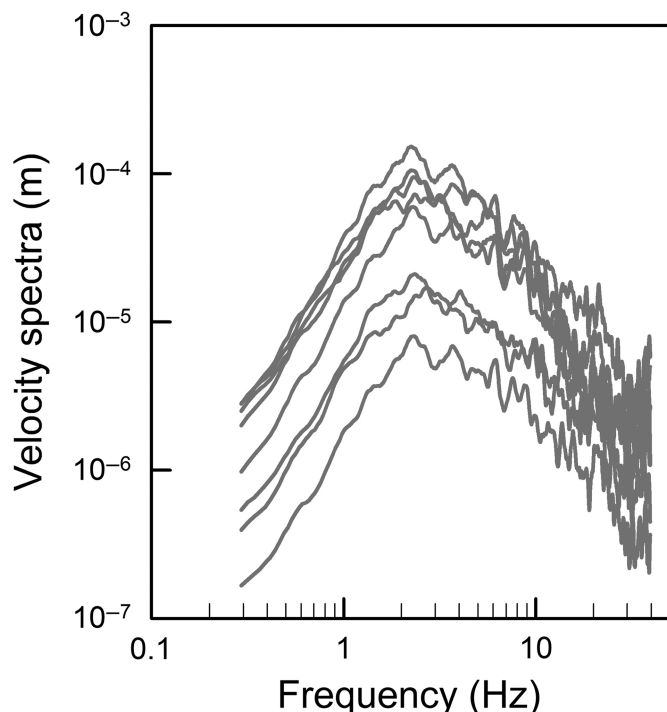


Figure 8. Velocity spectra of earthquake motions radiated from the eight aftershocks of group A1 (Table 1) and recorded at site N.TYNH (Table 2). The spectra are roots of the sum of squared spectral amplitudes of three-component earthquake motions.

was defined as CCC. The number given for each event in Figure 10 is the CCC.

It is not easy to determine how a large CCC corresponds to an accurate and reliable EGTD. However, our experience teaches us if the CCCs of all aftershocks of a group at an observation site exceed 0.75, and then the synthesized and recorded earthquake motions agree well for all aftershocks. Hence, we think that when the CCCs are above 0.75 for all aftershocks, the EGTD between an aftershock zone and an observation site is estimated with considerable accuracy.

Looking at the CCCs at observation site N.TYNH (Fig. 10a), although the CCC of aftershock 2 is slightly low at 0.83, the CCCs of the other aftershocks are 0.95 or higher. Based on the criterion described in the previous paragraph, the EGTDs between site N.TYNH and zone A1 are estimated accurately. In addition, the EGTDs between site N.SNWH and zone A1 are estimated accurately because the CCCs at site N.SNWH are above 0.9 for all the aftershocks (Fig. 10b). Hereafter as well, unless otherwise specified, earthquake motions and CCCs are displayed only for the east–west component.

Because the EGTDs at the closest site N.TYNH and the farthest site N.SNWH were verified to be estimated accurately by the visual inspection and the CCCs, we expected accurate estimation of the EGTDs at the other sites as well. Hence, we took the CCCs to examine the accuracy of the EGTDs at all sites for the three cutoff frequencies (Fig. 11). Comparing the

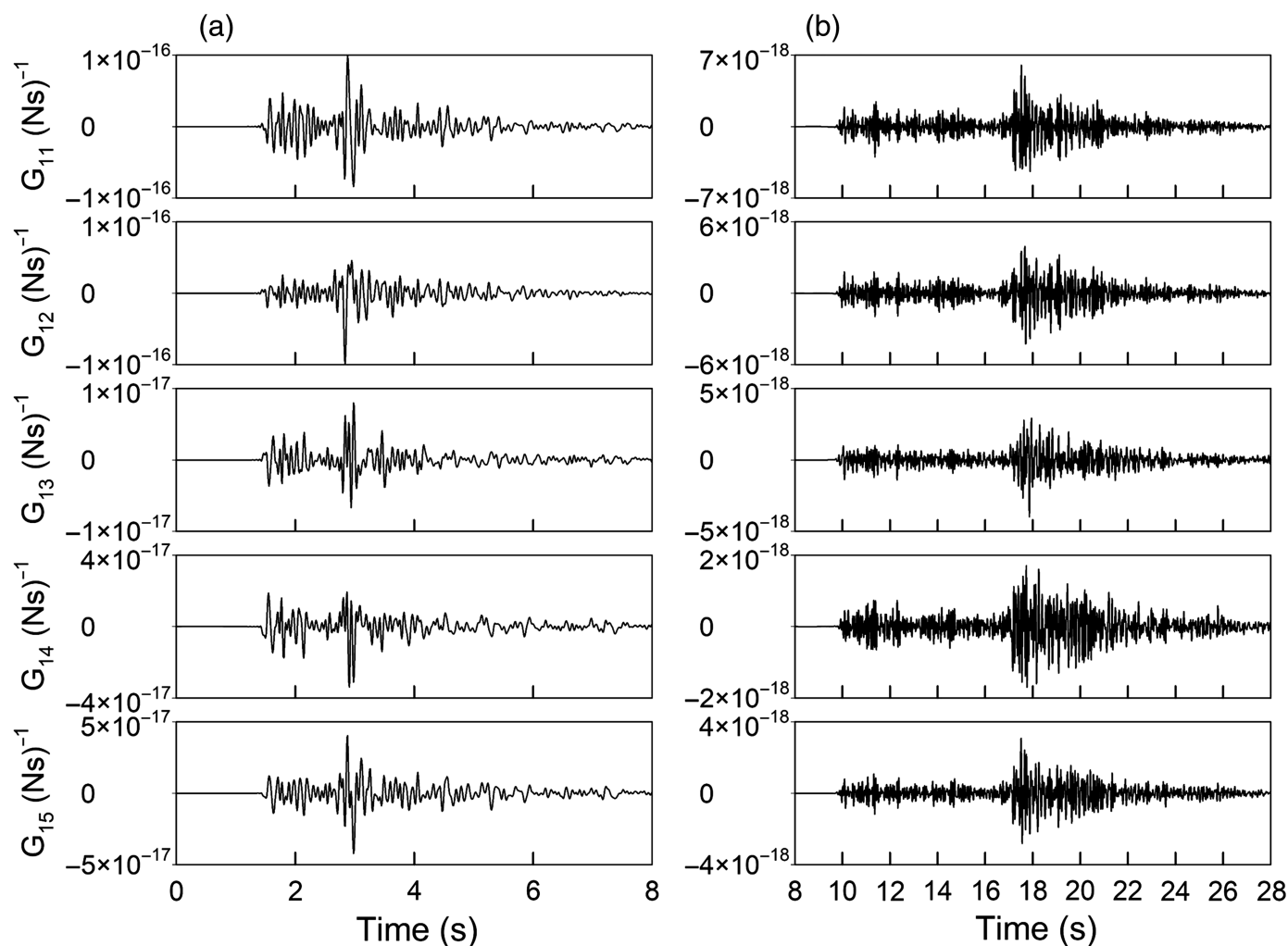
CCCs between the cutoff frequencies of 4, 8, and 16 Hz from the left, we recognize the CCCs decrease with the cutoff frequency increasing because the higher the cutoff frequency, the more difficult it is to meet the point-source condition. However, even for the cutoff frequency of 16 Hz, the CCCs are not necessarily low at every site, and all exceed 0.75. Thus, we think that the EGTDs between zone A1 and each of the nine sites are extracted with high accuracy for the cutoff frequency of 16 Hz or less.

The east–west-component earthquake motions synthesized using the EGTD at observation site N.MSMH have low reproducibility of the earthquake motion records for several aftershocks, as shown in Figure 11c. The synthetic earthquake motions of all aftershocks at this site are computed using the common EGTDs between zone A1 and site N.MSMH. Thus, the low reproducibility is due to the low accuracy of these EGTDs. Furthermore, the CCCs at the site are low only in the case of the cutoff frequency of 16 Hz.

The earlier results suggest that there may be something different in the high-frequency components above 8 Hz in the earthquake motion records at site N.MSMH. Comparing the eight aftershock records in the time domain, only the records for aftershock 3 are rich in high-frequency components. In addition, Fourier spectral amplitudes of the aftershock 3 are particularly large at frequencies above 8 Hz. Hence, we think that the high-frequency earthquake motions with larger amplitudes that can be corrected by the seismic moment of aftershock 3 deteriorated the accuracy of EGTD.

Let us assess the accuracy of the EGTDs in the third way: reproduction of the optimum STF. Specifically, by substituting the extracted EGTD into the right side of equation (4) and recording earthquake motions of a single component (e.g., east–west component) into the left side, we can derive the STF. Because earthquake motions radiated from a single aftershock consist of three components, three STFs are derived by the method. However, because there is only one STF for each aftershock, they should match each other and should moreover agree with the optimum STF of SSF when the EGTDs are extracted accurately. In other words, if the three derived STFs match each other, and they also match the optimum STF, we consider that the EGTDs are estimated with high accuracy. We employ this criterion to assess the accuracy of the EGTDs.

Figure 12 shows a comparison of the optimum STFs of the cut-off frequency of 16 Hz with the STFs derived from earthquake motion records of all aftershocks at site N.TYNH (left half of Table 2). In each panel, the three STFs derived from EW-, NS-, and UD-component earthquake motions are depicted with the red, green, and blue lines, respectively, and the optimum STF is depicted with the dashed line. Looking at each panel, we see that the STFs depicted by the three solid lines match each other and that they match the optimum STF as well. Hence, we think from the criterion that the EGTDs



between zone A1 and site N.TYNH are appreciably accurate. We applied this assessment method to the other sites and revealed that the degree of the reproduction of the optimum STFs at sites N.MSMH to N.SNWH is almost the same as that at site N.TYNH. Thus, we think that the EGTDs between zone A1 and each site are estimated accurately also by the method for reproduction of the optimum STF.

We notice that the derived three STFs begin to deviate from the optimum STF at around 1 s (Fig. 12). This is because the earthquake motion records were low-cut-filtered below 0.4 Hz to stabilize the derived STFs.

A closer look at the figure shows that the noises with monotonic frequency of 10–16 Hz are barely visible on the STFs of aftershocks 3, 4, and 7, whereas they are obviously visible on the STFs of the other aftershocks. Notably, noise is large for aftershock 2. Comparing the noise level in Figure 12 with the CCC in Figure 10a aftershock by aftershock, we notice that there is correspondence in the manner that the larger the CCC is, the smaller the noise level is.

This correspondence means that the CCCs and the derived STFs provide the same information regarding the accuracy of the EGTDs. This is reasonable because equation (3) shows

Figure 9. Examples of extracted EGTDs with a cut-off frequency of 16 Hz. (a) The EGTDs for east–west component between site N.TYNH and zone A1. (b) Those between site N.SNWH and zone A1. The smoothed step function (SSF) is adopted as the source time function (STF). Note that the scale of the time axes is different between the two panels.

that when the recorded ground motions match well with the synthetic ground motions, the derived STFs match well with the optimum STFs.

The discussion on EGTD extraction from aftershocks of group A1 in this section is summarized in two points. The extraction of high-frequency EGTDs is possible below the cutoff frequency of 16 Hz. The reproducibility of the STFs provides the same information about the accuracy of the extracted EGTDs as the reproducibility of the earthquake motion records.

Group A2

To examine the generality of the results for group A1, we performed the same analysis for group A2 as for group A1. Table 3 exhibits the source and fault parameters of the seven aftershocks of group A2. The first column of the right half of

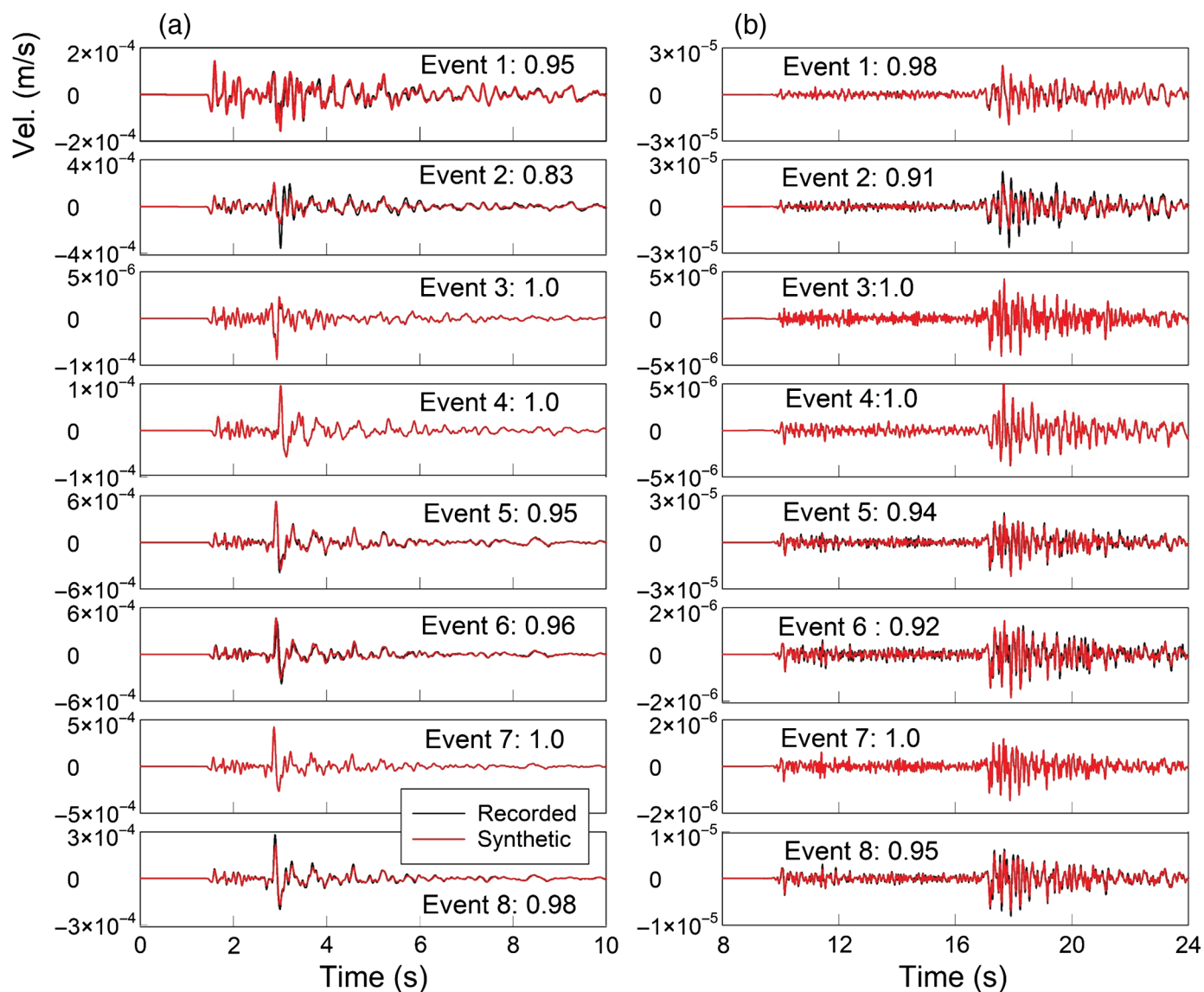


Table 2 exhibits Hi-net sites in which all aftershocks (events) in Table 3 were observed. The second column exhibits an average distance over the focal distances between each Hi-net site and aftershocks of group A2. The average distances are approximately in the same range as those of group A1. Thus, we again examine the possibility of estimating the EGTD for the same three cutoff frequencies.

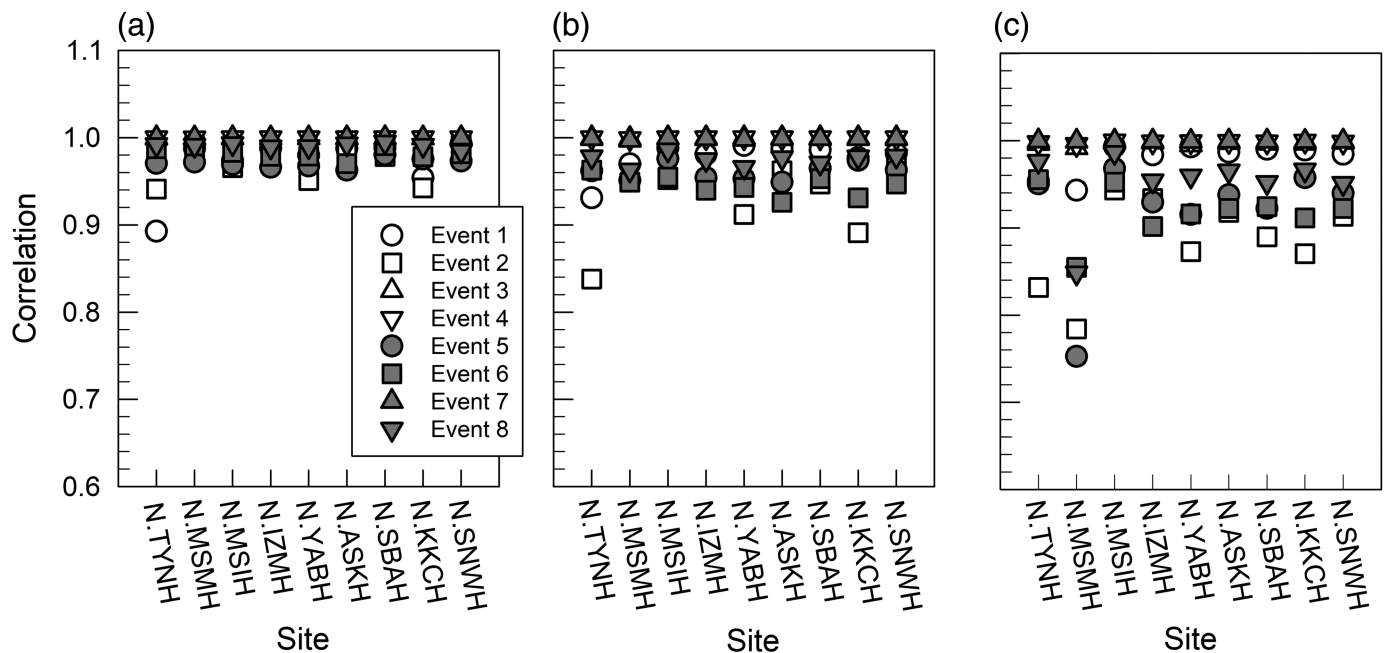
Figure 13 shows a comparison of the recorded earthquake motions low-pass-filtered below 16 Hz at the shortest distance site N.TYNH with the earthquake motions synthesized using the EGTDs. Although the synthetic motions of aftershocks 4 and 5 are significantly poor in the reproduction of the recorded motions, we can recognize by visual inspection and the CCC values that the synthetic earthquake motions of the other five aftershocks reproduce the recorded motions quite well.

Let us examine by the CCCs if the features in Figure 13 also appear at the other sites listed in Table 2 and for the three cutoff frequencies (Fig. 14). The figure reveals that for the three cutoff frequencies in common, the features again appear at all sites.

Figure 10. Comparison of recorded east–west-component earthquake motions (black lines) of the aftershocks of group A1 with earthquake motions (red lines) synthesized using the extracted EGTDs shown in Figure 8. (a) At site N.TYNH and (b) at site N.SNWH. The recorded earthquake motions are low-pass-filtered below 16 Hz. The number given for each event is the cross-correlation coefficient (CCC) between the recorded and synthetic earthquake motions. Note that the scale of the time axes is different between Panels (a,b).

That is, the CCCs of aftershocks 4 and 5 are low at all the sites for the three cutoff frequencies, whereas those of the other aftershocks are high, above 0.9, even for the cutoff frequency of 16 Hz. Despite the low CCC of the two aftershocks, the CCC with the cutoff frequency of 4 Hz is large, exceeding 0.83 at all sites, indicating that the EGTDs with this cutoff frequency are estimated with high accuracy between the individual sites and zone A2.

Low CCCs of aftershocks 4 and 5 at all sites indicate that the cause lies in the sources of the two aftershocks. Thus, replacing them with other aftershocks may improve the EGTDs.



Unfortunately, such manipulation is not allowed for lack of aftershocks, but if temporary aftershock observations had been made immediately after the occurrence of the Kumamoto earthquake, this manipulation might have become possible.

Here, we again evaluate the EGTDs by reproducing the optimum STF. Figure 15 shows a comparison of the STFs derived from the three-component earthquake motion records at site N.TYNH for the cutoff frequency of 16 Hz (three solid lines) with the optimum STF (dashed line). As expected from the results of the CCCs in Figure 14, the optimum STFs are reproduced almost perfectly for aftershocks 1, 3, 6, and 7, whereas the reproduction for aftershock 2 is somewhat poor, and for aftershocks 4 and 5 is even worse due to a large amount of noise. This again indicates that the reproducibility of the earthquake motion records (Fig. 13) and the optimum STF (Fig. 15) provide the same information about the accuracy of the EGTD. Notably, there is a large discrepancy in waveform between the optimum STF and the derived three STFs for aftershock 5, which suggests the necessity to replace it with another aftershock.

As mentioned earlier, the replacement is not strictly possible for the aftershocks of group A2, but it is possible approximately, fortunately, because several aftershocks in group B2 are very close to zone A2 (see Fig. 1b). Thus, we replaced aftershock 5 of group A2, which shows the worst CCC of the group, with the aftershock of group B2 that is the closest to zone A2. Hereafter, this aftershock group is referred to as the group A2'. The diameter of the group slightly increases to 1.1 km. Using group A2', we tried to demonstrate the improvement of the EGTDs by the replacement.

The CCCs of group A2' are shown in Figure 16. They are obviously improved for all three cutoff frequencies, compared with the corresponding CCCs shown in Figure 14. For group A2, the CCCs exceed 0.75 at every site only for the cutoff

Figure 11. The CCCs between east–west-component earthquake motions recorded and those synthesized using the EGTDs. The CCCs are taken for all aftershocks of group A1 at all sites listed in the left half of Table 2. (a–c) The CCCs for the cutoff frequencies of 4, 8, and 16 Hz are from the left.

frequency of 4 Hz, whereas for group A2', the CCCs at every site are 0.75 or more for the cutoff frequency of 8 and 4 Hz. This demonstrates that the replacement of aftershocks having low CCCs at all sites is one way to improve the EGTDs.

We summarize the results of groups A1 and A2 (and A2') in two points. First, the EGTDs with a cutoff frequency between 8 and 16 Hz can be estimated from earthquake motion records that are radiated from aftershocks located in a zone of approximately 1 km in diameter. Second, the accuracy of the extracted EGTD can be evaluated by reproducing either the earthquake motion records or the optimum STF.

Groups B1–B3

In the previous section, we showed that high-frequency EGTDs with a cutoff frequency of 8 Hz or higher could be estimated from the aftershocks distributed within approximate 1 km diameter zones. This raises the question of up to how large an aftershock zone the high-frequency EGTD can be estimated for. Hence, we prepared three aftershock groups with a larger zone having a 2 km diameter to answer the question.

The results of the EGTD estimation for the three groups revealed that there were many CCCs less than 0.75 in groups B2 and B3 even for the cutoff frequency of 4 Hz, whereas the CCCs of group B1 exceeded 0.75 only for that cutoff frequency. These results suggest that it is difficult to stably estimate the high-frequency EGTDs from aftershock groups with the 2 km diameter zones.

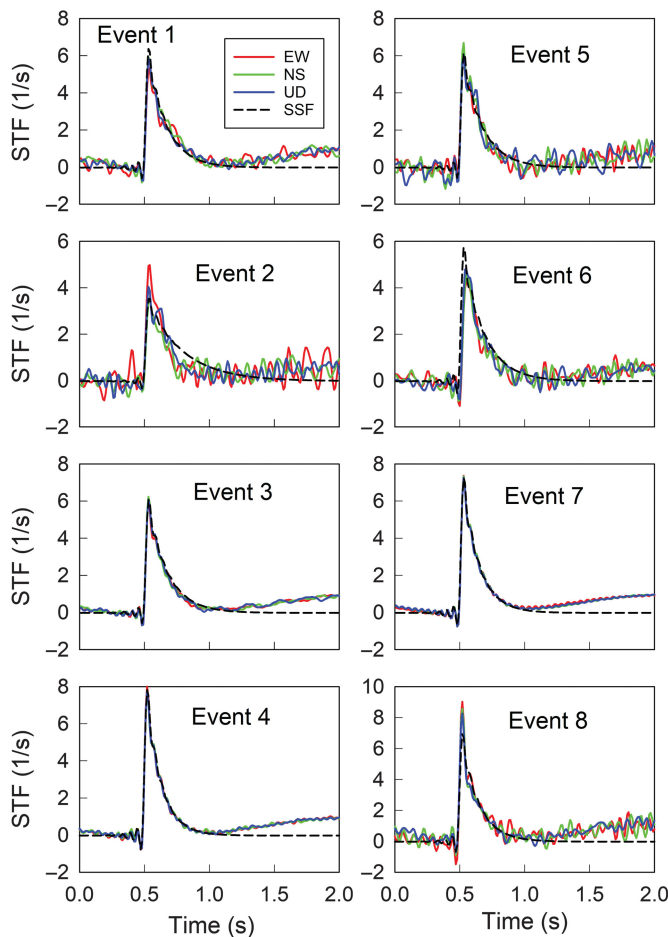


Figure 12. Comparison of the STFs derived from each of the three-component earthquake motion records of all aftershocks of group A1 at site N.TYNH with the optimum STF of SSF with the cut-off frequency of 16 Hz. The red, green, and blue lines, respectively, denote the STFs derived from the east-west-, north-south-, and up-down-component earthquake motion records low-pass-filtered below 16 Hz. The broken line is the optimum STF.

There are two reasons why the accuracy of the EGTs estimated from aftershocks in 2 km diameter zones is low. One is that the aftershock zone expands from 1 to 2 km in diameter, resulting in the limit high frequency (f_{lh}) reducing to a lower

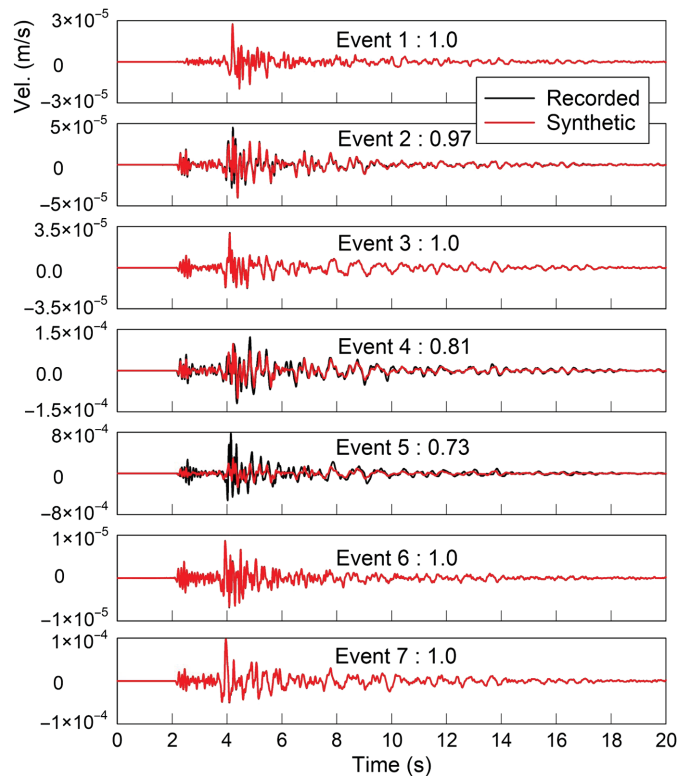


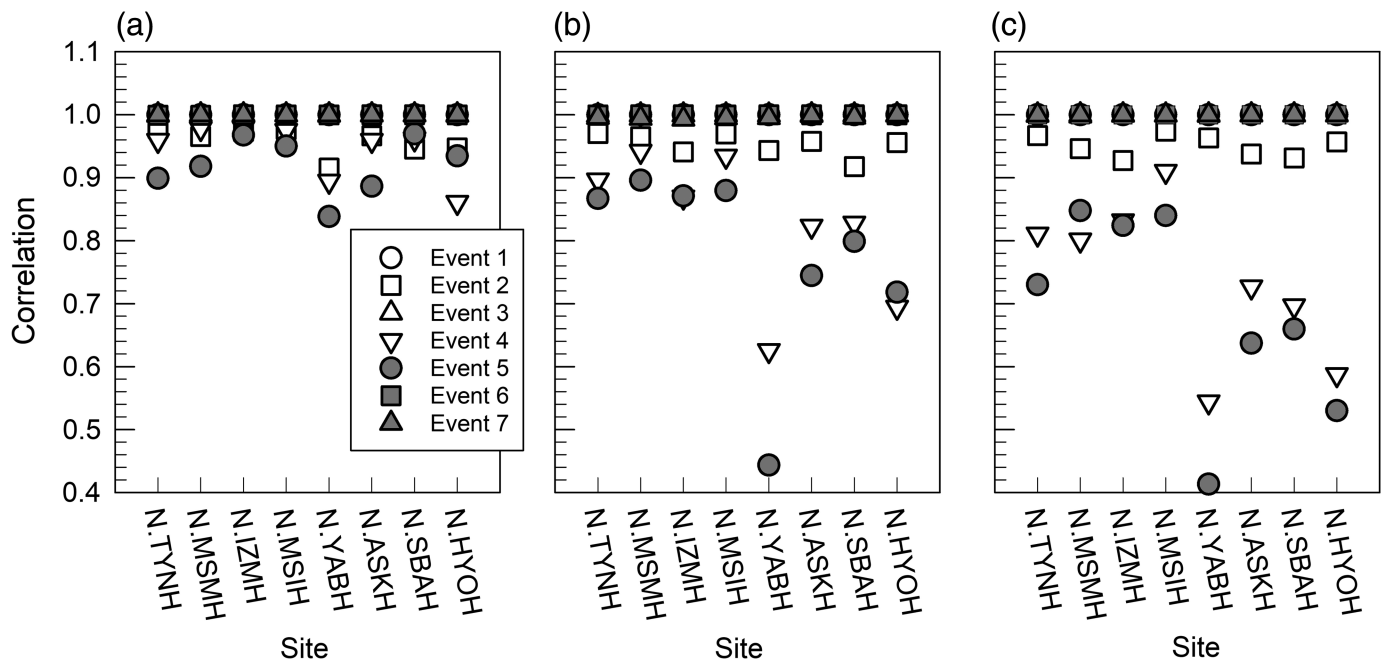
Figure 13. Comparison of recorded east-west-component earthquake motions (black lines) of the aftershocks of group A2 with earthquake motions (red lines) synthesized using the EGTs at site N.TYNH. The STF is SSF with a cut-off frequency of 16 Hz. The recorded earthquake motions are low-pass-filtered below 16 Hz. The number given for each event is the CCC between the recorded and synthesized earthquake motions.

frequency. Figure 17 shows the focal distance range in which a 2 km diameter zone can be considered as a point source for a given cutoff frequency. For example, when the cutoff frequency is 16 Hz, none of the aftershock zones can be regarded as a point source from observation sites shorter than 33.7 km in the focal distance, because the cutoff frequency f_{ch} depicted by the top dashed line lies above the limit high frequency (f_{lh}) depicted by the solid line. As shown in Table 4, the focal

TABLE 3
Source and Fault Parameters for the Aftershocks of Group A2

Event Number	M_j	M_0 (N-m)	f_c (Hz)	Latitude (°)	Longitude (°)	Depth (km)	ϕ (°)	Δ (°)	λ (°)	Quality	Dist. (km)
1	2.1	1.18×10^{13}	5.7	32.594	130.675	9.7	104	45	75	C	0.4
2	2.3	2.03×10^{13}	4.7	32.587	130.672	9.8	42	75	179	B	0.5
3	3.3	2.50×10^{14}	3.1	32.589	130.671	9.6	36	80	146	B	0.3
4	2.9	8.89×10^{13}	3.8	32.589	130.671	9.6	41	79	179	B	0.4
5	3.4	4.42×10^{14}	2.4	32.588	130.670	9.7	39	88	178	B	0.5
6	1.7	3.20×10^{12}	7.4	32.592	130.671	9.7	38	82	178	A	0
7	2.9	5.63×10^{13}	5.0	32.592	130.673	9.7	12	87	176	B	0.1

Dist. column shows the distance between each event (aftershock) and the reference event (event 6).



distances of almost all observation sites in the three groups are less than 33.7 km. Hence, the CCCs for the cutoff frequency of 16 Hz are unreliable. Similarly, in the case of the cutoff frequency of 8 Hz, the CCCs of observation sites located inside the focal distance of 16.9 km are unreliable. In the case of the cutoff frequency of 4 Hz, the sites more than 8.4 km in the focal distance can be used to estimate the EGTDs, resulting in the CCCs of all observation sites being reliable.

Therefore, the CCCs at the sites with focal distances above 16.9 km are re-examined only in the case of the cutoff frequency of 8 Hz. However, the conclusion regarding the estimation accuracy of the EGTD with the cutoff frequency of 8 Hz remains unchanged.

Another reason is that similar to group A2, each group has multiple aftershocks with low CCCs at all observation sites. Below, we mention the case of aftershock groups excluding aftershocks with low CCCs for the cutoff frequencies of 4 and 8 Hz. However, the name of the aftershock group remains unchanged. When the cutoff frequency is 4 Hz, the CCCs of all aftershocks are 0.8 or higher at all observation sites in the cases of groups B1–B3. Meanwhile, when the cutoff frequency is 8 Hz, the CCCs of all aftershocks are 0.75 or higher at all observation sites only in the case of group B2. Thus, excluding aftershocks with low CCCs at all sites from aftershock groups, the EGTD can be stably estimated with high accuracy for the cutoff frequency of 4 Hz from aftershocks located within 2 km diameter zones.

The summary of groups B1–B3 is that high-frequency EGTD cannot be stably estimated from earthquake motion records that radiated from aftershocks located within the 2 km diameter zone and recorded at sites with a focal distance of approximately 10 km or more. However, excluding aftershocks with low CCCs at all observation sites from the groups,

Figure 14. The CCCs between east–west-component earthquake motions recorded and those synthesized using the EGTDs. The legend of the symbols for the CCCs is in (a). The CCCs are taken for all aftershocks of group A2 at all sites listed in the right half of Table 2. (a–c) The cutoff frequencies of 4, 8, and 16 Hz are from the left.

the EGTDs can be estimated stably with high accuracy for the cutoff frequency of 4 Hz.

SELECTION OF THE STF

Comparison of the CCC

So far, we have adopted the SSF as the STF, but, as described in [Source and Fault Parameters and Aftershock Grouping](#) section 2, equation (3) tells us that any causal time function can be used as an alternative. However, the equation does not tell us whether the degree of the reproducibility of earthquake motions recorded by those synthesized using the EGTDs depends on the adopted STF. Hence, we investigate this research topic in this section. This topic is important in that the results make it clear whether the STF with the best reproducibility can be selected from among the currently available STFs.

We compare the reproducibility not between all currently available STFs but between the two STFs of SSF and TAF for a preliminary test. Specifically, we compare the CCCs taken from recorded and synthesized earthquake motions between the two STFs. For this topic, we use earthquake motion records from the aftershocks of groups A1 and A2 because high-frequency EGTDs were obtained from the aftershocks of the two groups alone. Because we have already taken the CCCs of groups A1 and A2 for the STF of SSF in the previous section, we take the CCCs only for the STF of TAF. The duration t_0 of TAF is half the length of the base.

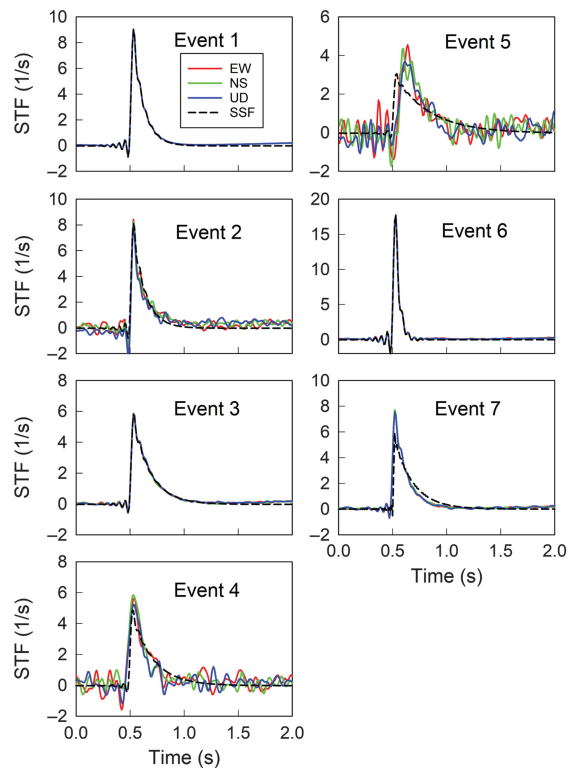


Figure 15. Comparison of the STFs derived from each of the three-component earthquake motions of all aftershocks of group A2 at site N.TYNH with the optimum STF of SSF having the cutoff frequency of 16 Hz. The red, green, and blue lines denote the STFs derived from the east-west-, north-south-, and up-down-component earthquake motion records low-pass-filtered below 16 Hz. The broken line is the optimum STF.

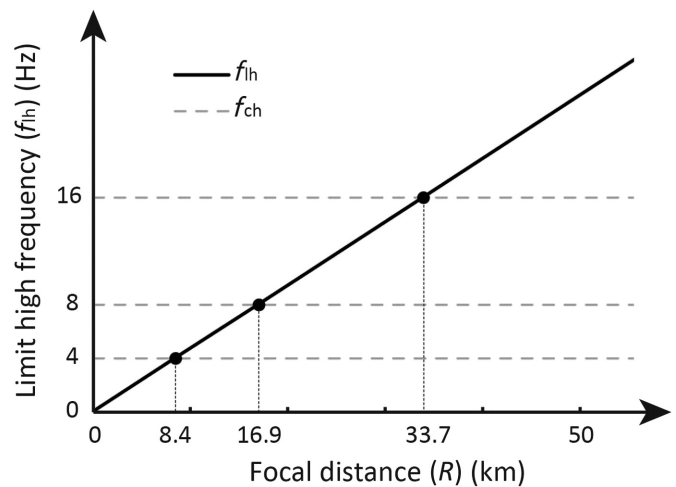


Figure 17. Diagram to determine a focal distance range to estimate the EGTs for a given cutoff frequency. The solid line denotes the limit high frequency (f_h) as a function of the focal distance, the three dashed lines are the cutoff frequencies of 4, 8, and 16 Hz, respectively, from the bottom.

We show a comparison between the earthquake motions recorded and synthesized at sites N.TYNH and N.SNWH when TAF is the STF (Fig. 18). This figure is the same as Figure 10 except that the STF is TAF instead of SSF. The synthetic motions (red lines) reproduce the recorded motions (black lines) well at the closest site N.TYNH (Fig. 18a) and the farthest site N.SNWH (Fig. 18b), indicating that the EGTs between each of the two sites and zone A1 are

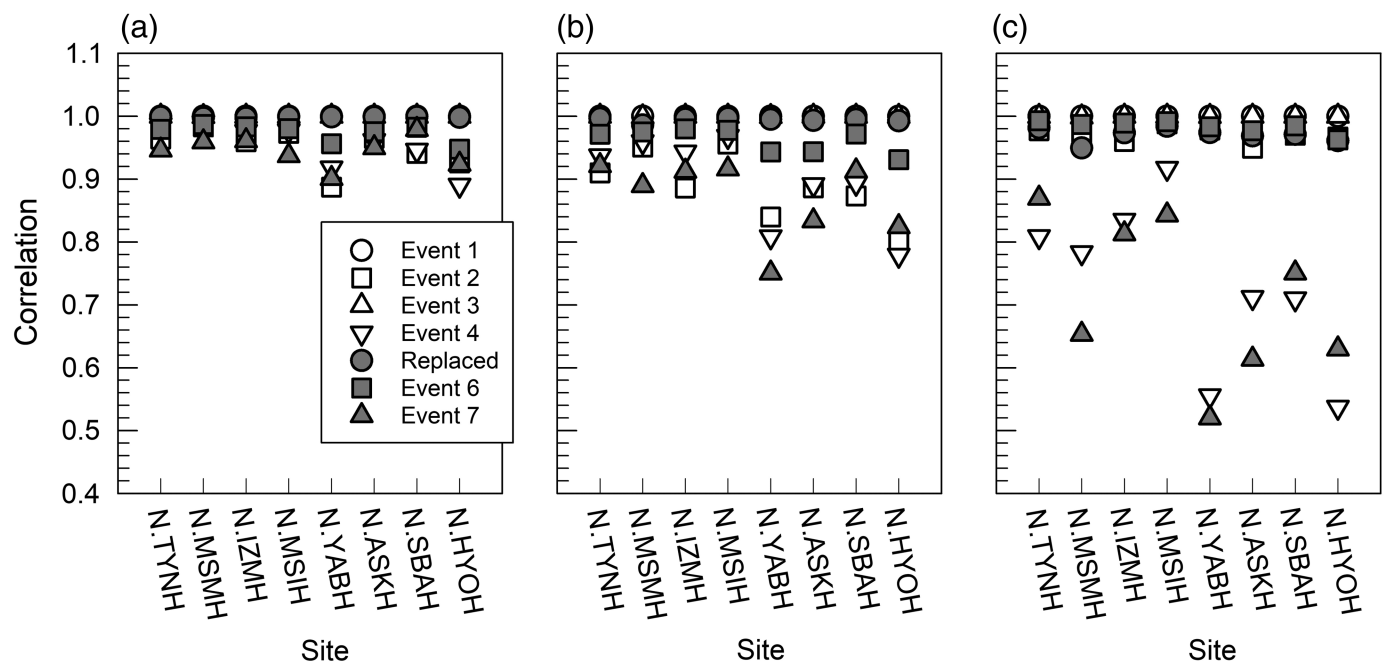


Figure 16. The CCCs between east-west-component earthquake motions recorded and those synthesized using the EGTs. The legend of the symbols for the CCCs is in (a). The CCCs are taken for all aftershocks of

group A2, which is replaced aftershock 5 with the aftershock of group B2 closest to the zone A2, at all sites listed in the right half of Table 2. (a–c) The cut-off frequency of 4, 8, and 16 Hz from the left.

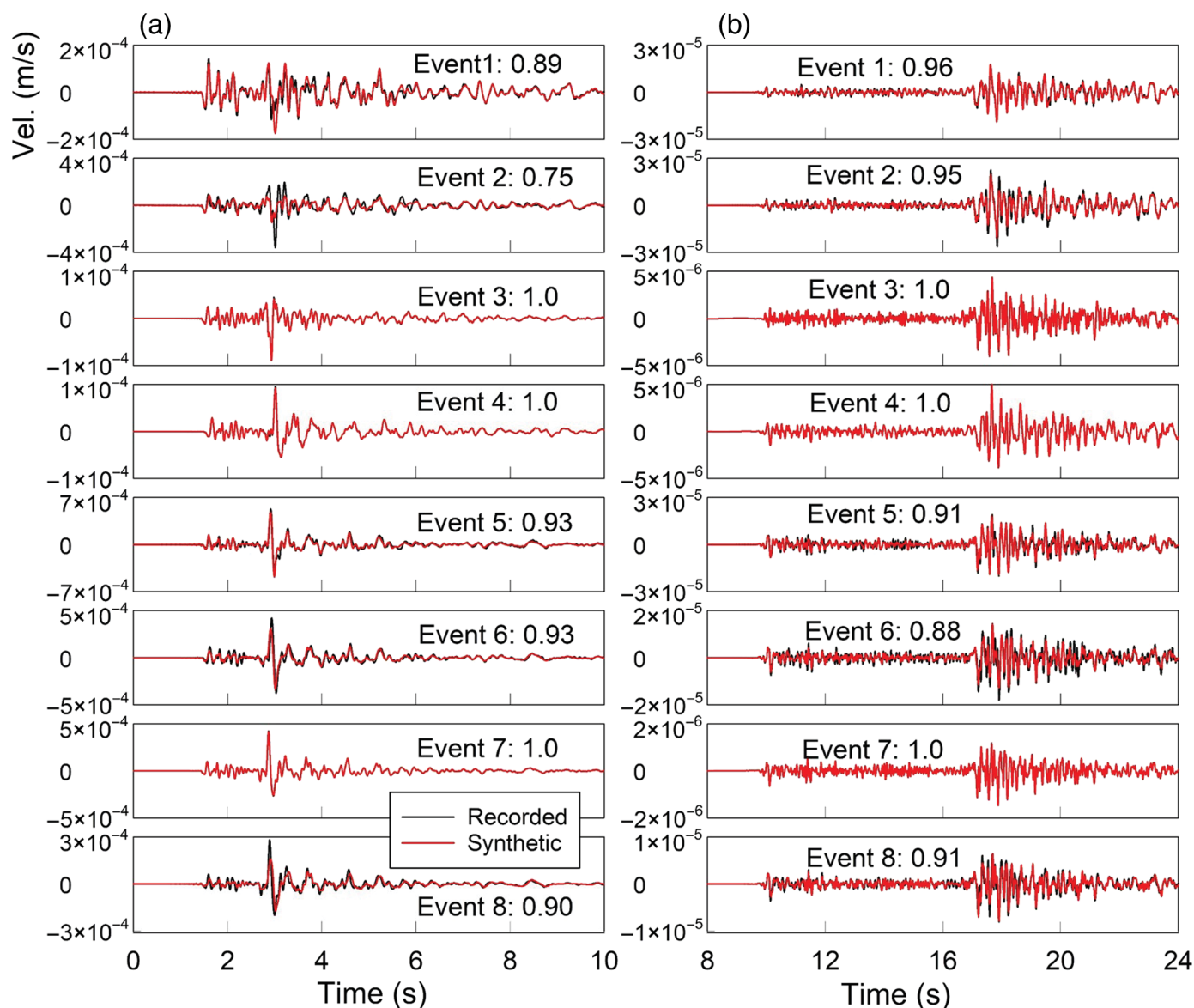
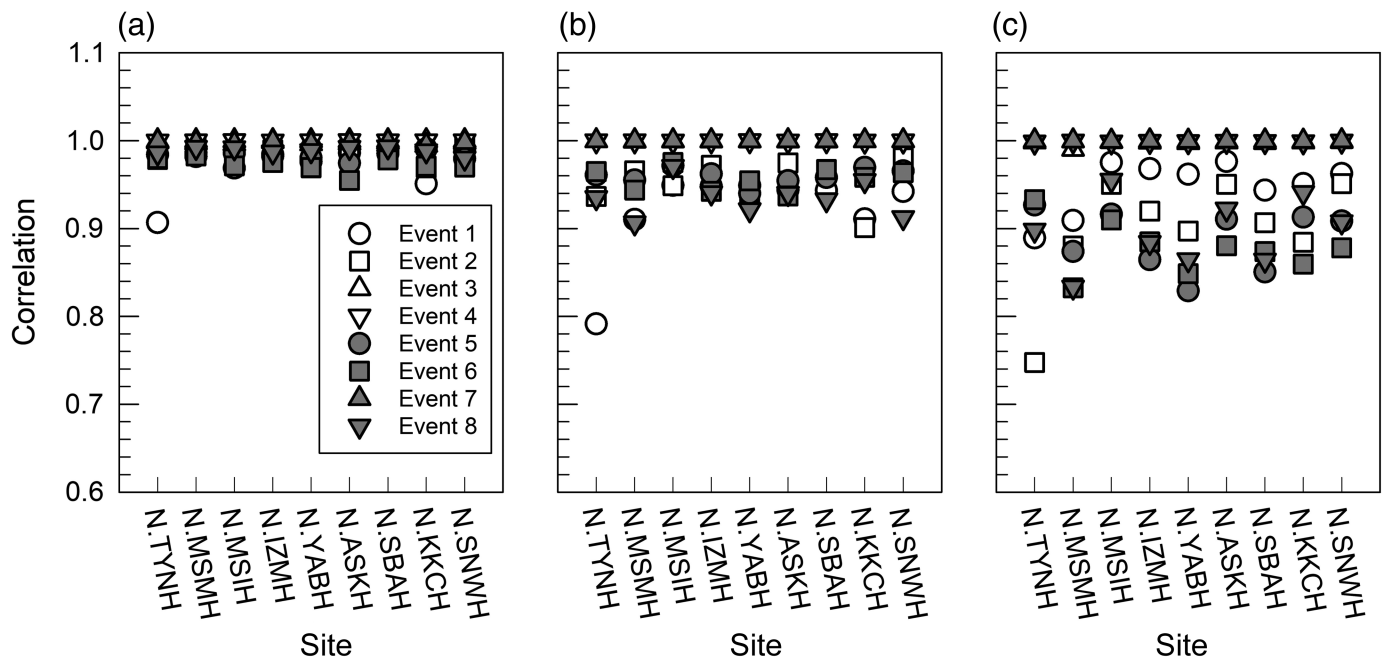


Figure 18. Comparison of recorded east-west-component earthquake motions (black lines) of the aftershocks of group A1 with earthquake motions (red lines) synthesized using the extracted EGTs. They were estimated assuming not SSF but TAF as STF. (a) At site N.TYNH, and (b) at

site N.SNWH. The recorded earthquake motions are low-pass-filtered below 16 Hz. The number given for each event is the cross-correlation coefficient (CCC) between the recorded and synthetic earthquake motions. Note that scale of the time axes is different between panels (a,b).

TABLE 4
Average Distances between the Sites and the Aftershocks of Groups B1–B3

Group B1		Group B2		Group B3	
Hi-net Site Name	Average Distance (km)	Hi-net Site Name	Average Distance (km)	Hi-net Site Name	Average Distance (km)
N.TYNH	8.8	N.MSMH	12.3	N.TYNH	12.5
N.MSMH	10.4	N.IZMH	20.5	N.MSMH	19.2
N.MSIH	20.3	N.YABH	24.6	N.MSIH	18.5
N.IZMH	23.4	N.ASKH	34.7	N.IZMH	30.3
N.YABH	22.3	N.SBAH	37.4	N.YABH	31.2
N.ASKH	40.4	N.HYOH	43.8	N.KKCH	37.2
				N.ASKH	47.3



estimated with high accuracy. This good reproduction suggests that the EGTDs may be estimated by any causal STF.

When the CCCs in Figure 18a are compared with the corresponding CCCs in Figure 10a, the CCCs in Figure 18a appear to be slightly smaller. This is also the case for the CCCs in Figure 18b. In other words, the reproducibility of the TAF is only slightly worse than the SSF at the two sites. We then examine whether this finding holds true at the other sites. Thus, let us compare the reproducibility between the STFs SSF and TAF by CCC at all sites from N.TYNH to N.SNWH. Figure 19 shows the CCC at all sites for the STF of TAF. The CCC of all sites is greater than 0.75 for the three cutoff frequencies. This indicates that the EGTDs below the cutoff frequency of 16 Hz are well estimated, as is the case when the STF is SSF. Thus, it is not easy to visually decide which of the two STFs—SSF and TAF—shows higher reproducibility. Hence, we make a quantitative comparison.

Here, we show that when the STF is TAF, the source functions derived from recorded earthquake motions provide the same information about the accuracy of the EGTDs as the CCCs. The STFs derived from the three-component records at site N.TYNH and the optimum STF are exhibited in each panel of Figure 20. The display is the same as Figure 12. For all aftershocks, the derived STFs generally match each other and match the optimum source function as well. However, the degree of agreement varies for each aftershock. The derived STFs match each other almost perfectly for aftershocks 3, 4, and 7 and match well with the optimum STF. Meanwhile, the degree of agreement is somewhat poor for aftershocks 5, 6, and 8 and even worse for aftershocks 1 and 2. A comparison of the degree of reproduction of the optimum STFs in the figure with the value of the CCC shown in Figure 18a aftershock by aftershock reveals that there is good correspondence. Thus, also in the case

Figure 19. The CCCs between east–west-component earthquake motions recorded and those synthesized using the EGTDs. They were estimated assuming not SSF but TAF as STF. The CCCs were taken for all aftershocks of group A1 at all sites listed in the left half of Table 2. (a–c) The CCCs for the cutoff frequencies of 4, 8, and 16 Hz are from the left.

of the STF of TAF, the accuracy of the estimated EGTDs can be evaluated by reproducing either the earthquake motion records or the optimum STF. In addition, we can confirm the delay (aftershock 6) and advance (aftershock 9) in this figure.

We took the ratios of the CCCs for TAF to those for SSF (Fig. 21). Although the higher the cutoff frequency, the greater the variation, the ratios are distributed around unity at every site for the three cutoff frequencies in common. Average values over the ratios of aftershocks at each site (white diamonds) are nearly unity at all sites for the cutoff frequencies of 4 and 16 Hz. The ratios for the cutoff frequency of 8 Hz look less than unity, but the average value at each site is approximately 0.97. Thus, we think that the degree of the agreement of the recorded and synthetic earthquake motions is the same for the two STFs. Furthermore, the same comparison was made for group A2, and the results were the same as for group A1. Therefore, we think that it is difficult to select the STF giving the best reproducibility among the available STFs.

Comparison of EGTD

The two STFs—SSF and TAF—are different, but, as already shown, the extracted EGTDs reproduced the recorded earthquake motions equally well. This means that the extracted EGTDs differ between the two STFs, which is unacceptable because they are influenced only by the properties of propagating media. Therefore, it is important to show the difference

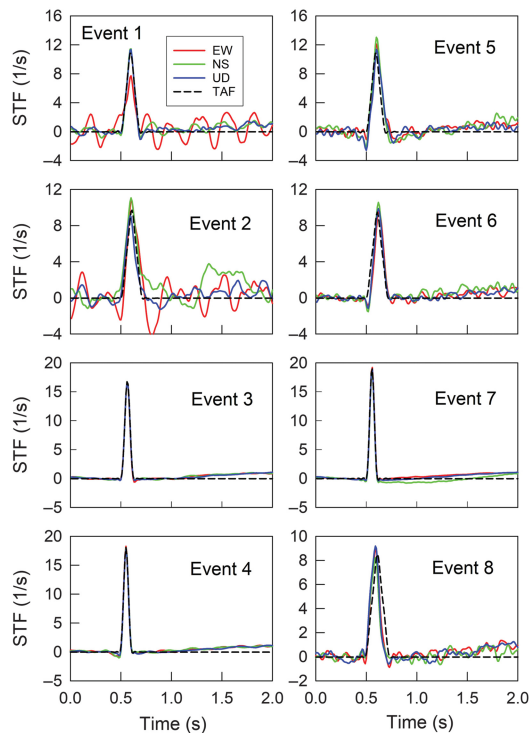


Figure 20. Comparison of the STFs derived from each of the three-component earthquake motion records of all aftershocks of group A1 at site N.TYNH with the optimum STF of TAF with the cut-off frequency of 16 Hz. The red, green, and blue lines, respectively, denote the STFs derived from the east-west-, north-south-, and up-down-component earthquake motion records lowpass-filtered below 16 Hz. The broken line is the optimum STF.

between the two types of EGTs in waveform and amplitude to understand the need for the adoption of a true STF, especially in high-frequency EGT estimation.

We show the EGTs of east-west component between site N.TYNH and zone A1 when SSF and TAF are adopted as the STFs (Fig. 22). The cutoff frequency is 4 Hz. The gray and black solid lines represent the EGTs for SSF and TAF, respectively. We can see that the two corresponding EGTs are quite different in both amplitude and waveform. The dependency of the estimated EGTs on the STF means that they should be carefully used when applied in practice. To avoid dependency, it is necessary to extract the EGTs by adopting an STF that is based on or consistent with the physics of fault rupture. Hence, an STF based on the dynamics of fault rupture is a critical issue for estimating high-frequency EGTs and deserves further studies.

DISCUSSION AND CONCLUSION

We use the EGT in two ways: reproduction of recorded earthquake motions and prediction of earthquake motions due to future earthquakes. In this study, we investigated the feasibility of estimating the high-frequency EGTs for the reproduction of ground motion from aftershocks of the 2016 Kumamoto earthquake in the JMA magnitude scale range $1.5 < M_j < 3.5$ within zones of 1 and 2 km in diameter. We summarize the results in three points.

First, we demonstrated that high-frequency EGTs can be accurately and stably estimated for the cutoff frequency between 8 and 16 Hz from earthquake ground motions that

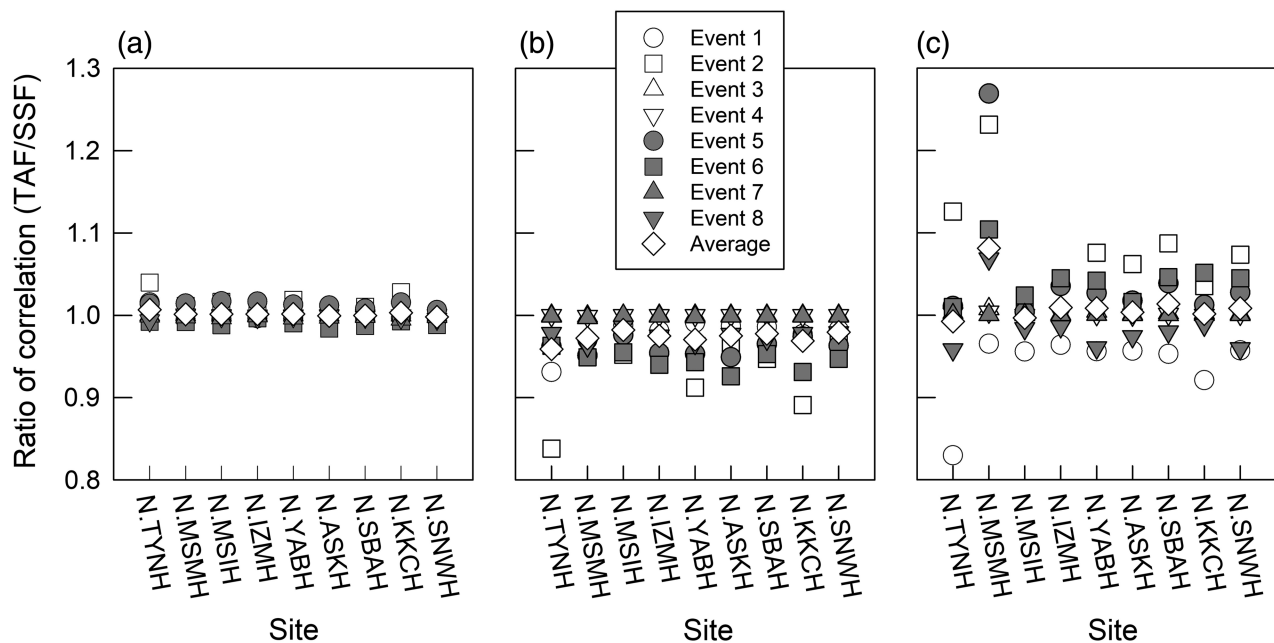


Figure 21. Ratios of the CCCs for the STF TAF are shown in Figure 19 compared to those for the STF SSF in Figure 11. (a–c) The cutoff

frequencies of 4, 8, and 16 Hz. The legend for the symbols is in (b).

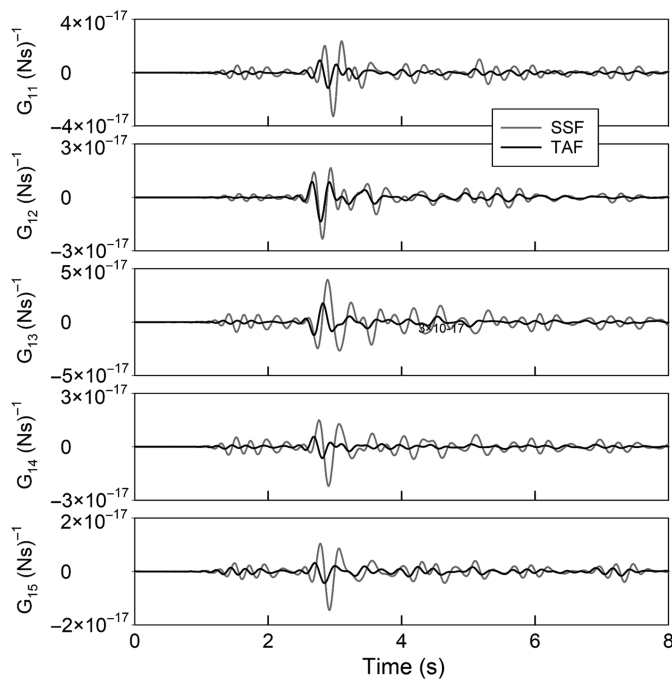


Figure 22. Comparison of two types of the EGTDs at site N.TYNH. Both are estimated from aftershocks of group A1 for the cutoff frequency of 4 Hz. The gray and black traces in each box are east–west-component EGTDs for the STF of the SSF and the TAF.

are radiated from aftershocks located within the zone of 1 km diameter and are recorded at sites distant more than 10 km from the zone, using the optimized source and fault parameters. Meanwhile, it is difficult to stably estimate them from earthquake motions radiated from aftershocks within the zone of 2 km in diameter, even for the cut-off frequency of 4 Hz. However, excluding aftershocks with low CCCs at every site from aftershock groups, the EGTDs with the cutoff frequency of 4 Hz can be stably estimated with high accuracy from aftershocks located within a 2 km diameter zone.

Second, the accuracy of the extracted EGTD can be evaluated by reproducing either earthquake motion records or the STF. Finally, the two types of EGTDs that are estimated by adopting two STFs—SSF and TAF—reproduce the earthquake motion records with equal accuracy but are quite different in waveform and amplitude from each other.

The last point indicates the following two issues: difficulty in selecting the STF giving the best reproducibility among currently available STFs based on the reproduction accuracy of earthquake motion records and the necessity to adopt an STF that is based on or consistent with the dynamics of fault rupture such as the regularized Yoffe function proposed by Tinti, Fukuyama, *et al.* (2005).

Our study indicates that it is possible to stably obtain high-frequency EGTDs from observed earthquake motions and is a promising tool for future studies on attenuation,

amplification, and scattering along the propagation path. Our results also stimulate us to proceed with the next issues such as the prediction of strong ground motions for future earthquakes and the kinematic inversion using high-frequency EGTD.

In the verification of extracted EGTDs, we found a few aftershocks with low reproducibility at all sites. Examples are aftershock 2 in group A1 (see Fig. 10) and aftershocks 4 and 5 in group A2 (see Fig. 13). Elucidating the causes of poor reproducibility of these aftershocks is a prominent issue for more accurate high-frequency EGTD extraction.

Low reproducibility at all sites means that the causes lie in earthquake sources. One potential cause is that the GA did not find the source and fault parameters accurately enough because the search ranges of these parameters are too narrow. Thus, we doubled the search ranges of the five parameters, but the CCCs of aftershock 2 in group A1 and aftershocks 4 and 5 in group A2 were hardly improved as a result.

Another potential cause is that we did not optimize the epicenter location and the time of occurrence by the GA so there may be still errors in them. We added only the occurrence time to the GA parameters and searched the optimum values of the six parameters by the GA, but here again, there was little improvement in the CCCs. We precluded the epicenter location from the search parameters because when it was included in them, some of the aftershocks were excluded from the aftershock group due to migration of the hypocenters.

After all, the changes in the search range and the search parameters in the GA do not contribute to the improvement of the EGTDs at all. This suggests that more elaborate expedition methods are required to obtain more accurate parameters.

DATA AND RESOURCES

We collected earthquake data sourced from the Japan Meteorological Agency (JMA) Unified Earthquake Catalogue available at <https://hinetwww11.bosai.go.jp/auth/JMA/?LANG=en> (last accessed July 2017). We downloaded waveform data from the National Research Institute for Earth Science and Disaster Resilience (NIED) and JMA from <https://hinetwww11.bosai.go.jp/auth/download/cont/?LANG=en> (last accessed November 2018). Figures 1 and 3 were made using the Generic Mapping Tool (GMT) developed by Wessel and Smith (doi: 10.1029/98EO00426). The other codes used in this article came from published sources listed in the references.

DECLARATION OF COMPETING INTERESTS

The authors acknowledge that there are no conflicts of interest recorded.

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REFERENCES

- Aki, K., and P. G. Richards (1980). *Quantitative Seismology. Theory and Methods*, W. H. Freeman and Company, San Francisco, California.
- Andrews, D. J. (1986). Objective determination of source parameters and similarity of earthquakes of different size, in *Earthquake Source Mechanics*, S. Das, J. Boatwright, and C. H. Scholz (Editors), American Geophysical Union, Washington, D.C., 259–267.
- Brune, J. N. (1970). Tectonic stress and the spectra of seismic shear waves from earthquakes, *J. Geophys. Res.* **75**, 4997–5009, doi: [10.1029/JB075i026p04997](https://doi.org/10.1029/JB075i026p04997).
- Frohlich, C. (1992). Triangle diagrams: Ternary graphs to display similarity and diversity of earthquake focal mechanisms, *Phys. Earth Planet. In.* **75**, 193–198, doi: [10.1016/0031-9201\(92\)90130-N](https://doi.org/10.1016/0031-9201(92)90130-N).
- Hardebeck, J. L., and P. M. Shearer (2002). A new method for determining first-motion focal mechanisms, *Bull. Seismol. Soc. Am.* **92**, 2264–2276, doi: [10.1785/0120010200](https://doi.org/10.1785/0120010200).
- Hardebeck, J. L., and P. M. Shearer (2008). HASH: A FORTRAN program for computing earthquake first-motion focal mechanism-V1.2-January 31, available at <http://quake.wr.usgs.gov/research/software/#HASH> (last accessed May 2014).
- Hartzell, S. (1992). Site response estimation from earthquake data, *Bull. Seismol. Soc. Am.* **82**, 2308–2337.
- Hirata, N., and M. Matsu'ura (1987). Maximum-likelihood estimation of hypocenter with origin time eliminated using nonlinear inversion technique, *Phys. Earth Planet. In.* **47**, 50–61.
- Ide, S., and M. Takeo (1997). Determination of constitutive relations of fault slip based on seismic wave analysis, *J. Geophys. Res.* **102**, 379–391, doi: [10.1029/97JB0265](https://doi.org/10.1029/97JB0265).
- Ito, Y., T. Okada, T. Matsuzawa, N. Umino, and A. Hasegawa (2001). Estimation of stress tensor using aftershocks of 15 September 1998 M5.0 Sendai, NE Japan, earthquake, *Bull. Earthq. Res. Inst.* **76**, 51–59, doi: [10.15083/0000032626](https://doi.org/10.15083/0000032626).
- Mikumo, T. (1994). Dynamic fault rupture of moderate size earthquakes inferred from the results of kinematic wave form inversion, *Ann. Geophys.* **37**, 1377–1389, doi: [10.4401/AG-4140](https://doi.org/10.4401/AG-4140).
- Ohori, M., and Y. Hisada (2011). Comparison of the empirical Green's spatial derivative method and empirical Green's function method, *Bull. Seismol. Soc. Am.* **191**, 2872–2886, doi: [10.1785/0120100114](https://doi.org/10.1785/0120100114).
- Plicka, V., and J. Zahradnik (1998). Inverting seismograms of weak events for empirical Green's tensor derivatives, *Geophys. J. Int.* **132**, 471–478, doi: [10.1046/J.1365-246X.1998.00513](https://doi.org/10.1046/J.1365-246X.1998.00513).
- Pulido, N., and K. Irikura (2000). Estimation of dynamic rupture parameters from the radiated seismic energy and apparent stress, *Geophys. Res. Lett.* **27**, no. 23, 3945–3948, doi: [10.1029/2000GL011658](https://doi.org/10.1029/2000GL011658).
- Pulido, N., L. Dalguer, and H. Fujiwara (2006). Strong motion simulation on a dynamic fault rupture process and empirical Green's tensor derivatives, *Fall Meeting of the Seismological Society of Japan*, D018 pp.
- The Research Group for the 2003 Hinagu Fault Seismic Expedition (2008). Seismic expedition in the Hinagu fault area, Kyushu Island, Japan, *Bull. Earthq. Res. Inst. Univ. Tokyo* **83**, 103–130, doi: [10.15083/0000032461](https://doi.org/10.15083/0000032461) (in Japanese).
- Tinti, E., E. Fukuyama, P. Alessio, and M. Cocco (2005). A kinematic source-time function compatible with earthquake dynamics, *Bull. Seismol. Soc. Am.* **95**, 1211–1223, doi: [10.1785/0120040177](https://doi.org/10.1785/0120040177).
- Tinti, E., P. Spudich, and M. Cocco (2005). Earthquake fracture energy inferred from kinematic models on extended faults, *J. Geophys. Res.* **110**, no. B12, doi: [10.1029/2005JB003644](https://doi.org/10.1029/2005JB003644).
- Uchide, T., and K. Imanishi (2018). Underestimation of microearthquake size by the magnitude scale of the Japan Meteorological Agency: Influence on earthquake statistics, *J. Geophys. Res.* **123**, 606–620, doi: [10.1002/2017jb014697](https://doi.org/10.1002/2017jb014697).
- Vavryčuk, V., and P. Adamová (2020). Non-double-couple moment tensors of earthquake calculated using empirical Green's functions, *Seismol. Res. Lett.* **91**, 390–398, doi: [10.1785/0220190154](https://doi.org/10.1785/0220190154).
- Waldhauser, F., and W. L. Ellsworth (2000). A double-difference earthquake location algorithm: Method and application to the northern Hayward fault, *Bull. Seismol. Soc. Am.* **90**, 1353–1368, doi: [10.1785/0120000006](https://doi.org/10.1785/0120000006).

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