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Supporting Information for

Bayesian self-adapting fault slip inversion with Green's functions uncertainty and application on the 2016 M_w7.1 Kumamoto earthquake

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Introduction

This file of Supporting Information contains additional mathematical formulation supporting the utilized acceptance probabilities of the trans-dimensional steps. And Fig. S1 includes results from the "dry-run" inversion test for the SIV benchmark settings with fixed (constant) misfit function. This standard test reveals the effective priors on some model parameters of the finite-extent source, including marginal prior PDF on slip.

Text S1. Metropolis-Hastings acceptance probabilities of trans-dimensional steps

The Metropolis-Hastings acceptance probability is the key to ensure that the samples of the MCMC follow the target posterior probability density $p(\boldsymbol{m}|\boldsymbol{d_{obs}})$. Green (1995, 2003) show that the chain of model samples converges to the trans-dimensional posterior PDF if

$$\alpha(\boldsymbol{m} \to \boldsymbol{m}') = \min\left(1, \frac{p(k')}{p(k)} \frac{p(\boldsymbol{w'}_{k'}|k')}{p(\boldsymbol{w}_k|k)} \frac{p(\boldsymbol{d_{obs}}|\boldsymbol{m}')}{p(\boldsymbol{d_{obs}}|\boldsymbol{m})} \frac{q(\boldsymbol{m}|\boldsymbol{m}')}{q(\boldsymbol{m}'|\boldsymbol{m})} |\mathbf{J}|\right). \quad \text{(S1)}$$

See Eq. (14) in the main text for the description of the quantities. The acceptance probabilities for the "birth" and "death" trans-dimensional moves $\alpha_B(\boldsymbol{m} \to \boldsymbol{m}')$ and $\alpha_D(\boldsymbol{m} \to \boldsymbol{m}')$ are given in analogy to Sambridge et al. (2006) by Eqs (17) and (18) in the main text. Here we perform a simplification of these general equations concerning our particular formulation.

Let us first define some auxiliary probabilities and probability densities. The probability density of birth (spontaneous occurrence) of a new spline point on a random location on the fault of finite-size $x_\Sigma \times y_\Sigma$ (independent of k) can be expressed as $(x_\Sigma y_\Sigma)^{-1}$. However, for mathematical convenience, let us for a moment assume that the spline points can occur on an underlying grid of *N* finite positions (as in Bodin et al., 2012). Then, for k spline points, there are $\frac{N!}{k!(N-k)!}$ possible configurations on the underlying grid. And then, the probability on having a given configuration is its inverse. The probability of death (removal) of a given spline point from all k's present is then k^{-1}, while the probability of birth (add one from $N-k$ possible) is $(N-k)^{-1}$. This allows us to easily formulate probabilities of "birth" and "death" moves of random spline points. Nevertheless, it is merely for mathematical convenience, and in the practical application, there is no need to use an actual underlying discrete grid in generating spline point positions (for details see Bodin & Sambridge, 2009; Bodin et al., 2012; Sambridge et al., 2006).

Further, the prior PDFs in Eqs (17) and (18) can be simplified assuming efficiently homogenous prior PDF (i.e., a weakly informative proper prior) on model parameters under the same state, i.e., $p(\boldsymbol{w'}_k|k) = p(\boldsymbol{w}_k|k)$. Then, the prior PDF on model parameters under the state $k+1$ is the prior PDF at state k augmented by the prior of a random occurrence of one spline point on the fault, i.e., $p(\boldsymbol{w'}_{k+1}|k+1) = p(\boldsymbol{w}_k|k)(x_\Sigma y_\Sigma)^{-1}$.

<u>Birth trans-dimensional move:</u>

For the birth move, i.e., $\boldsymbol{m} = (k, \boldsymbol{w}_k) \xrightarrow{birth} \boldsymbol{m'} = (k+1, \boldsymbol{w'}_{k+1})$, the prior and proposal terms in Eq. (17) can be expresed as

$$\frac{\text{Prior}(\boldsymbol{w}')}{\text{Prior}(\boldsymbol{w})} \equiv \frac{p(\boldsymbol{w'}_{k+1}|k+1)}{p(\boldsymbol{w}_k|k)} = \frac{p(\boldsymbol{w}_k|k)\frac{1}{x_\Sigma y_\Sigma}\frac{\frac{1}{N!}(k+1)!(N-k-1)!}{\frac{1}{N!}k!(N-k)!}}{p(\boldsymbol{w}_k|k)} = \frac{1}{x_\Sigma y_\Sigma}\frac{k+1}{N-k} \quad \text{(S2)}$$

$$\frac{\text{Proposal}(\boldsymbol{m}'\to\boldsymbol{m})}{\text{Proposal}(\boldsymbol{m}\to\boldsymbol{m}')} \equiv \frac{q^D_{k+1}}{q^B_k} = \frac{1}{\frac{1}{x_\Sigma y_\Sigma}}\frac{\frac{1}{k+1}}{\frac{1}{N-k}} = \frac{x_\Sigma y_\Sigma}{1}\frac{N-k}{k+1} \quad \text{(S3)}$$

$$\frac{\text{Prior}(\boldsymbol{w}')\cdot\text{Proposal}(\boldsymbol{m}'\to\boldsymbol{m})}{\text{Prior}(\boldsymbol{w})\cdot\text{Proposal}(\boldsymbol{m}\to\boldsymbol{m}')} \equiv \frac{p(\boldsymbol{w'}_{k+1}|k+1)}{p(\boldsymbol{w}_k|k)}\frac{q^D_{k+1}}{q^B_k} = \frac{x_\Sigma y_\Sigma}{x_\Sigma y_\Sigma}\frac{k+1}{N-k}\frac{N-k}{k+1} = 1 \quad \text{(S4)}$$

In Eq. (S4) we show that there is a balance between prior and proposal in the "birth" move. The final acceptance probability for the "birth" move $\alpha_B(\boldsymbol{m} \to \boldsymbol{m}')$ is

$$\alpha_B(\boldsymbol{m} \to \boldsymbol{m}') = \min\left(1, \frac{p(k+1)}{p(k)} \frac{p(\boldsymbol{d_{obs}}|\boldsymbol{m}')}{p(\boldsymbol{d_{obs}}|\boldsymbol{m})}\right), \quad \text{(S5)}$$

where $p(\boldsymbol{d_{obs}}|\boldsymbol{m})$ is the likelihood function and $p(k)$ is the prior probability on model states. We prescribe it as a reciprocal distribution $p(k) \propto k^{-1}$, working as Occam's razor. Such distribution would result in improper probability in $\mathbb{R}$; however, it is proper (i.e., normalizable to 1) in our case of a countable collection of model states $k \in \mathcal{K}$. Then,

$$\alpha_B(\boldsymbol{m} \to \boldsymbol{m}') = \min\left(1, \frac{k}{k+1} \ \frac{p(\boldsymbol{d_{obs}}|\boldsymbol{m'})}{p(\boldsymbol{d_{obs}}|\boldsymbol{m})}\right). \qquad \text{(S6)}$$

Death trans-dimensional move:

For the death move, i.e., $\boldsymbol{m} = (k, \boldsymbol{w}_k) \xrightarrow{death} \boldsymbol{m}' = (k-1, \boldsymbol{w}'_{k-1})$, the prior and proposal terms in Eq. (18) can be expresed as

$$\frac{\text{Prior}(\boldsymbol{w'})}{\text{Prior}(\boldsymbol{w})} \equiv \frac{p(\boldsymbol{w'}_{k-1}|k-1)}{p(\boldsymbol{w}_k|k)} = \frac{p(\boldsymbol{w'}_k|k)}{p(\boldsymbol{w'}_k|k)\frac{1}{x_\Sigma y_\Sigma}} \frac{\frac{1}{N!}(k-1)!(N-k+1)!}{\frac{1}{N!}k!(N-k)!} = \frac{x_\Sigma y_\Sigma}{1} \frac{N-k+1}{k} \qquad \text{(S7)}$$

$$\frac{\text{Proposal}(\boldsymbol{m'} \to \boldsymbol{m})}{\text{Proposal}(\boldsymbol{m} \to \boldsymbol{m'})} \equiv \frac{q^B_{k-1}}{q^D_k} = \frac{\frac{1}{x_\Sigma y_\Sigma}}{1} \frac{\frac{1}{N-k+1}}{\frac{1}{k}} = \frac{1}{x_\Sigma y_\Sigma} \frac{k}{N-k+1} \qquad \text{(S8)}$$

$$\frac{\text{Prior}(\boldsymbol{w'}) \cdot \text{Proposal}(\boldsymbol{m'} \to \boldsymbol{m})}{\text{Prior}(\boldsymbol{w}) \cdot \text{Proposal}(\boldsymbol{m} \to \boldsymbol{m'})} \equiv \frac{p(\boldsymbol{w'}_{k-1}|k-1)}{p(\boldsymbol{w}_k|k)} \frac{q^B_{k-1}}{q^D_k} = \frac{x_\Sigma y_\Sigma}{x_\Sigma y_\Sigma} \frac{N-k+1}{k} \frac{k}{N-k+1} = 1 \qquad \text{(S9)}$$

In Eq. (S9) we show that there is a balance between prior and proposal in the "death" move. The final acceptance probability for the "death" move $\alpha_D(\boldsymbol{m} \to \boldsymbol{m}')$ is

$$\alpha_D(\boldsymbol{m} \to \boldsymbol{m}') = \min\left(1, \frac{p(k-1)}{p(k)} \ \frac{p(\boldsymbol{d_{obs}}|\boldsymbol{m'})}{p(\boldsymbol{d_{obs}}|\boldsymbol{m})}\right). \qquad \text{(S10)}$$

Finally, as for the "birth" move, we prescribe the reciprocal prior on model states $p(k) \propto k^{-1}$ working as Occam's razor, and then Eq. (S10) leads to

$$\alpha_D(\boldsymbol{m} \to \boldsymbol{m}') = \min\left(1, \frac{k}{k-1} \ \frac{p(\boldsymbol{d_{obs}}|\boldsymbol{m'})}{p(\boldsymbol{d_{obs}}|\boldsymbol{m})}\right). \qquad \text{(S11)}$$

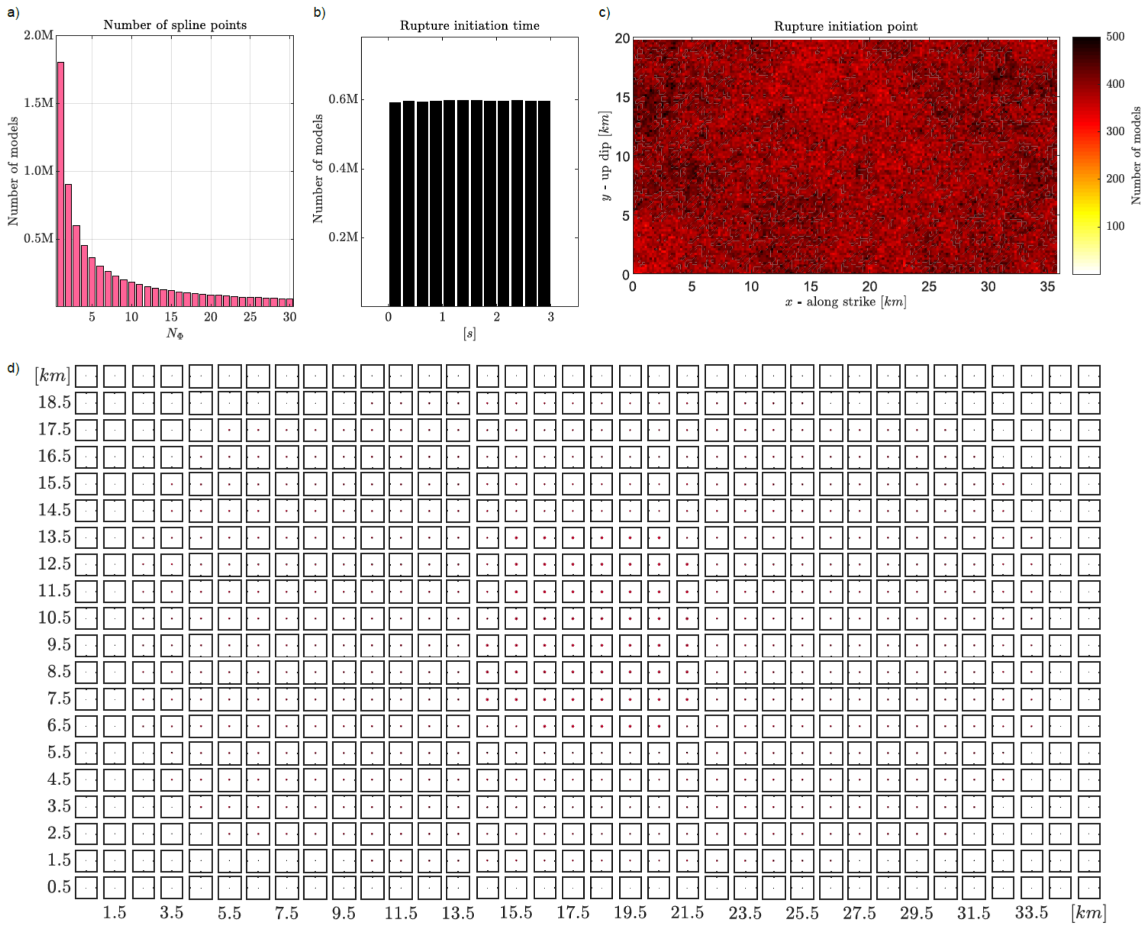

Figure S1. The effective prior from the "dry-run" test (SIV benchmark). This figure shows the results of an additional test with the SIV benchmark settings, where we fixed the misfit function to a constant (dry-run test without data). Such a test reveals the effective prior of model parameters. Panel **a)** shows a histogram of the number of spline points (i.e., the prescribed prior reciprocal distribution $p(N_\Phi) \propto {N_\Phi}^{-1}$, working as Occam's razor). Panels **b)** and **c)** show efficiently homogenous prior probabilities on rupture initiation time and position, respectively. Panel **d)** shows marginal prior on the spatial distribution of the slip on the fault (RPH plot with slip axes limits [-1,1] *m*). Note that the marginal prior on slip is close to zero but weakly concave in the middle of the fault.