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Abstract  Two possible extensions of the commented article are suggested, with the aim to more efficiently couple the source and path simulation on one side, and the site-effect simulation on the other: (1) New excitation lines, surrounding the sedimentary basin, reduce the necessary size of the 2D finite-difference model of the site. They also better represent the 3D nature of the excitation, including the focal-mechanism effect. (2) New (2.5D approximate) equations of motion improve the way the 3D excitation is propagated within the 2D site model. The two suggested improvements can help in solving problems when a purely 2D treatment is oversimplified, but a true 3D treatment is too expensive.

Introduction

Computer simulations of earthquake ground motions at sedimentary basins are of great practical value. However, often the excitations by plane waves that are used are oversimplified. In this light, the discussed article represents a significant step toward more realistic models. It enables 2D finite-difference (FD) models to be excited by wave motions obtained from the independent modal summation (MS) method. This method provides complete wave fields, including surface waves, generated in a horizontally layered crustal structure by a point source of an arbitrary focal mechanism. The applications presented by Fäh et al. (1994) are convincing and encouraging. They stimulated me to discuss possible extensions of their method.

Fäh et al. (1994) perform 2D FD simulations in a plane, say x-z. The source in their MS method is a true 3D point source, located in the same x-z plane. No sources located off the x-z plane are possible. Coupling between the MS and FD solutions is by the algorithm of Alterman and Karal (1968), employed along two vertical excitation lines placed “in front” of the investigated basin (Fig. 1a). As emphasized by Fäh et al. (1994), the excitation lines (and, therefore, the entire FD model) should be deep enough to properly illuminate the basin by the incoming wave field. Also mentioned by the authors is another drawback of their excitation method, that the more distant receivers (e.g., X' in Fig. 1a) do not get enough information about the 3D nature of the incoming wave field, e.g., about the 3D spreading taking place between X and X'.

Therefore, our objectives are as follows: (1) to improve the input of the 3D source and path effects into the FD model, including also (at least approximately) the sources located off the FD x-z plane, and (2) to improve the way the 3D input motion is propagated within the FD model. For these purposes we suggest new excitation lines and new equations of motion.

New Excitation Lines

We suggest the lines surrounding the sedimentary basin as in Figure 1b, defining two regions, A and B. We assume the complete wave field in the presence of the basin, \( \tilde{\eta} \), to be represented as \( \tilde{\eta} = \tilde{\eta}^k + \tilde{\eta}^r \). Here, \( \tilde{\eta}^k \) is a known part that would exist in the “background” half-space in absence of the basin (the free-surface effects included), and \( \tilde{\eta}^r \) is the remaining (scattered) part. The algorithm of Alterman and Karal (1968) makes it then possible to compute \( \tilde{\eta} \) and \( \tilde{\eta}^r \) by the FD method in regions A and B, respectively. For this purpose, \( \tilde{\eta}^k \) (not \( \tilde{\eta} \)) must be known on the excitation lines. The \( \tilde{\eta}^k \) can be computed by any method properly reflecting the assumed half-space structure, e.g., the MS method in case of a 1D (layered) half-space, the FD method for the 2D half-space, the ray method for the 3D one. It is essential that region A contains all “sources” of the scattered wave field \( \tilde{\eta}^r \). (That is why a pure incidence wave cannot be taken for \( \tilde{\eta}^k \).) However, as the only source of \( \tilde{\eta}^r \) is the basin, region A can be the smallest rectangle including the basin.

Therefore, with the new excitation lines as in Figure 1b, the FD models can be shallower than in the case of Figure 1a. Moreover, the 3D nature of the excitation between receivers X and X' is taken into account better than in Figure 1a. Note also that, at least in some models, \( \tilde{\eta}^r \) is relatively weak at the outer edges of the FD region B. For example, this might be the case of a soft basin on a compact bedrock. Then, taking into account that in region B of the FD model \( \tilde{\eta}^r \) only is computed, some models with the excitation lines
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of Figure 1b could perhaps be treated with the approximate condition \( \tilde{r} = 0 \) on the “left,” “right,” and “bottom” edges of the FD region B, avoiding the need for the always problematic “nonreflecting” conditions.

Anyway, our main stimulation for the new excitation lines is the challenge of the off-plane source (Figs. 1c and 1d). Importantly, the lines surrounding the basin enable prescribing the excitations that, for different points in the \( x-z \) plane, include their different hypocentral distances \( D \), and different azimuths, i.e., properly reflect the 3D spreading as well as the focal-mechanism effect. Although exciting the basin in the \( x-z \) plane only, the moving load does simulate the off-plane source (Pedersen et al., 1994). As the FD method requires the excitation at a relatively fine space and time grid, which is impractical (time consuming when computing \( \tilde{n}^2 \), and memory consuming in the FD method), a suitable interpolation of \( \tilde{n}^2 \) is useful.

New Equations of Motion

Having the input motion prescribed in the \( x-z \) plane, we face the problem of how to propagate it in that plane, keeping some of its 3D features, while the medium in the FD region is 2D. If a second-order FD method is used, we wish to consider three planes only (Fig. 2), \( y = 0 \), and \( \pm \Delta y \), while the wave field at \( \pm \Delta y \) has to have a proper connection with the field at \( y = 0 \). We suggest employing the following approximation. Consider hypocentral distances \( D \) comparable to the length \( L \) of the FD cross section, \( D \sim L \). The incoming wave field approaches different points of the FD region with different incidence angles and azimuths, hence also the \( y \) component of the slowness varies with \( x \) and \( z \), \( s = s(x, 0, z) \). On the other hand, as \( \Delta y \ll D \), the slownesses at \( \pm \Delta y \) are nearly identical, and \( s(x, \pm \Delta y, z) \approx s(x, 0, z) \). Then, any of the displacement components \( u, v, w \) at a point \( (x, \pm \Delta y, z) \) can be locally approximated by a pure shift, e.g., \( u(x, \pm \Delta y, z; t) = u(x, 0, z; t \pm \Delta y s(x, 0, z)) \), and the differentiation with respect to \( y \) transforms into the time differentiation: \( \partial u(x, 0, z; t) / \partial y = u(x, 0, z; t) s(x, 0, z) \), etc. for \( v \) and \( w \). We assume the same also for any \( x \) and \( z \) derivatives (\( \partial u / \partial x, \partial w / \partial z \), etc.) whenever they have to be differentiated with respect to \( y \). An obvious simplification comes with a very distant source, \( D >> L \), where a single slowness value \( s \) independent of \( x, z \) can be used.

At this moment, recalling the assumed independency of the Lamé parameters \( \lambda, \mu \), and density \( \rho \) on \( y \), the 3D equations of motion (e.g., Frankel and Vidale, 1992) simplify a lot. Starting with the equation updating \( u \), the only terms appearing in addition to those of the 2D equations are as follows:

\[
\frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + \mu \frac{\partial s}{\partial y},
\]

\[
\frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial x} + \mu \frac{\partial v}{\partial z} \right) = \mu \frac{\partial v}{\partial x} + \mu \frac{\partial v}{\partial z},
\]

\[
\frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial y} \right) + \lambda \frac{\partial v}{\partial y} + \lambda \frac{\partial v}{\partial z} = \frac{\partial (\lambda v)}{\partial z} s.
\]

Three and five analogous terms also appear in the update of

Figure 1. Schematic illustration of the model excitation: (a) the old one; (b) through (d) the suggested new one.

Figure 2. Schematic illustration of the approximate 2D computation partially accounting for the 3D wave features.
$w$ and $v$, respectively. Finally, the 3D equations of motion get the following form:

\[
\begin{align*}
(p - \mu s^2)\ddot{u} &= f_{2D}(u, w; x, z) + \mu \frac{\partial^2}{\partial x^2} \ddot{u} + \left( \frac{\partial^2}{\partial x \partial z} \ddot{u} \right), \\
(p - \lambda s^2)\ddot{w} &= f_{2D}(u, w; x, z) + \mu \frac{\partial^2}{\partial z^2} \ddot{w} + \left( \frac{\partial^2}{\partial x \partial z} \ddot{w} \right), \\
(p - (\lambda + 2\mu)s^2)\ddot{v} &= f_{2D}(v; x, z) + \left( \lambda + \frac{\partial^2}{\partial x^2} + 4 \frac{\partial^2}{\partial z^2} \right) \ddot{v}.
\end{align*}
\]

Here, $f_{2D}$ denotes the usual 2D spatial terms. The resulting equations couple all three displacement components, as desired. The only exception is the case of $s(x, 0, z) = 0$ for all $x$ and $z$, corresponding to the source located in the $x$-$z$ plane, where the present equations transform to those of $P$-$SV$ and $SH$ waves.

A numerical FD solution in the time domain will require simultaneously updating the displacements and their first time derivatives. As the displacement update means computing $\ddot{u}(t)$, the first derivatives are updated at practically no extra cost: $\ddot{u}(t + \Delta t) \approx \ddot{u}(t) + \ddot{u}(t)\Delta t$. Therefore, the main increase in the computer requirements, as compared to the 2D case, is the memory for the time derivatives.

A crucial point of the present method is the requirement of the a priori assessment of $s(x, 0, z)$. With complete wave fields, this problem is difficult. However, we believe that strong ground motion applications could be performed even with the simplest estimation of $s(x, 0, z)$ by means of a local plane-wave approximation to the direct $S$ wave, hence estimating $s$ by means of the local incidence angle, azimuth, and the shear-wave velocity. In a more sophisticated approach, the slowness variation with time may be prescribed, too: $s(x, 0, z; t)$.

Finite-extent seismic sources, located off the $x$-$z$ plane, can be modeled as a sum of point subevents (Fig. 3). Then, however, each subevent should be processed in its own FD run [with its proper $s(x, 0, z)$], and the summation over the subevents can only be performed at the end. The only special case permitting a finite-extent source to be summed up before the FD run is the source completely embedded in the $x$-$z$ plane (i.e., $s = 0$), as already used by Fäh et al. (1994).

**Conclusion**

This comment has discussed possible extensions of the Fäh et al. (1994) article in two respects: new excitation lines, improving the input of 3D source and path effects, and new equations of motion. The equations approximately represent 3D wave fields in a single cross section of a 2D medium (a 2.5D case). The suggested method could be of practical value in problems where a purely 2D treatment is oversimplified, but a true 3D treatment is expensive.

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**References**


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