On possible convective regimes of subsurface ocean beneath Europa's ice shell

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Model I

A classical Boussinesq approximation controlled by three non-dimensional parameters: Rayleigh (Ra), Prandtl (Pr) and Ekman (Ek) numbers [Christensen and Wicht, 2007]:

$$\nabla\cdot\vec{v} = 0 \ , \tag{1}$$

$$\frac{1}{Pr}\left(\frac{\partial\vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v}\right) = -\nabla p + \nabla^2\vec{v} - Ra\,T\,\vec{e}_r - \frac{2}{Ek}\vec{e}_z\times\vec{v} \ , \tag{2}$$

$$\frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T = \nabla^2 T \ , \tag{3}$$

completed by stress-free, impenetrable and isothermal boundaries [Soderlund, 2019, Kvorka and Čadek, 2022]

$$\vec{e}_r\cdot\boldsymbol{\sigma}_{bnd} = (\vec{e}_r\cdot\boldsymbol{\sigma}_{bnd}\cdot\vec{e}_r)\vec{e}_r \ , \ \ T_{bnd} = \mathrm{const} \ . \tag{4}$$

Model II

Rayleigh number, Ra

- $Ra = \alpha_w g_o \Delta T D^3 / \nu_w \kappa_w$
- cannot be measured directly due to dependence on ΔT
- estimates rely on extrapolation from DNS: 10^{20}–10^{22}

Ekman number, Ek

- $Ek = \nu_w / \Omega D^2$
- direct estimates do not rely on DNS: 10^{-12}–10^{-11}

Prandtl number, Pr

- $Pr = \nu_w / \kappa_w$
- direct estimates do not rely on DNS: 10–11

α_w - thermal expansivity, g_o - grav. acceleration at the outer boundary, ΔT - superadiabatic temperature difference, D - thickness of the ocean, ν_w - kinematic viscosity, κ - thermal diffusivity, Ω - rotational period

Model III

Flux-based Rayleigh number, Ra_q

- $Ra_q = \frac{1}{4\pi r_i r_o} \frac{\alpha_w g_o Q}{\rho_w C_{p,w} \Omega^3 D^2}$
- alternative to Ra, however, ΔT is replaced by Q - the convective heat flow through the ocean
- direct estimates do not rely on DNS: $(1.7\text{–}6.8)\times 10^{-8}$

Important concept

Seeking for a scaling law of a form $Ra \sim Pr^a Ek^b Ra_q^c$ in the DNS would allow us to narrow the estimate of Ra, Ro_c and Ro_{loc}.

Results I

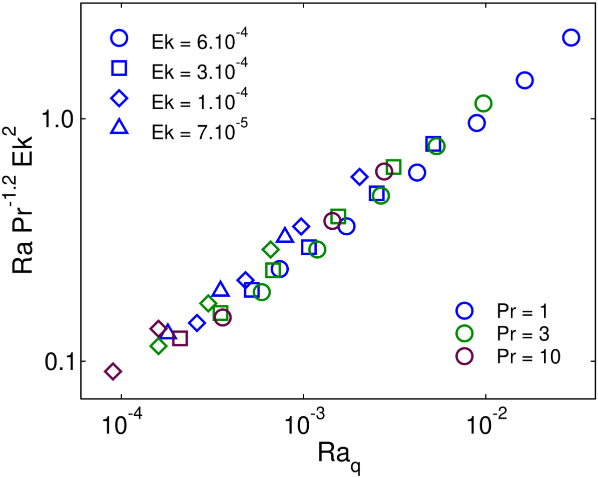

Figure: Rayleigh number scaling as obtained from numerical models. Best least squares fit: $Ra = 13.48\ Pr^{1.2}\ Ek^{-2}\ Ra_q^{0.55}$. Extrapolation to Europa: $1.8{\times}10^{20} \le Ra \le 1.1{\times}10^{21}$, $0.034 \le Ro_c \le 0.05$, $82 \le Ro_{loc} \le 211$.

Results II

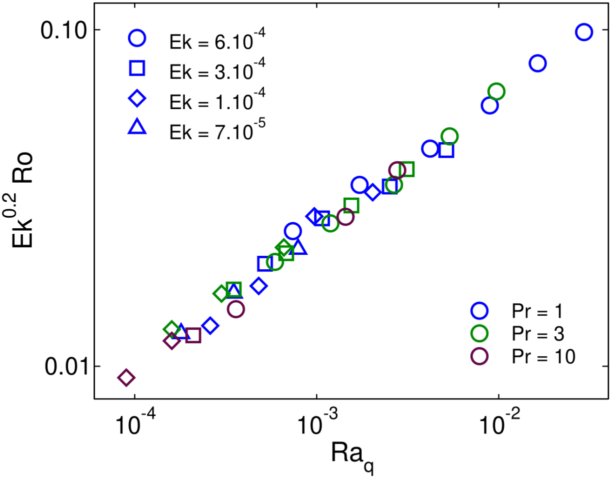

Figure: Rossby number scaling as obtained from numerical models. Best least squares fit: $Ro = 0.40\ Ek^{-0.2}\ Ra_q^{0.40}$. Extrapolation to Europa's set of parameters yields the rms velocity approximately 0.08 – 0.29 m/s.

Ra_q estimates

Two quantities need to be estimated:

- the thickness of the ocean, $D = 95–149\,\mathrm{km}$ [Petricca et al., 2023]
- the heat flow generated by the core, $Q = 0.26–3.2\,\mathrm{TW}$ [Běhounková et al., 2021, Vance et al., 2018]

Sampling

The space of parameters is sampled by 24 possibilities differing in ocean's thickness $D = 95; 103; 115; 120; 124; 128; 149\,\mathrm{km}$ and the heat flux generated in the interior of Europa $q = 10; 47; 125\,\mathrm{mW/m^2}$.

Distribution of the zonal flows: Europa

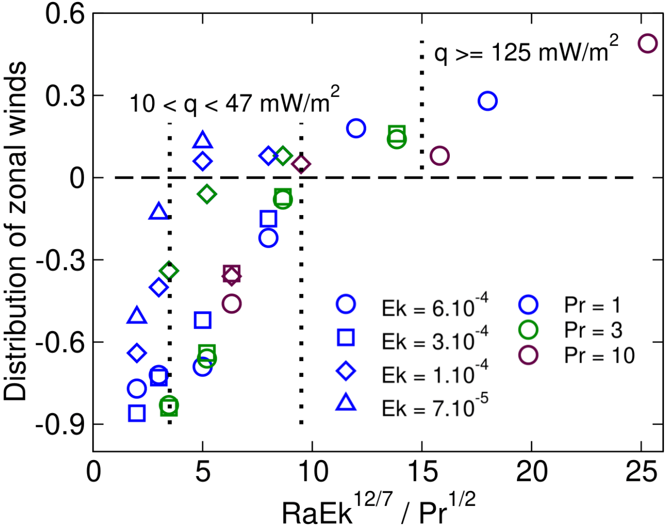

Figure: Distribution of zonal winds in the case of Europa.

Conclusions: Europa

- most probably dominant equatorial jet (opposite to Vance et al., 2021; possible according to Soderlund, 2019)
- mean rms velocity estimate 0.08–0.18(0.29) m/s (in a good agreement with Jansen et al., 2023)
- velocities used for magnetic field generation in Vance et al. [2021] are most probably overestimated

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