On possible convective regimes of subsurface ocean beneath Europa's ice shell and forlook to Ganymede

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21.03.2024

Model I

A classical Boussinesq approximation controlled by three non-dimensional parameters: Rayleigh (Ra), Prandtl (Pr) and Ekman (Ek) numbers [Christensen and Wicht, 2007]:

$$\nabla \cdot \vec{v} = 0 \ , \tag{1}$$

$$\frac{1}{Pr}\left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}\right) = -\nabla p + \nabla^2 \vec{v} - Ra\, T \vec{e}_r - \frac{2}{Ek} \vec{e}_z \times \vec{v} \ , \tag{2}$$

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \nabla^2 T \ , \tag{3}$$

completed by stress-free, impenetrable and isothermal boundaries [Soderlund, 2019, Kvorka and Čadek, 2022]

$$\vec{e}_r \cdot \boldsymbol{\sigma}_{bnd} = (\vec{e}_r \cdot \boldsymbol{\sigma}_{bnd} \cdot \vec{e}_r) \vec{e}_r \ , \ \ T_{bnd} = \mathrm{const} \ . \tag{4}$$

Model II

Rayleigh number, Ra

- $Ra = \alpha_w g_o \Delta T D^3 / \nu_w \kappa_w$
- cannot be measured directly due to dependence on ΔT
- estimates rely on extrapolation from DNS: 10^{20}–10^{22}

Ekman number, Ek

- $Ek = \nu_w / \Omega D^2$
- direct estimates do not rely on DNS: 10^{-12}–10^{-11}

Prandtl number, Pr

- $Pr = \nu_w / \kappa_w$
- direct estimates do not rely on DNS: 10–11

α_w - thermal expansivity, g_o - grav. acceleration at the outer boundary, ΔT - superadiabatic temperature difference, D - thickness of the ocean, ν_w - kinematic viscosity, κ - thermal diffusivity, Ω - rotational period

Model III

Flux-based Rayleigh number, Ra_q

- $Ra_q = \frac{1}{4\pi r_i r_o} \frac{\alpha_w g_o Q}{\rho_w C_{p,w} \Omega^3 D^2}$
- alternative to Ra, however, ΔT is replaced by Q - the convective heat flow through the ocean
- direct estimates do not rely on DNS: $(1.7\text{–}6.8)\times 10^{-8}$

Important concept

Seeking for a scaling law of a form $Ra \sim Pr^a Ek^b Ra_q^c$ in the DNS would allow us to narrow the estimate of Ra, Ro_c and Ro_{loc}.

Results I

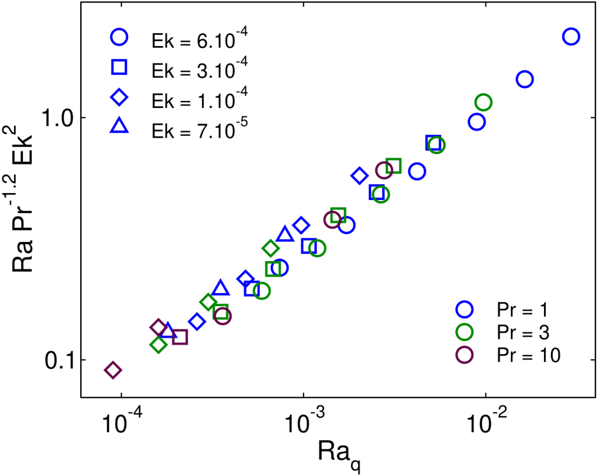

Figure: Rayleigh number scaling as obtained from numerical models. Best least squares fit: $Ra = 13.48\ Pr^{1.2}\ Ek^{-2}\ Ra_q^{0.55}$. Extrapolation to Europa: $1.8\times10^{20} \leq Ra \leq 1.1\times10^{21}$, $0.034 \leq Ro_c \leq 0.05$, $82 \leq Ro_{loc} \leq 211$.

Results II

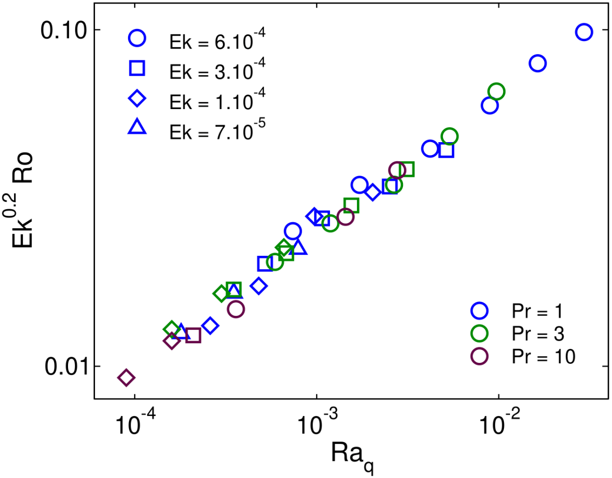

Figure: Rossby number scaling as obtained from numerical models. Best least squares fit: $Ro = 0.40\ Ek^{-0.2}\ Ra_q^{0.40}$. Extrapolation to Europa's set of parameters yields the rms velocity approximately 0.08 – 0.29 m/s.

Ra_q estimates

Two quantities need to be estimated:

- the thickness of the ocean, $D = 95\text{–}149\,\text{km}$ [Petricca et al., 2023]
- the heat flow generated by the core, $Q = 0.26\text{–}3.2\,\text{TW}$ [Běhounková et al., 2021, Vance et al., 2018]

Sampling

The space of parameters is sampled by 24 possibilities differing in ocean's thickness $D = 95; 103; 115; 120; 124; 128; 149\,\text{km}$ and the heat flux generated in the interior of Europa $q = 10; 47; 125\,\text{mW/m}^2$.

Distribution of the zonal flows: Europa

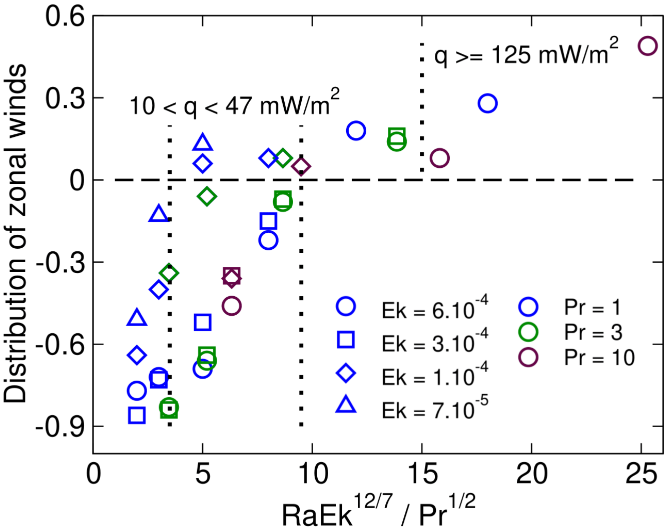

Figure: Distribution of zonal winds in the case of Europa.

Conclusions: Europa

- most probably dominant equatorial jet (opposite to Vance et al., 2021; possible according to Soderlund, 2019)
- mean rms velocity estimate 0.08–0.18(0.29) m/s (in a good agreement with Jansen et al., 2023)
- velocities used for magnetic field generation in Vance et al. [2021] are most probably overestimated

Influence of ocean's geometry I

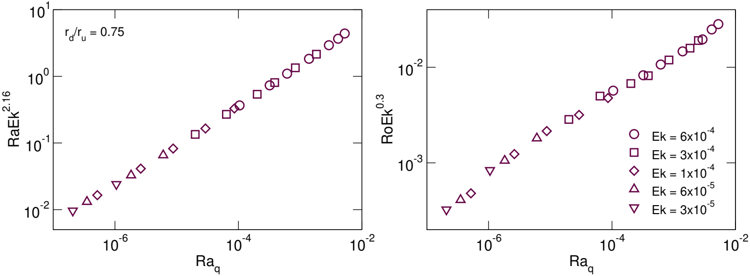

Figure: $Ra = 94.34 Ek^{-2.16} Ra_q^{0.60}, \; Ro = 0.24 Ek^{-0.3} Ra_q^{0.42}$.

Influence of ocean's geometry II

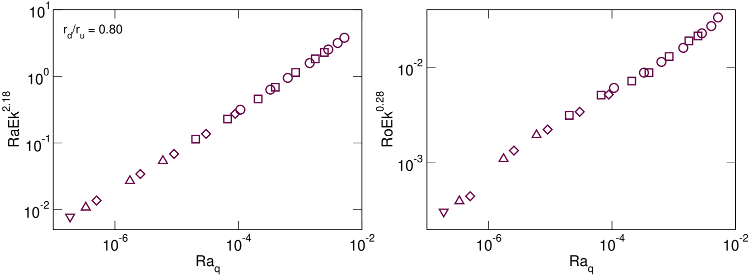

Figure: $Ra = 82.40 Ek^{-2.18} Ra_q^{0.60}$, $Ro = 0.29 Ek^{-0.3} Ra_q^{0.43}$.

Influence of ocean's geometry III

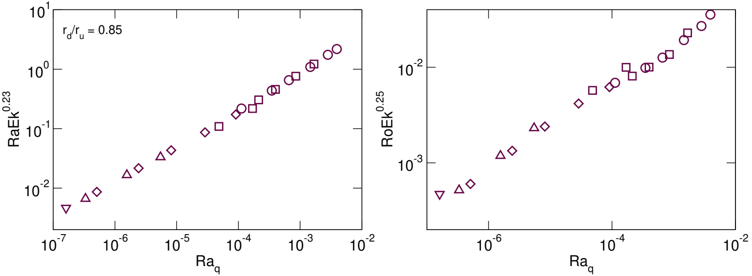

Figure: $Ra = 54.64 Ek^{-2.23} Ra_q^{0.61}$, $Ro = 0.31 Ek^{-0.25} Ra_q^{0.42}$.

Ganymede

Ganymede ocean's radius ratio: 0.80 – 0.99 [Vance et al., 2018]

- except the cases of a thin ocean, the dataset covers possibilities for Ganymede
- in the case of a thick ocean (493 km) and high heat flux (107 mW/m^2), *Ro* is roughly 4 m/s
- in the case of a thinner ocean (200 km) and smaller heat flux (15 mW/m^2), *Ro* is roughly 80 cm/s
- in conclusion, there is a possibility of measurable magnetic field in the case of Ganymede
- Vance et al., 2021 might have overestimated the case of Ganymede

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