π

Drobné

Machin

$$\frac{\pi}{4} = \arctan\frac{1}{2} + \arctan\frac{1}{3} = 2\arctan\frac{1}{2} - \arctan\frac{1}{7} = 2\arctan\frac{1}{3} + \arctan\frac{1}{7} = 4\arctan\frac{1}{5} - \arctan\frac{1}{239}$$

Nekonečné součty

Gregory, Leibniz

$$\frac{\pi}{4} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

Newton

$$\frac{\pi}{6} = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{4n+1}(n!)^2(2n+1)} = \frac{1}{2} + \frac{1}{3\cdot 2^4} + \frac{3}{5\cdot 2^8} + \frac{5}{7\cdot 2^{11}} + \cdots$$

$$\frac{\pi}{24} = \frac{\sqrt{3}}{32} - \sum_{n=0}^{\infty} \frac{(2n)!}{2^{4n+2}(n!)^2(2n-1)(2n+3)} = \frac{\sqrt{3}}{32} + \frac{1}{12} - \frac{1}{5\cdot 2^5} - \frac{1}{28\cdot 2^7} - \frac{1}{72\cdot 2^9} - \cdots$$

Euler

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

$$\frac{\pi^4}{90} = \sum_{n=1}^{\infty} \frac{1}{n^4} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots$$

$$\frac{\pi^6}{945} = \sum_{n=1}^{\infty} \frac{1}{n^6} = 1 + \frac{1}{2^6} + \frac{1}{3^6} + \frac{1}{4^6} + \cdots$$

a další

$$\pi = 3 + 4\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n(2n+1)(2n+2)}, \quad \pi = 4 - 2\sum_{n=0}^{\infty} \frac{n!}{\prod_{j=0}^{n}(2j+3)} = 4 - 2\left(\frac{1}{3} + \frac{1}{3\cdot 5} + \frac{2}{3\cdot 5\cdot 7} + \cdots\right)$$

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

$$\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

Bailey, Borwein, Plouffe („BBP“)

$$\pi = \sum_{n=0}^{\infty} \frac{(-1)^n}{4^n}\left(\frac{2}{4n+1} + \frac{2}{4n+2} + \frac{1}{4n+3}\right)$$

$$\pi = \sum_{n=0}^{\infty} \frac{1}{16^n}\left(\frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6}\right)$$

Ramanujan

$$\frac{1}{\pi} = \sum_{n=0}^{\infty} \binom{2n}{n}^3 \frac{42n+5}{2^{12n+4}} = \frac{5}{2^4} + \frac{2^3\cdot 47}{2^{16}} + \frac{6^3\cdot 89}{2^{28}} + \frac{20^3\cdot 131}{2^{40}} + \cdots$$

$$\frac{1}{\pi} = \frac{\sqrt{8}}{9801}\sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4}\frac{1103+26390n}{396^{4n}} = \frac{\sqrt{8}}{9801}\left(1103 + \frac{54986}{2049271488} + \cdots\right)$$

Nekonečné součiny

Wallis

$$\frac{\pi}{2} = \prod_{n=1}^{\infty} \frac{(2n)^2}{(2n-1)(2n+1)} = \frac{2\cdot 2}{1\cdot 3}\cdot\frac{4\cdot 4}{3\cdot 5}\cdot\frac{6\cdot 6}{5\cdot 7}\cdot\frac{8\cdot 8}{7\cdot 9}\cdots$$

Viète

$$\frac{2}{\pi} = \sqrt{\frac{1}{2}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}}\cdots$$

Řetězové zlomky

Brouncker a dva další

$$\frac{4}{\pi} = 1+\cfrac{1}{2+\cfrac{9}{2+\cfrac{25}{2+\cfrac{49}{2+\cdots}}}}\ , \quad \pi = 3+\cfrac{1}{6+\cfrac{9}{6+\cfrac{25}{6+\cfrac{49}{6+\cdots}}}}\ , \quad \frac{4}{\pi} = 1+\cfrac{1}{3+\cfrac{4}{5+\cfrac{9}{7+\cfrac{16}{9+\cdots}}}}$$

Integrály

$$\pi = \int_0^1 \frac{16x-16}{x^4-2x^3+4x-4}\,dx$$

$$\pi = \frac{22}{7} - \int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx\ , \quad \pi = \frac{355}{113} - \frac{1}{3164}\int_0^1 \frac{x^8(1-x)^8(25+816x^2)}{1+x^2}\,dx$$

A také

$$\left(1+\frac{1}{\pi}\right)^{\pi+1} < \pi\ , \quad (3,1,4) \equiv ((1,5,9)+(2,6,5)) \mod 10$$

Verše

Lín a kapr u hráze prohlédli si rybáře, udici měl novou, jikrnáči neuplovou.
How I wish I could calculate pi.
May I have a large container of coffee?
How I want a drink, alcoholic of course, after the heavy lectures involving quantum mechanics.
Poe, E.: Near a Raven. Midnights so dreary, tired and weary.
Silently pondering volumes extolling all by-now obsolete lore.
During my rather long nap-the weirdest tap!
An ominous vibrating sound disturbing my chamber's antedoor.
'This,' I whispered quietly, 'I ignore.'
Perfectly, the intellect remembers: the ghostly fires, a glittering ember.
Inflamed by lightning's outbursts, windows cast penumbras upon this floor.
Sorrowful, as one mistreated, unhappy thoughts I heeded:
That inimitable lesson in elegance--Lenore--
Is delighting, exciting... nevermore.

Odkazy

Bible, 1. Královská 7, 23	http://www.biblenet.cz/app/bible/Kgs1/chapter/7
Vzorce	http://functions.wolfram.com/Constants/Pi
	http://mathworld.wolfram.com/PiFormulas.html
	http://www.pi314.net
	http://en.wikipedia.org/wiki/List_of_formulae_involving_%CF%80
Verše	http://mathworld.wolfram.com/PiWordplay.html
	http://paginas.fe.up.pt/~fsilva/port/pi2.html
Numerické recepty	http://www.nrbook.com/a/bookcpdf/c20-6.pdf
	http://www.nrbook.com/a/bookfpdf/f20-6.pdf
Mazda	http://en.wikipedia.org/wiki/File:Geek_car.jpg